Liquidity Risk and Distressed Equity

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Abstract

I show theoretically and empirically that cash holdings used to offset liquidity risk can help rationalize the anomalous returns of distressed equity. In my model, levered firms with financing constraints can default because of liquidity or solvency, but firms seek to manage their cash to avoid the former. An insolvent but liquid firm has a large fraction of its assets in cash, which makes its equity beta low and helps rationalize low expected returns. Using data on rated US firms between 1970 and 2013, I find empirical evidence consistent with my theoretical predictions: i) the average insolvent firm holds cash that meets or exceeds its current liabilities; ii) firm-specific betas and risk-adjusted returns decline both as firms become less solvent and as cash levels decline; and iii) a portfolio long firm with high cash and short firms with low cash has increasing returns for less solvent firms, while a portfolio long firms with high solvency and short firms with low solvency has increasing returns for firms with less cash. My results suggest that there is no distress anomaly for insolvent but liquid firms.

Keywords: Distress risk, equity valuation, liquidity risk, cash holdings

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Introduction

Recent studies of distressed equity rely on capital structure theory to rationalize why high default risk predicts low future returns, i.e. the "distress anomaly" documented by Dichev (1998), Griffen and Lemmon (2002), Campbell, Hilscher, and Szilagyi (2008), and others. Garlappi and Yan (2011) show that potential shareholder recovery upon resolution of distress increases the value of the default option and lowers expected returns near the default boundary. Opp (2013) argues that faster learning about firm solvency in aggregate downturns has the same effect, while McQuade (2013) argues that distressed equity hedges against persistent volatility risk and therefore commands lower expected returns.

In this paper, I argue that cash holdings used to offset liquidity risk can help rationalize the anomalous returns of distressed equity. In contrast to extant rationales of the anomaly, my model features levered firms with financing constraints that can default because of liquidity or solvency, but firms seek to manage their cash to avoid the former. The model predicts that an insolvent but liquid firm has a large fraction of its assets in cash, which makes its equity beta low and helps rationalize low expected returns. Using data on rated US firms between 1970 and 2013, I find empirical evidence consistent with my theoretical predictions.

The model features a representative levered firm that generates uncertain earnings. The firm has common equity and coupon-bearing debt in its capital structure. Because of capital market frictions, the firm has no access to external financing, and can, in particular, not issue additional equity to cover coupon payments. Default is costly and can occur because of liquidity or solvency. To reduce the risk of the former, the firm retains some earnings as precautionary cash. How much of earnings to retain and how much to distribute to shareholders is determined by the following trade-off: An extra dollar in dividends increases equity value conditional on survival, while an extra dollar in cash holdings increases the probability of survival. Equity holders have the option to strategically declare insolvency and withdraw the firm’s cash holdings when its asset value falls sufficiently low relative to its liabilities.

It is optimal for the firm to aim at a target level of cash that eliminates its liquidity risk (Proposition 1) and only distribute dividends when its cash is at or above the target level (Proposition 2). The target cash level decreases as the firm becomes less solvent because a less solvent firm can only survive smaller earnings-shortfalls and thus demands less cash. In equilibrium, cash is at its target level (i.e. the firm is liquid) and equity is the sum of current cash holdings, expected excess earnings, and the option to declare insolvency (Proposition 3). Equity holders optimally declare the firm insolvent and withdraw its cash holdings when its asset value consists mostly of cash. In line with the model, I find for my sample of rated US firms that average cash levels decline as firms become less solvent, but that the average insolvent firm still holds cash that meets or exceeds its current liabilities.

The paper’s main contribution is to shed new light on the distress anomaly by characterizing the cross-sectional relation between cash levels and equity returns as solvency varies.

The equilibrium expected return on the firm’s equity is determined by its conditional equity beta, i.e. equity’s sensitivity to systematic earnings risk, which depends on the target cash level and the probability of insolvency (Proposition 4). The model predicts that the equity beta of the liquid firm is hump-shaped in its probability of insolvency (Corollary 4.1). The intuition is as follows. When the liquid firm is solvent, its assets consist mostly of expected earnings and are therefore sensitive to systematic risk, which implies a high beta that increases in the probability of insolvency. However, as the liquid firm nears insolvency, its cash becomes an increasingly larger fraction of its assets, and, because cash is insensitive to systematic risk, this implies a low equity beta that decreases in the probability of insolvency. This provides a theoretical rationalization of the distress anomaly for insolvent but liquid firms.

The model produces the following testable predictions. Because the target cash level declines with the probability of insolvency, the model predicts that equity betas are not only hump-shaped in the probability of in-
solvency, but also in the target cash level. Furthermore, because of these hump-shaped relationships, a portfolio long firms with high cash and short firms with low cash (HCmLC) will have increasing expected returns as firms become less solvent, and, conversely, that a portfolio long firms with high solvency and short firms with low solvency (HSmLS) will have increasing expected returns as cash levels decline.

Consistent with the models predictions I find for my sample of rated US firms that i) firm-specific equity betas and risk-adjusted returns decline as firms become less solvent and as cash levels decline, ii) the HCmLC portfolio strategy has increasing risk-adjusted returns as firms become less solvent, and iii) the HSmLS portfolio strategy has increasing risk-adjusted returns as cash levels decrease.

In sum, my results suggest that cash holdings used to offset liquidity risk can help rationalize the anomalous returns of distressed equity, and show that there is no distress anomaly for insolvent but liquid firms.

Related literature

The distress risk anomaly has been investigated empirically by Dichev (1998), Griffen and Lemmon (2002), Vassalou and Xing (2004), Campbell et al. (2008), and the references therein.

The hump-shaped relationship between equity beta and probability of default was first directly documented Garlappi, Shu, and Yan (2008). The models of Garlappi and Yan (2011), Opp (2013), and McQuade (2013) rationalize the hump-shaped beta by means of shareholder recovery, shareholder learning, and persistent volatility risk. I complement these theories by arguing that cash holdings used to offset liquidity risk reduce equity betas for liquid firms nearing insolvency, which yields the novel testable predictions for which I find empirical support.

The equity valuation model presented in this paper builds on the classical time-homogenous framework of e.g. Leland (1994), but augments it with a target cash level similar to Gryglewicz (2011), who studies the optimal capital structure of a firm facing liquidity and solvency concerns. A related framework is considered by Davydenko (2013), who studies whether corporate defaults are driven by insolvency or illiquidity.

Technically, Gryglewicz (2011) specifies the firm’s cumulated earnings as an arithmetic Brownian motion—following the classical pure-liquidity models of e.g. Jeanblanc-Picqué and Shiryaev (1995) and Radner and Shepp (1996)—but allows the drift component of this process to be randomized over a known two-point distribution. The added uncertainty about expected earnings implies a state-dependent assets value, which—contrary to the pure-liquidity models—allows for strategic insolvency.

My model deviates from the framework of Gryglewicz (2011) by specifying cumulated earnings as a geometric Brownian motion instead of an arithmetic Brownian motion with randomized drift. This again implies a state-dependent value for the firm’s assets, which ensures strategic insolvency, but has several implications for the model.

First, it simplifies the model’s information structure, which allows me to easily add a systematic risk component to the firm’s earnings process and therefore to study equity returns and their determinants. Second, I show that in my model, the firm’s asset value process is also a geometric Brownian motion, thus making my specification of cumulated earnings consistent with the classical asset-value based models of e.g. Black and Scholes (1973), Merton (1974), Black and Cox (1976), Leland (1994), Fan and Sundaresan (2000), Goldstein, Ju, and Leland (2001), Duffie and Lando (2001), etc.

This paper is also related to the literature on the determinants and implications of corporate cash holdings. Relevant empirical studies include Opler et al. (1999), Bates et al. (2009), Acharya et al. (2012), and Davydenko (2013). Relevant, theoretical studies, include Décamps, Mariotti, Rochet, and Villenueve (2011), and Bolton, Chen, and Wang (2011, 2013).
1 An equity valuation model with liquidity risk

This section develops an equity valuation model for a levered firm with financing constraints that can default because of liquidity or solvency. The firm retains some of its earnings as cash to avoid the former. In equilibrium, the firm only defaults because of solvency, and in that case its equity beta is low because cash is a large fraction of its asset value.

1.1 A financially constrained firm

I consider a levered firm generating uncertain cash flows in a continuous-time economy with infinite time-horizon, \([0, \infty)\). Corporate earnings are taxed at the rate \(\tau \in (0, 1)\), with a full loss offset, and the instantaneous risk-free interest rate, \(r\), is assumed to be constant.

The firm’s liabilities consist of common equity stock and, due to tax benefits, consol (infinite maturity) bonds with total coupon rate \(k\) per time unit. The number of shares outstanding and the total coupon level are assumed to be predetermined before time zero.

The firm’s assets are fixed throughout its lifetime and continuously generate earnings (revenue net of expenses). For my purpose of modeling cash holdings, it is convenient to specify the stock rather than the flow of earnings. The model’s main state variable is therefore the firm’s cumulated earnings before interest and taxes (EBIT) up to time \(t\), which I denote \(X_t\). I assume that \(X_t\), under a physical probability measure, \(P\), is a geometric Brownian motion with dynamics

\[
\frac{dX_t}{X_t} = \mu^P X_t dt + \sigma X_t dW^P_t. \tag{1}
\]

Here, \(W^P_t\) is a standard \(P\)-Brownian motion that drives the firm’s total (idiosyncratic and systematic) earnings risk. Instantaneous earnings (per \(dt\)) are thus given by the increment \(dX_t\), which can be either positive (a profit) or negative (a loss), depending on the realization of the total earnings shock, \(dW^P_t\). The drift parameter, \(\mu^P\), is the \(P\)-expected growth rate of earnings, while the diffusion parameter, \(\sigma\), is the volatility rate of earnings. After taxes and coupons, the firm’s instantaneous net earnings are \((1 - \tau)(dX_t - kdt)\), which are at the discretion of equity holders until an eventual liquidation of the firm. In liquidation, bond holders recover a fraction of the market value of the firm’s earnings-generating assets (which is derived below), while the remainder is lost to bankruptcy costs.

I assume that due to capital market frictions, the firm has no access to external financing. Coupon payments after time zero must therefore be internally financed through earnings. Consequently, the firm can be in distress, and ultimately be liquidated, for two reasons: Either due to illiquidity, because its earnings are insufficient to service a coupon payment, or due to insolvency, because the value of its earnings-generating assets does not outweigh future coupon payments.

In an economy without financing constraints, illiquidity never occurs because the firm finances any earnings shortfalls by issuing additional equity—specifically in the form of negative dividends as in e.g., Black and Cox (1976) and Leland (1994). Since this is possible as long as equity value is positive, the firm is only in distress due to insolvency. By contrast, when financing constraints prohibit additional external financing, the firm has a precautionary motive to reduce dividends and retain some earnings as cash holdings, i.e. non-productive liquid assets, which serve to offset liquidity risk.

The managers of the financially constrained firm, which are assumed to act in the best interest of eq-

\[3\] In contrast to Gryglewicz (2011), I specify cumulated earnings in (1) as a geometric Brownian motion rather than an arithmetic Brownian motion with a randomized drift. This still implies a state-dependent value for the firm’s earnings-generating assets, which ensures the existence of a strategic insolvency-trigger (see footnote 7), but simplifies the model’s information structure and allows me to focus on systematic risk in my analysis of expected equity returns. Moreover, I show below that the implied value of earnings-generating assets is again a geometric Brownian motion, which makes the model consistent with classical capital structure models (see footnote 8).

\[4\] This could, for instance, be due to the debt-overhang problem of Myers (1977) or the information asymmetry problems of Leland and Pyle (1977) and Myers and Majluf (1984). The assumption of no external financing can be replaced by the milder assumption of sufficiently high issuance costs, but this does not alter the qualitative nature of the results as long as i) liquidation of the firm is costly and ii) only a fraction of future earnings can be pledged as collateral for external financing. See Acharya et al. (2012) for a further discussion.

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osity holders, determine the firm’s optimal cash-dividend policy by the following trade-off: An additional dollar in dividends increases equity value conditional on survival, while an additional dollar in cash holdings increases the probability of survival. Under an optimal policy, the firm’s liquidity risk will be eliminated, but it may be strategically liquidated if equity holders deem it insolvent. I assume that the firm’s managers have full discretion over costless distribution of cash holdings to equity holders at any time before liquidation, and that there are no bond covenants limiting such payouts. This makes equity holders’ strategic insolvency-decision independent of the firm’s cash holdings. In reality, it may be difficult for bond holders to implement—let alone enforce—covenants restricting payouts.

### 1.2 Optimal cash-dividend policy

I now derive the financially constrained firm’s optimal policies for holding cash and paying out dividends.

Conditional on survival, the firm receives earnings and pays bond holders the tax-deductible coupon. Net earnings can then be paid out as dividends to equity holders or, to reduce liquidity risk, be retained as cash holdings. I assume cash holdings earn the risk-free rate, $r$, for instance through investment in short-term marketable securities with a tax credit, and equity holders choose payouts whenever they weakly prefer so.\(^5\)

Let $C_t$ be the firm’s cash holdings at time $t$ and let $D_t$ be its cumulated dividend payouts up to time $t$. Then

$$dD_t = (1 - \tau)(dX_t - k dt) - dC_t + rC_t dt. \quad (2)$$

Because the firm has no access to external financing, $C_t$ and $D_t$ are nonnegative for all $t > 0$.\(^8\) Illiquidity occurs when cash holdings hit zero, i.e. at time $\tau_C = \inf_{t>0}[C_t \leq 0]$. Since expected earnings depend on $X_t$, equity holders will strategically declare the firm insolvent when $X_t$ falls below an optimally chosen trigger, $X^*$, i.e. at time $\tau_X = \inf_{t>0}[X_t \leq X^*].$\(^3\)

#### Managers’ optimization problem

The firm’s managers choose the cash-dividend policy that maximizes the market value of equity. To calculate market values, I assume that the economy’s stochastic discount factor, $\Lambda_t$, is given by the dynamics

$$d\Lambda_t = -r\Lambda_t dt - \eta\Lambda_t dZ^p_t. \quad (3)$$

Here, $Z^p_t$ is a standard $\mathbb{P}$-Brownian motion that drives systematic earnings risk and has correlation $\rho$ with the firm’s total earnings risk, $W^p_t$, as given in (1). The constant $\eta$ is then the market price of systematic earnings risk. In the appendix, I detail how (3) defines a risk-neutral pricing measure, $\mathbb{Q}$, under which $X_t$ is a geometric Brownian motion with dynamics

$$dX_t = \mu^Q X_t dt + \sigma X_t dW^Q_t, \quad (4)$$

where $\mu^Q = \mu^P - \eta\rho\sigma$ is the risk-neutral growth rate of earnings and $W^Q_t = W^p_t + \eta\rho t$ is a standard $\mathbb{Q}$-Brownian motion. Assuming $r > \mu^Q > 0$, (4) implies that the time-$t$ market value of the firm’s earnings-generating assets is

$$A(X_t) = \mathbb{E}_t^Q \left[ \int_t^\infty e^{-r(u-t)}(1 - \tau)dX_u \right] = (1 - \tau) \frac{\mu^Q X_t}{r - \mu^Q},$$

which is again a geometric Brownian motion.\(^8\) In the following, I focus on the case of $A(X_t) > (1 - \tau)X_t$, which is equivalent to the parameter restriction $\mu^Q > \frac{1}{2}r$. Otherwise, it would be optimal for equity holders to liquidate the firm at zero time—see the discussion of “asset shifting” due to too low earnings in Acharya et al. (2012).

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\(^3\) The existence of the strategic insolvency trigger, $X^*$, follows from the assumption that $X_t$ is a geometric Brownian motion. If $X_t$ were an arithmetic Brownian motion with constant drift $\mu$, earnings-generating asset value would be constant and insolvency would be exogenously determined at time zero by the sign of $\mu - k$.

\(^8\) By Itô’s Lemma and (4), $A(X_t)$ has $\mathbb{Q}$-dynamics

$$dA(X_t) = \mu^Q A(X_t)dt + \sigma A(X_t)dW^Q_t.$$

This makes the specification of cumulated earnings in (4) (and, equivalently, in (1)) consistent with standard capital structure models assuming that the firm’s asset value is a geometric Brownian motion—e.g. Black and Scholes (1973), Merton (1974), Black and Cox (1976), Leland (1994), Fan and Sundaresan (2000), Goldstein et al. (2001), Duffie and Lando (2001), etc.

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5 The assumption that cash earns $\tau$ simplistically implies a zero carry-cost of holding cash—such costs are, however, easily introduced in the present model without qualitatively altering the results.

6 Usually, negative negative cash holdings are interpreted as the firm drawing on a credit line, while negative dividend payouts are interpreted as capital injections by equity holders. Since I assume no external financing beyond time zero, the firm has access to neither, and therefore its cash holdings and dividend payouts have to remain nonnegative at any time prior to liquidation.
The market value of the firm’s equity is the $Q$-expected, discounted value of future dividend payouts until either illiquidity or strategic insolvency, plus a liquidation payout of any remaining cash holdings—the latter is a limiting consequence of the assumption of unrestricted payout before liquidation. Given the strategic insolvency trigger, $X^*$, and for current (time $t$) values $X$ and $C$ of cumulated earnings and cash holdings, equity value is thus given by

$$E(X, C) = \sup_D \mathbb{E}_\mathbb{Q}^{\tau_C,X,C} \left[ \int_t^{\tau} e^{-r(u-t)} dD_u + e^{-r(\bar{\tau}-t)} C_{\bar{\tau}} \right], \tag{5}$$

where $\bar{\tau} = \tau_C \wedge \tau_X$ is the firm’s liquidation time. The supremum is taken over all dividend payout policies, $D$, which are nonnegative, non-decreasing, satisfy (2), and are adapted to the filtration generated by the cumulated earnings-process in (4).

**Target cash holdings**

To solve the manager’s optimization problem, note that higher cash holdings lower lower liquidity risk but also dividend payouts. I therefore conjecture that an optimal cash-dividend policy involves a target level of cash holdings, denoted $C(X)$, which is the smallest amount of cash large enough to eliminate liquidity risk when cumulated earnings are at the level $X$.

To characterize $C(X)$, note that the cumulated dividend process in (2) is positive and non-decreasing at all $t > 0$ if and only if i) the drift of $D_t$ is nonnegative and ii) the volatility of $D_t$ is zero. By these requirements, $C(X)$ satisfies a differential equation with a lower bound to all its solutions. The following proposition gives the smallest solution. (All proofs are in the appendix.)

**Proposition 1** (Target cash holdings). **Suppose the firm has access to external financing after time zero—i.e. it cannot issue additional bonds and its cumulated dividend process in (2) is nonnegative and non-decreasing. Given the coupon level, $k$, and the strategic insolvency trigger, $X^*$, the smallest level of cash holdings large enough to eliminate liquidity risk when cumulated earnings are at the level $X$ is given by

$$C(X) = (1 - \tau) \left[ X - X^* + \frac{k}{r} \right], \tag{6}$$

The target cash level is the after-tax earnings above strategic insolvency trigger, $(1 - \tau)(X - X^*)$, plus the present value of future after-tax coupons, $(1 - \tau)k/r$.\(^9\)

Note that since $X \geq X^*$ and cash holdings earn $r$, the interest on the target level is always sufficient to cover a coupon payment. The target level is thus the smallest amount of cash large enough to ensure both nonnegative dividends as well as continued coupon payments even if an earnings shock brings $X$ down to $X^*$. Consequently, by holding at least $C(X)$ in cash as $X$ varies, the financially constrained firm’s liquidity risk eliminated and it is only liquidated if equity holders deem it insolvent.\(^10\)

Note that $C$ increases in $X$ but decreases in $X^*$ because as the firm becomes less solvent (lower $X$ or higher $X^*$), it can only withstand a smaller earnings shock before being declared insolvent, so the optimal policy is to reduce cash holdings accordingly. By the same logic, $C$ decreases as excess earnings, $X - X^*$, decrease. Consequently, a firm with cash at the target level and cumulated earnings close to the point of strategic insolvency holds as little cash as possible.

Finally, the target cash level increases with the coupon rate, $k$, because higher coupons imply a higher liquidity risk, and decreases with the risk-free rate, $r$, because higher interest implies higher compounding of current cash holdings and future retained earnings.

**Optimal dividend payouts**

Given the target cash level of Proposition 1, I conjecture that the optimal dividend policy adjusts cash holdings, $C_t$, towards the target level, $C(X_t)$, as $X_t$ fluctuates.

\(^9\)An inspection of the proof of Proposition 1 reveals that the form in (6) is independent of the assumption that the cumulated earnings process, $X_t$, is a geometric Brownian motion (see (1) or (4)). In fact, the form of (6) is for a general cumulated earnings process, as long as such a process is consistent with the existence of a strategic solvency trigger, $X^*$. As argued in footnote 7, this is in particular the case when $X_t$ is a geometric Brownian motion, but not if it were an arithmetic Brownian motion with constant drift.

\(^10\)An intuitive way to derive (6) is to note that for $C(X_t)$ to eliminating liquidity risk for any $X_t \geq X^*$, it is reasonable to assume that $rC(X_t) dt \geq (1 - \tau)k dt$. Given this, (2) implies that $dD_t$ is nonnegative for all $0 < t \leq \tau_X$ if and only if $dC(X_t) \leq (1 - \tau)k dt$, implying

$$C(X_t) \geq C(X_{t_0}) + (1 - \tau) \left[ X_t - X_{t_0} \right] \geq (1 - \tau) \frac{k}{r} + (1 - \tau) \left[ X_t - X^* \right].$$

Choosing the smallest level satisfying this restriction gives (6).
To see this, note that if $C_t < C(X_t)$, it is suboptimal for the firm to pay out dividends, as this would make it vulnerable to liquidity risk, and all earnings should be retained until cash holdings again reach the target level. Conversely, if $C_t > C(X_t)$, dividend payouts are too low, in that cash holdings are above the target level, and it is optimal to distribute the residual $C_t - C(X_t)$ to equity holders. The conjectured dividend payout policy is therefore given by

$$dD^*_t = \begin{cases} 0 & \text{if } C_t < C(X_t) \\ (rC(X_t) - (1 - \tau)k)dt & \text{if } C_t = C(X_t) \\ C_t - C(X_t) & \text{if } C_t > C(X_t). \end{cases}$$

(7)

The intermediate case follows by applying Itô’s Lemma to the target cash level (6) and using the dynamics of the dividend payout process in (2), and may also be written

$$dD^*_t = r(1 - \tau)(X_t - X^*) dt.$$  

(8)

From (7), when $C_t = C(X_t)$, instantaneous dividends are the interest earned on cash holdings net of an after-tax coupon. From (8), this is equivalent to the interest earned on after-tax excess earnings. It follows from either form that dividends equal zero at the strategic insolvency-trigger, $X^*$.

The following proposition asserts that the dividend policy conjectured in (7) (which depends on the target cash level of Proposition 1) does maximize equity value.

**Proposition 2 (Optimal dividend payouts).** Suppose the firm has no access to external financing after time zero and that its equity value function in (5) is twice continuously differentiable. Then the dividend payout policy in (7) is optimal, in that it attains the supremum in (5).

Intuitively, the dividend policy in (7) maximizes equity value because it optimally exploits that an extra dollar in cash holdings is, at least equal, at least worth its face value to equity holders: $\frac{\partial E(X, C)}{\partial C} \geq 1$.

(11)

From (7), earnings are retained whenever an additional dollar in cash decreases liquidity risk (i.e. when $\frac{\partial E(X, C)}{\partial C} > 1$), while earnings are paid out whenever an additional dollar in cash does not further reduce liquidity risk (i.e. when $\frac{\partial E(X, C)}{\partial C} = 1$).

1.3 Equilibrium equity value

The cash-dividend policy of Propositions 1 and 2 implies that when cash holdings are at or above the target level, liquidity risk is eliminated, and the firm is only liquidated when equity holders strategically deem it insolvent. In the following, I study the correlation between the equity value and target cash holdings for varying levels of solvency.

To this end, I focus on the equilibrium case where the firm’s cash holdings are at the target level, i.e. when $C_t = C(X_t)$. Indeed, if $C_t \neq C(X_t)$, the optimal dividend policy in (7) dictates that all variations in cash holdings are due to the adjustment towards the target level.

In this case, equity is a claim on the firm’s assets (both earnings-generating and liquid) that pays the continuous flow of dividends in (8) until strategic insolvency. Let $E(X) = E(X, C(X))$ be the value of equity under this dividend policy. Using the $Q$-dynamics of $X_t$ in (4), an application of Itô’s Lemma to the discounted gains process of equity gives that $E(X)$ solves the ordinary differential equation

$$rE(X) = \frac{1}{2}\sigma^2 X^2 E_{XX}(X) + \mu^2 X E_X(X) + (1 - \tau)r(X - X^*).$$

(9)

As earnings approach infinity, the value of the firm’s total assets—i.e. cash holdings plus earnings-generating assets plus the tax-benefit of debt—converges towards the sum of risk-free equity and debt. This implies the asymptotic value matching condition

$$E(X) \rightarrow C(X) + (1 - \tau)\frac{\mu X}{r - \mu r} + \frac{k}{\gamma - \frac{\mu}{r}}$$

(10)

for $X \rightarrow \infty$. Here, the first three terms in the limit are the firm’s total assets and the fourth is risk-free debt.

On the other hand, as earnings approach the point of strategic insolvency, the value of earnings-generating assets disappears and equity is only a claim on the firm’s cash holdings. This, combined with the assumption of unrestricted payout before strategic insolvency, implies
the asymptotic limited liability condition
\[ E(X) \searrow C(X^\ast) \text{ for } X \searrow X^\ast. \]  
(11)

The boundary conditions (10)–(11) imply the solution to the differential equation (9) given in the following proposition.

**Proposition 3** (Equilibrium equity value). Suppose \( r > \mu^Q > 0 \) and that the firm’s cash holdings are at the target level. Given the coupon, \( k \), and the insolvency-trigger, \( X^\ast \), the market value of equity when cumulated earnings are at the level \( X \) is given by
\[ E(X) = C(X) + (1 - \tau) \left[ \frac{\mu^Q x}{r - \mu^Q} - \frac{k}{r} \right] \]
\[ + (1 - \tau) \left[ \frac{k}{r} - \frac{\phi^Q x^\ast}{r - \mu^Q} \right] \pi^Q(X), \]  
(12)
where \( \pi^Q(X) = (X/X^\ast)^\phi^Q \) and where \( \phi^- \) is given by
\[ \phi^- = \frac{\sigma^2 - 2\mu^Q - \sqrt{(\sigma^2 - 2\mu^Q)^2 + 8r\sigma^2}}{2\sigma^2} < 0. \]

The equilibrium value of equity is the sum of three terms. The first is the face value of the target cash level. The second is the present value of expected earnings (i.e. the market value of earnings-generating assets) net of future coupon payments. The third term is the value of the limited liability put option to default on the firm’s debt when cumulated earnings fall to the strategic insolvency-trigger. Finally, the factor \( \pi^Q(X) \) goes to 0 as \( X \) approaches infinity and goes to 1 as \( X \) approaches \( X^\ast \), and may thus, similar to Garlappi and Yan (2011), be interpreted as the firm’s instantaneous \( Q \)-probability of insolvency.

**Strategic insolvency decision**

The strategic insolvency-trigger, \( X^\ast \), is the value of cumulated earnings solving the smooth-pasting condition,
\[ \frac{\partial E(X)}{\partial X} \bigg|_{X=X^\ast} = \frac{\partial C(X)}{\partial X} \bigg|_{X=X^\ast}, \]
which follows from the limited liability condition in (11). This states that equity holders strategically declare the firm insolvent at the point where an extra dollar in earnings increases equity value by no more than the increase in cash holdings. The solution is
\[ X^\ast = \frac{k r - \mu^Q}{r} \phi^- \rightarrow 1. \]
(13)

Note that since \( \frac{\phi^-}{\phi^-} \in (0, 1) \), it follows by the expression for the target cash holdings in (6) that
\[ C(X^\ast) = (1 - \tau) \frac{k}{r} > (1 - \tau) \frac{\mu^Q X^\ast}{r - \mu^Q} = A(X^\ast). \]  
(14)

Hence, equity holders strategically declare the firm insolvent when cash holdings make up a greater fraction of total asset value than earnings-generating assets. This, as will be emphasized in the analysis of expected equity returns, reduces the exposure of the firm’s equity to systematic risk when the firm approaches insolvency.

### 1.4 Expected returns and target cash

In this section, I study the effects of target cash on the firm’s expected equity returns. Specifically, I derive expressions for the firm’s equity beta and study how it is affected by target cash holdings for varying levels of solvency.

**Systematic risk and equity returns**

Applying Itô’s Lemma to the equilibrium equity value, \( E(X_t) \), and using the differential equation (9), the excess return on equity, under \( Q \), is given by
\[ \frac{dE(X_t)}{E(X_t)} + \frac{dD_t^Q}{E(X_t)} - rdt = X_t \frac{E(X_t)}{E(X_t)} \sigma dW_t^Q. \]

Using the translation \( W_t^Q = W_t^Q + \eta \sigma t \), it follows that, conditional on \( X_t \), the \( \mathbb{P} \)-expected, instantaneous excess return on equity may be expressed as
\[ \rho_t^E - r = \psi^E_t \eta \rho \sigma, \]  
(15)
where \( \psi^E_t = X_t \frac{E(X_t)}{E(X_t)} = \frac{\partial \log E(X_t)}{\partial \log X_t} \) is the earnings sensitivity (or elasticity) of equity. The relation (15) readily implies that for a given level of correlated earnings volatility, \( \rho \sigma \), and systematic volatility, \( \eta \), higher earnings sensitivity implies higher expected return.
To characterize the market price of systematic risk, \( \eta \), I assume that there exists a traded, diversified portfolio with value \( M_t \), subject only to the systematic risk component of the stochastic discount factor (3). The return on this portfolio, under \( \mathbb{P} \), is then given by

\[
\frac{dM_t}{M_t} = r^M dt + \sigma^M dZ^t_t.
\]

Since \( Z^t_t = Z^0_t + \eta t \) and since the \( \mathbb{Q} \)-expected, instantaneous return on the portfolio has to be the risk-free rate, it follows that \( \eta = -\frac{\mu^M}{\sigma^M} \), i.e. the expected excess return on the portfolio relative to its volatility, or its Sharpe ratio. The return with value \( M_t \) assumes that there exists a traded, diversified portfolio (CAPM)

The relation (16) is a conditional capital asset pricing model (CAPM), stating that, given \( X_t \), the expected excess return on the firm’s equity is proportional to the expected excess return on the diversified portfolio. The proportionally factor is the asset beta, \( \beta^X \), scaled by the earnings sensitivity, \( \psi^E_t \). In this sense, \( \psi^E_t \beta^X \) is this model’s conditional equity beta at time \( t \).

The following proposition gives an expression for the sensitivity, \( \psi^E_t \), that highlights the effects of the firm’s cash holdings and probability of insololvency on expected equity returns.

**Proposition 4** (Earnings sensitivity of equity). Given the coupon, \( k \), and the time-\( t \) value of cumulated earnings, \( X_t \), the earnings sensitivity of equity, \( \psi^E_t \), can be written as

\[
\psi^E_t = 1 - \frac{C(X_t)}{E(X_t)} + (1 - \tau) \frac{X_t}{E(X_t)} + (1 - \tau) \frac{k/r}{E(X_t)} - (1 - \tau)(1 - \phi^{-}) \frac{\psi^E_t \beta^X}{E(X_t)}.
\]

**Proposition 4** benchmarks equity’s earnings sensitivity to 1, which corresponds to the earnings sensitivity of an unlevered firm (i.e. when \( k = 0 \)) and would imply an equity beta as in the unconditional CAPM. Furthermore, to measure the deviations from this benchmark, the sensitivity is decomposed into four terms.

The term labeled “Cash” is the firm’s cash-to-equity ratio, and represents the percentage value of target cash to equity holders (relative to current value of their initial equity investment). Intuitively, the term is negative and reflects a discount (relative to the benchmark, i.e. the unconditional CAPM beta) because higher target cash means, all else equal, lower sensitivity to earnings risk. By (16), the isolated effect of a higher target cash is thus lower beta and lower expected returns.

The term labeled “Earnings” is a proxy for the firm’s total earnings yield (i.e. the inverse of the price-to-earnings ratio) from the time of initial investment until time \( t \). It represents the percentage value of the earnings claim owned by equity holders. Intuitively, the term is positive and reflects a premium (relative to the benchmark) because a higher claim on the firm’s earnings will, all else equal, imply higher earnings sensitivity. The isolated effect of a higher earnings yield will thus imply higher beta and higher expected returns.

The “Leverage”-term is a proxy for the firm’s debt-to-equity ratio, and represents the percentage value of the firm’s financial obligations. Intuitively, the term is positive and reflects a premium because higher coupon payments will, all else equal, imply more sensitivity to earnings risk. The isolated effect of more leverage is therefore higher beta and higher expected returns.

Finally, the term labeled “Insolvency” is a proxy for the firm’s debt-to-equity ratio, and represents the percentage value of the firm’s debt at the strategic insololvency trigger, \( X^* \). It represents the value of the limited liability provision imbedded in the equity claim. Intuitively, it is negative and reflects a discount because as the firm becomes less solvent, the value of its earnings-generating asset declines and the equity claim approaches the value of the firm’s cash (cf. (11)), which, all else equal, lowers equity’s sensitivity to earnings risk. The isolated effect of a higher value for the default option is thus lower equity beta and lower expected returns.
Cash holdings and earnings sensitivity

The above considerations give an assessment of the model’s isolated effects on equity beta and expected returns. More interestingly, however, is the relationship between target cash holdings and the earnings sensitivity (i.e. the behavior of equity beta and expected returns) for varying levels of solvency.

**Corollary 4.1** (Target cash and earnings sensitivity). Suppose \( r > \mu^Q > \frac{1}{2}r \), let \( \pi^Q(X) = (X/X^*)^\phi \) be the Q-probability of insolvency as in Proposition 3, and let \( A(X_i) = (1 - \tau)E^X_{i} \) be the market value of earnings-generating assets. Then the target cash holdings, \( C(X_i) \), and the earnings sensitivity of equity, \( \psi_i^E \), have the following properties:

1) As \( \pi^Q(X_i) \to 0 \), cash holdings are worth less than earnings-generating assets, while earnings sensitivity is higher than for an unlevered firm and increases in the probability of insolvency:

\[
C(X_i) < A(X_i), \quad \psi_i^E > 1, \quad \frac{d\psi_i^E}{d\pi^Q(X_i)} > 0.
\]

2) As \( \pi^Q(X_i) \to 1 \), cash holdings are worth more than earnings-generating assets, while earnings sensitivity is lower than for an unlevered firm and decreases in the probability of insolvency:

\[
C(X_i) > A(X_i), \quad \psi_i^E < 1, \quad \frac{d\psi_i^E}{d\pi^Q(X_i)} < 0.
\]

The first part of the corollary states that when the firm is solvent, its earnings-sensitivity is high (relative to an unlevered firm) and an increase in the probability of insolvency will make the equity claim more sensitive to earnings risk. This is because for high levels of solvency, the firm’s earnings-generating assets are a larger fraction of its total asset value than its cash holdings, which magnifies the effect of an increase in the probability of insolvency on the equity claim. By (15) and (16), expected returns and equity beta are thus high and increasing in the probability of insolvency when the firm is solvent.

By contrast, the second part of the corollary states that when the firm has a high probability of insolvency, its earnings-sensitivity is actually low (relative to an unlevered firm) and declining for a further decrease in solvency. The reason for this is the presence of cash in the firm’s capital structure. As insolvency becomes increasingly apparent, the value of the earnings-generating assets declines and cash holdings become an increasingly larger fraction of total asset value. This implies that for low levels of solvency, equity will to a greater degree be a claim on the firm’s cash, which decreases its sensitivity to earnings risk. Therefore, (15) and (16) imply that expected returns and equity beta are low and decreasing in the probability of insolvency for low levels of solvency.

Because target cash holdings, \( C(X) \), monotonically decrease in the probability of insolvency, \( \pi^Q(X) \), Corollary 4.1 has the following implications:

- The earnings-sensitivity, \( \psi_i^E \), is also hump-shaped in the target cash level: For high levels of target cash, \( \psi_i^E \) is greater than 1 and increasing in \( C(X) \), while for low levels of target cash, \( \psi_i^E \) is less than 1 and decreasing in \( C(X) \).

- A portfolio strategy long firms with high cash and short firms with low cash (hereafter a HCmLC portfolio) will have negative (positive) expected returns for high (low) levels of solvency.

- A portfolio strategy long firms with high solvency and short firms with low solvency (hereafter a HSmlLS portfolio) will have negative (positive) expected returns for high (low) levels of cash.

The first implications follows from the inverse relationship between target cash and probability of insolvency. The performance of the HCMcL strategy follows because for high levels of solvency, equity beta increases as cash decreases, so the long leg of HCMcL will have lower expected returns than the short leg. The situation is, however, reversed for low levels of solvency, where equity beta decreases as cash decreases, so the long leg of HCMcL will have higher expected returns than the short leg. The same logic applies to the HSmlLS strategy.
Figure 1. Numerical illustration: Earnings-sensitivity of equity. This figure shows a numerical illustration of the earnings-sensitivity of equity. Top left: Earnings-generating asset value (black curve) and cash holdings (purple curve) as a function of the probability of insolvency with an indication of the strategic insolvency trigger (black dashed line) and the lower boundary of cash holdings (dashed purple line). Top right: Earnings sensitivity of equity as a function of the probability of insolvency with an indication of the sensitivity of an unlevered firm at 1 (dashed line). Bottom left: Earnings sensitivity of equity as a function of the probability of insolvency (from 0 to 0.4) with an indication of the high-cash minus-low cash strategy (dashed purple line segments). Bottom right: Earnings sensitivity of equity as a function of cash holdings with an indication of the high solvency minus low solvency strategy (dashed purple line segments). The parameters are $\mu^Q = 0.04$, $\sigma = 0.15$, $k = 4.5$, $\tau = 0.15$, $r = 0.06$, and $X_0 = 58.82$.

Numerical illustration

Figure 1 illustrates the results of Corollary 4.1. I consider a representative firm given by the following parameters:

$$\mu^Q = 0.04, \sigma = 0.15, k = 4.5, \tau = 0.15, r = 0.06.$$  

The initial earnings level is set at $X_0 = 58.82$, so that initial earnings-generating asset value is $A(X_0) = 100$. Insolvency is then triggered when cumulated earnings hit $X^* = 30.10$ (corresponding to $A(X^*) = 51.17$). Initial equity value is then $E(X_0) = 125.36$ while initial cash holdings are $C(X_0) = 88.65$.

The top panels shows the firm’s earnings-generating assets, target cash holdings, and earnings-sensitivity of equity as a function of the probability of insolvency. The bottom panels show the earnings-sensitivity of equity as a function of probability of insolvency (right panel) and target cash (left plot) with an indication of the HCmLC and HSmLS strategies.

The top panel illustrates the mechanism behind the hump-shaped equity beta in the probability of insolvency. When the firm is solvent, it has a high target cash level, but its earnings-generating assets are larger than target cash. Consequently, equity’s earnings sensitivity is above 1 (i.e. above the earnings-sensitivity of an unlevered firm) for high levels of solvency. However, as the firm approaches insolvency, target cash holdings de-
cline, but earnings-generating asset value falls as well until it declines below target cash, in accordance with the relation in (14). Hence, the earnings sensitivity of equity falls below that of an unlevered firm for high levels of insolvency probability. By the relation in (16), the firm’s equity beta and its expected returns are higher than those of an unlevered firm when it is solvent, but are, in fact, lower than those of an unlevered firm when it approaches insolvency.

The lower left panel shows the performance of the HCmLC portfolio strategy for varying levels of solvency. Because earnings-sensitivity initially increases in the probability of insolvency, the short leg of HCmLC has higher expected returns than the long leg for high levels of solvency, so HCmLC has negative expected returns for high levels of solvency. This is, however, reversed for low levels of solvency, where the long leg has higher expected returns than the short leg. The lower right panel shows the expected returns on the HSmLS portfolio strategy. Because earnings-sensitivity increases as cash decreases from high levels, the short leg of the strategy has higher expected returns than the long leg for high levels of cash. The situation is, however, reversed for low levels of cash.

1.5 Discussion and testable predictions

Corollary 4.1 effectively predicts that equity betas and expected returns should be hump-shaped in both solvency and liquidity. As discussed in the paper’s introduction, such a relation was first derived by Garlappi and Yan (2011) in a model based on shareholder recovery in default—that is, when equity holders anticipate a renegotiation of debt-terms or even a violation the absolute priority rule after a default. In their model, the firm is not financially constrained, and therefore does not face liquidity risk, while the recovery value that ultimately implies the hump-shape is specified exogenously.

By contrast, the driving force behind Corollary 4.1 is the precautionary motive for holding cash, which implies that it is optimal for the financially constrained firm to offset its liquidity risk. Equity value in my model will thus not only be determined by the firm’s dividend payouts, but also by the endogenously determined target cash held by the firm, which becomes a larger fraction of total asset value as the firm becomes less solvent, thus lowering equity beta.

While the mathematical mechanisms behind the hump-shape is the same in both models—namely that equity value is determined by a non-zero underlying asset close to default—the economic mechanisms are different, and, thus, yield different testable predictions. In Garlappi and Yan (2011), equity risk declines close to default because equity holders anticipate a substitution of levered equity with unlevered asset value. In my model, equity risk declines because cash holdings reduce the systematic risk of the underlying. The novel prediction of my model are thus that cash holdings decrease equity betas and expected returns as solvency decreases; that the HCmLC portfolio earns increasing average returns as solvency decreases; and that the HSmLS portfolio earns increasing average returns as liquidity decreases.

2 An empirical study of liquidity risk and distressed equity

This section presents an empirical study of the model’s predictions for the effects of cash used to offset liquidity risk on the returns of distressed equity. I use firm-level data on US stocks prices, accounting numbers, and credit ratings to estimate firm-specific betas and calculate cross-sectional portfolio returns.

2.1 Data

I search for data in the intersection of industrial firms with stock prices in the CRSP database, accounting fundamentals in the Compustat North American database, and credit ratings or default records in the Moody’s DRS (Default Risk Service) database.

For every US debt issuer in DRS’ “industrial” category with an available third party identifier, I search for the corresponding security-level PERMNO-identifiers in the daily CRSP file and in the quarterly and yearly Compustat files, taking name changes, mergers, accusations, and parent-subsidiary relations into account, and excluding issuers which I cannot reliably match. I only include common stocks (CRSP’s SHRC 10-11) and I
exclude utilities and financial firms (CRSP’s SIC codes 4900-4999 and 6000-6999). The final sample has 3,947 unique firms, spanning 15,079,329 firm-days (720,371 firm-months) over the period from January 1970 to December 2013.

Because distress risk may ultimately result in a default or a bankruptcy, I track these events for the firms in the sample. I identify a default or bankruptcy event if it is recorded in either DRS, CRSP (DLSTCD 400-490 or 574, or SECSTAT ‘Q’), or Compustat (DLRSN 2-3 or STALTQ ‘TL’), and I count multiple events for the same firm occurring within a month as a single event. This results in a total of 874 events incurred by 683 firms, of which 529 events were identified solely through DRS, 134 solely through CRSP, and 137 solely through Compustat—the remaining 87 events were identified simultaneously by two or more sources.

I use the stock data to calculate market equity values, \( ME \) (the product of CRSP’s PRC and SHROUT, adjusted by their cumulative adjustment factors), and I accumulate daily log-returns (ln of 1 plus CRSP’s RET) over a 20 trading day rolling window to obtain monthly returns. I require at least 10 trading days to calculate a monthly return, and I use delisting returns (CRSP’s DLRET) whenever possible.

When possible, I substitute yearly accounting numbers for missing quarterly accounting numbers. These are then used to calculate quarterly book equity, \( BE = AT - LT \) (Compustat’s total assets, ATQ, minus total liabilities, LTQ), cash-ratios measuring balance sheet liquidity, proxies for solvency, and, finally, regression controls like firm size and dividends.

I align the quarterly accounting data and the daily stock data as follows: On a given trading day, the corresponding accounting numbers are the latest ones available prior to that day. All raw variables and ratios are winsorized at the 1% and the 99% quantiles to remove the influence of near-zero divisions, recording errors, and outliers.

### 2.2 Liquidity and solvency

The model predicts that cash holdings used to offset liquidity risk correlate with equity prices and expected returns in manner that depends on solvency. In this subsection, I present the variables which I employ to measure liquidity and solvency.

#### Liquidity measures

I use three cash-based variables to measure a firm’s liquid assets relative to its current liabilities. First, I use the current ratio, \( CA/CL \) (Compustat’s current assets, ACTQ, divided by current liabilities, LCTQ), which measures a firm’s total liquid holdings as a fraction of its short-term liabilities. Second, I use the the quick ratio, \( QA/CL \) (Compustat’s current assets, ACTQ, minus inventories, INVTQ, the difference divided by current liabilities, LCTQ), which measures assets that can “quickly” be converted into cash in order to pay off short-term liabilities. The current- and quick ratios are similar, but the quick ratio is more conservative, in that it excludes inventories from current assets. In either case, a ratio below 1 indicates that the firm’s liquid assets are insufficient to meet short-term liabilities, i.e. illiquidity. Finally, I use the ratio of working capital to total assets (Compustat’s current assets, ACTQ, minus current liabilities, LCTQ, the difference divided by total assets, ATQ), measuring the firm’s net liquid assets as a percentage of total book assets. A negative working capital ratio thus indicates illiquidity.

#### Solvency measures

To measure solvency, I use Moody’s senior unsecured long-term credit ratings (provided in DRS) as well as two balance-sheet based variables: Leverage and interest coverage. The ratings give a categorical measure of solvency, based on an overall assessment of firm’s ability to honor its financial obligations with an original maturity of one year or more. I measure leverage using either the book leverage ratio, \( LT/AT \), or the market leverage ratio, \( LT/(LT + ME) \), which are a stock variable measuring total liabilities as a fraction of either book or market assets. Finally, the interest coverage ratio, \( OI/IX \) (Compustat’s EBITDA-variable, OIBDPQ, over interest expense, XINTQ), is a flow variable measuring the firm’s ability to generate earnings in excess of its interest expense.
Figure 2. Liquidity measures sorted across solvency measures. This figure shows the three liquidity measures (current ratio, $CA/CL$, quick ratio, $QA/CL$, and working capital, $WC/AT$) plotted against the three solvency measures (deciles of interest coverage, $OI/IX$, deciles of book leverage, $LT/AT$, and Moody’s credit rating). Solid lines indicate means within groups while dashed lines indicate medians. In all panels, a horizontal move to the right corresponds to lower solvency, while a vertical move downwards corresponds to lower liquidity.

Liquidity and solvency

Figure 2 illustrates how liquidity varies with solvency.

In the leftmost column, the three liquidity measures are plotted against the interest coverage ratio. Firms generating the highest earnings relative to their interest expense hold liquid assets that are 2-3 times their current liabilities and their working capital is about 35% of book asset. Liquidity declines as interest coverage declines, except for the firms with lowest interest coverage, where liquid assets again rise to 1.5-2 times current liabilities and working capital again rises to around 20% of book assets. Importantly, liquid assets exceed current liabilities and working capital is positive across all levels of interest coverage. This corroborates the findings of Acharya et al. (2012) and supports the model’s assumption that levered firms hold cash to offset the risk of an earnings-shortfall.
The middle column shows the liquidity measures across book leverage (the plots are essentially identical for market leverage). Firms with the lowest leverage hold 2.5-3.5 times as much liquid assets as their current liabilities and have a working capital ratio of about 40%. As leverage increases, liquidity decreases monotonically. Still, the most levered firms have liquid assets that equal or are slightly above their current liabilities and a positive working capital ratio just short of 10%. This indicates that, on average, even the most levered firms are still liquid.

Finally, the rightmost column shows the liquidity measures across credit ratings. The upper investment-grade firms (from Aaa to Aa) hold liquid assets that are 1-1.5 times their current liabilities and have a working capital ratio of around 15%. With such high ratings, these firms need to worry less about financing constraints, which is reflected in their relatively modest reserves of liquid assets. As ratings move away from investment grade, liquidity increases and reaches its highest level for upper speculative grade firms (from Ba to B), who hold liquid assets in the range of 1.3-2 times their current liabilities and have a working capital ratio of around 20%. For these firms, access to external financing is likely to be somewhat constrained, which is reflected in their larger reserves of liquid assets. While liquidity generally declines as ratings deteriorate into lower speculative-grade (from Caa to C) and default (D), even the lowest rated firms have liquid assets that either slightly exceed or just fall short of their current liabilities, once more corroborating the impression from the middle column that even the riskiest firms are, on average, liquid.

In sum, Figure 2 indicates that firms use liquidity reserves to offset the risk of an earnings-shortfall; that liquidity generally declines with leverage, but that even the most levered firms have liquid assets; and that firms in the middle of the rating scale are the ones who hold the most liquid assets, but that even the firms with lowest ratings have liquid asset that either slightly exceed or just fall short of their current liabilities.

### 2.3 Liquidity risk and equity returns

In this section, I test the model’s prediction regarding the correlation between balance-sheet liquidity and equity returns. The model predicts that i) firm-specific betas and cross-sectional returns are hump-shaped in solvency and liquidity, ii) a portfolio strategy long firms with high cash and short firms with low cash (HCMLC) has increasing expected returns as firms become less solvent, and iii) a portfolio strategy long firms with high solvency and short firms with low solvency (HSmLS) has increasing expected returns as cash levels decline.

#### Firm-specific betas

Figure 3 plots firm-specific betas across the solvency and liquidity measures. The firm-specific betas are calculated at the monthly frequency by regressing daily excess returns on the daily excess returns of the CRSP value-weighted “market” index (available in the CRSP file or on prof. Ken French’s website). I require a minimum of 10 trading days to estimate a monthly beta. To reduce the influence of outliers, I exclude the lowest and highest monthly beta for each firm. Finally, following Vasicke (1973) and Frazzini and Pedersen (2014), I adjust firm $i$’s estimated monthly beta, $\hat{\beta}_i$, towards the cross-sectional mean for the corresponding month, $\bar{\beta}_r$, by setting

$$\bar{\beta}_i = w_i \hat{\beta}_i + (1 - w_i) \bar{\beta}_r,$$

Here, $w_i = 1 - \nu_i^2 / (\nu_i^2 + \nu_r^2)$ is a Bayesian adjustment factor calculated using the variance of the estimated betas for firm $i$, $\nu_i^2$, and the cross-sectional variance of the estimated betas at month $t$, $\nu_r^2$. It places more weight on the firm’s beta estimates when their variance is small or when the cross-sectional variance is high.\(^\text{12}\)

The top panel shows the firm-specific betas across the solvency measures. In all three plots, and consistent with the model’s prediction that equity betas decline for low levels of solvency, the most solvent firms also have the highest betas, and betas decline almost monotonically as firms become less solvent. The most solvent firms with a high credit rating, low leverage, and high interest coverage, have average betas between 0.9 and

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\(^\text{12}\)I have also produced a version of Figure 3 where I replace the firm-specific adjustment factors, $w_i$, with their average across firms and time, 0.57, and where the cross-sectional mean beta is set as $\hat{\beta}_i = 1$ for all months. This mimics the simplified adjustment of firm-specific betas used by Frazzini and Pedersen (2014). The resulting figure is very similar to Figure 3.
Figure 3. Firm-specific betas across solvency and liquidity measures. This figure shows firm-specific CAPM-betas plotted against the three solvency measures (top panels: Moody’s credit rating, deciles of market leverage, \(LT/(LT+ME)\), and deciles of interest coverage, \(OI/IX\)) and the three liquidity measures (bottom panels: Deciles of current ratio, \(CA/CL\), quick ratio, \(QA/CL\), and working capital, \(WC/AT\)). Firm-specific betas are estimated at the monthly frequency by regressing daily excess returns on the daily excess returns of the CRSP value-weighted “market” index. Each monthly beta is estimated using a minimum of 10 trading days. To reduce the influence of outliers, the lowest and highest monthly beta for each firm is excluded, and monthly betas are adjusted towards the cross-sectional mean monthly beta as in Vasicek (1973) and Frazzini and Pedersen (2014). Solid lines indicate means within groups while dashed lines indicate medians. In the top (bottom) panels, a horizontal move to the right corresponds to higher solvency (liquidity) risk.

At the beginning of each month, I sort firms into portfolios according to their solvency or liquidity levels. The portfolios are value-weighted, refreshed every month, and rebalanced every month to maintain the value-weighting. To ensure that the results are not driven by outliers, the highest and lowest realized return is excluded for each portfolio. The tables report the time-series average of the portfolio returns over the 1-month US T-bill as well as alphas, estimated as the intercepts from time-series regressions of excess returns on the “market” index (MKT); the size (SMB) and value (HML) factors of Fama and French (1993); and the momentum factor (UMD) of Carhart (1997).

In Table 1, panel A shows portfolios formed on credit ratings, while panels B and C show portfolios formed on deciles of market leverage or interest coverage. In all three panels, firms in the high end of the solvency scale (high rating, low leverage, or high interest coverage) have lower expected returns and alphas (abnormal returns) for firms in either type of distress. To verify this, Table 1 shows excess returns and alphas (abnormal returns) for portfolios formed on the three solvency measures, while Table 2 shows the corresponding results for portfolios formed on the three liquidity measures.
verage) have positive and significant excess returns and alphas. At the other end of the solvency scale, however, excess returns and alphas turn insignificant and even significantly negative for the least solvent firms. Furthermore, the decline in excess returns and alphas is almost monotonic when using leverage and interest coverage to proxy for solvency. This is consistent with the model’s prediction that returns decline in the probability of insolvency for low levels of solvency.

Table 2 shows similar results for the portfolios formed on the deciles of the three liquidity measures: The current ratio in panel A, the quick ratio in panel B, and the working capital ratio in panel C. In all three panels, excess returns and alphas are significantly positive for high levels of liquidity and generally decline as the liquidity measures decline. This is consistent with the model’s prediction that the declining returns on distressed equity is prevalent across both the solvency- and the liquidity dimension.

High cash minus low cash across credit ratings

The model predicts that a portfolio strategy long firms with high cash and short firms with low cash will have increasing average returns as solvency decreases. To test this prediction, Table 3 shows results for double-sorted portfolios according to credit ratings and liquidity measures. To ensure a sufficient number of firms in each portfolio, I re-code the original 9 credit ratings into three groups: Aaa-A, Baa-B, and Caa-C. I form the portfolios using conditional sorts—first into the three credit ratings portfolios, and then into three portfolios based on terciles of liquidity measures. I then calculate, for each credit rating group, the difference between the returns on the “high cash” portfolio and the “low cash” portfolio (hereafter, a HCmLC portfolio).

Consistent with the models prediction, the HCmLC portfolios have monotonically increasing returns, alphas, and Sharpe ratios as all three liquidity measures decrease.

High solvency minus low solvency across liquidity measures

The model also predicts that a portfolio strategy long firms with high solvency and short firms with low solvency will have increasing average returns as cash holdings decrease. To test this prediction, Table 4 shows results for double-sorted portfolios formed first on liquidity terciles and then on credit rating groups: Aaa-A, Baa-B, and Caa-C. For each liquidity tercile, I calculate the difference between the returns on the Aaa-A portfolio and the Caa-C portfolio (hereafter, a HSmLS portfolio).

Consistent with the models prediction, the HCmLC portfolios have monotonically increasing returns, alphas, and Sharpe ratios as all three liquidity measures decrease.

Concluding remarks

This paper has shown, theoretically and empirically, that cash holdings used to offset liquidity risk can help rationalize the anomalous returns of distressed equity. In my model, levered firms with financing constraints can default because of liquidity or solvency, but firms seek to manage their cash to avoid the former. Using data on rated US firms between 1970 and 2013, I find empirical evidence consistent with my theoretical predictions: i) the average insolvent firm holds cash that meets or exceeds its current liabilities; ii) firm-specific betas and risk-adjusted returns decline as firms become less solvent and as cash levels decline; and iii) a portfolio long firm with high cash and short firms with low cash has increasing returns for less solvent firms, while a portfolio long firms with high solvency and short firms with low solvency has increasing returns for firms with less cash. In sum, my results suggest that there is no distress anomaly for insolvent but liquid firms.
Table 1. Monthly excess returns of portfolios formed on solvency measures. This table shows excess returns, alphas, and related quantities for portfolios, formed on solvency measures. At the beginning of each calendar month, I assign stocks into portfolios using the previous month’s market equity values, estimated every calendar month, and average the monthly portfolio returns (in percentages) in excess of the 1-month US Treasury bill rate. Volatility is the annualized standard deviation of the monthly excess return. Sharpe ratio is the monthly average excess return divided by the monthly standard deviation. The intercepts from time-series regressions of monthly excess returns on the excess returns of the value-weighted CRSP market index (Mkt), the three and four-factor alphas as well as the Carhart (1997) factor (UMD). Percentages in superscript give t-statistics. For the excess returns and the alphas, the null value is zero, while it is one for the beta. Statistical significance at the 5% level is indicated in bold.

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Panel A</th>
<th>Panel B</th>
<th>Panel C</th>
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<tbody>
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<td></td>
<td>Excess return (avg. mon. %)</td>
<td>Three-factor alpha (ann. %)</td>
<td>Market Leverage</td>
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<td></td>
<td>CAPM beta (portfolio)</td>
<td>CAPM beta (portfolio)</td>
<td>Sharpe Ratio (ann.)</td>
</tr>
<tr>
<td>Aaa</td>
<td>0.81±0.36</td>
<td>0.57±0.17</td>
<td>0.54±0.11</td>
</tr>
<tr>
<td></td>
<td>(0.92±0.42)</td>
<td>(1.06±0.37)</td>
<td>(0.62±0.24)</td>
</tr>
<tr>
<td>Ba</td>
<td>0.90±0.30</td>
<td>0.55±0.27</td>
<td>0.55±0.27</td>
</tr>
<tr>
<td></td>
<td>(0.42±0.28)</td>
<td>(0.64±0.35)</td>
<td>(0.42±0.28)</td>
</tr>
<tr>
<td>Caa</td>
<td>0.83±0.27</td>
<td>0.52±0.22</td>
<td>0.52±0.22</td>
</tr>
<tr>
<td></td>
<td>(0.27±0.09)</td>
<td>(0.37±0.08)</td>
<td>(0.27±0.09)</td>
</tr>
<tr>
<td>Average</td>
<td>0.83±0.27</td>
<td>0.52±0.22</td>
<td>0.52±0.22</td>
</tr>
<tr>
<td>T-Bill</td>
<td>0.00±0.00</td>
<td>0.00±0.00</td>
<td>0.00±0.00</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.53±0.12</td>
<td>0.46±0.12</td>
<td>0.46±0.12</td>
</tr>
<tr>
<td></td>
<td>(1.02±0.35)</td>
<td>(1.02±0.35)</td>
<td>(1.02±0.35)</td>
</tr>
<tr>
<td>Months</td>
<td>496</td>
<td>305</td>
<td>305</td>
</tr>
</tbody>
</table>

Notes: (1) The Sharpe ratio is defined as the excess return divided by the standard deviation of the excess returns. (2) The t-statistics for the intercepts from time-series regressions of monthly excess returns on the excess returns of the value-weighted CRSP market index (Mkt) are given in superscript. (3) The t-statistics for the three and four-factor alphas as well as the Carhart (1997) factor (UMD) are given in superscript.
Table 2. Monthly excess returns of portfolios formed on liquidity measures. This table shows excess returns, alphas, and related quantities for portfolios formed on liquidity measures. At the beginning of each calendar month, I assign firms into portfolios according to deciles of current ratio (Panel A), quick ratio (Panel B), or working capital ratio (Panel C) for the previous month. The portfolios are value-weighted using the previous month’s market equity values, refreshed every calendar month, and rebalanced every calendar month to maintain value-weighting. The highest and lowest realized return is excluded for each portfolio. Excess return is the time-series average of the monthly portfolio returns (in percentages) in excess of the 1-month US Treasury bill rate. Volatility is the annualized standard deviation of the monthly excess return (in percentages). Sharpe ratio is the annualized average excess return divided by the annualized volatility. CAPM alpha and CAPM beta are the intercepts and slope estimates from a time-series regression of monthly excess returns on the excess returns of the value-weighted CRSP “market” index (MKT). Three- and four-factor alphas are the intercepts from time-series regressions of monthly excess returns on the three Fama and French (1993) factors (MKT, SMB, and HML) or these three factors as well as the Carhart (1997) factor (UMD). Parentheses in subcaption give t-statistics. For the excess returns and the alphas, the null value is zero, while it is one for the beta. Statistical significance at the 5% level is indicated in bold.

### Panel A

<table>
<thead>
<tr>
<th>Current ratio</th>
<th>High</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess Return (avg. mon. %)</td>
<td>1.174.26</td>
<td>1.244.96</td>
<td>1.154.76</td>
<td>1.115.32</td>
<td>1.125.26</td>
<td>1.115.22</td>
<td>0.964.56</td>
<td>0.995.30</td>
<td>0.985.03</td>
</tr>
<tr>
<td>CAPM alpha (ann. %)</td>
<td>0.745.46</td>
<td>0.825.94</td>
<td>0.765.97</td>
<td>0.735.58</td>
<td>0.755.67</td>
<td>0.735.39</td>
<td>0.615.07</td>
<td>0.695.74</td>
<td>0.665.52</td>
</tr>
<tr>
<td>Three-factor alpha (ann. %)</td>
<td>1.009.23</td>
<td>1.009.86</td>
<td>0.878.42</td>
<td>0.778.96</td>
<td>0.758.71</td>
<td>0.759.51</td>
<td>0.587.85</td>
<td>0.628.45</td>
<td>0.576.72</td>
</tr>
<tr>
<td>Four-factor alpha (ann. %)</td>
<td>0.985.93</td>
<td>1.005.94</td>
<td>0.935.92</td>
<td>0.795.99</td>
<td>0.785.82</td>
<td>0.795.82</td>
<td>0.595.89</td>
<td>0.615.21</td>
<td>0.605.94</td>
</tr>
<tr>
<td>CAPM beta (portfolio)</td>
<td>1.206.70</td>
<td>1.155.77</td>
<td>1.085.56</td>
<td>1.015.63</td>
<td>1.025.89</td>
<td>1.035.91</td>
<td>0.995.72</td>
<td>0.865.10</td>
<td>0.885.40</td>
</tr>
<tr>
<td>Volatility (ann. %)</td>
<td>21.66</td>
<td>19.73</td>
<td>19.00</td>
<td>16.84</td>
<td>16.84</td>
<td>16.84</td>
<td>16.57</td>
<td>14.69</td>
<td>15.37</td>
</tr>
<tr>
<td>Sharpe Ratio (ann.)</td>
<td>0.65</td>
<td>0.75</td>
<td>0.72</td>
<td>0.79</td>
<td>0.80</td>
<td>0.79</td>
<td>0.69</td>
<td>0.80</td>
<td>0.76</td>
</tr>
<tr>
<td>Months</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
</tr>
</tbody>
</table>

### Panel B

<table>
<thead>
<tr>
<th>Quick ratio</th>
<th>High</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess Return (avg. mon. %)</td>
<td>1.354.54</td>
<td>1.234.98</td>
<td>1.175.16</td>
<td>1.105.27</td>
<td>1.034.99</td>
<td>1.025.04</td>
<td>1.015.17</td>
<td>0.944.83</td>
<td>1.065.50</td>
</tr>
<tr>
<td>CAPM alpha (ann. %)</td>
<td>0.936.72</td>
<td>0.857.08</td>
<td>0.777.67</td>
<td>0.737.64</td>
<td>0.677.34</td>
<td>0.657.97</td>
<td>0.699.26</td>
<td>0.637.68</td>
<td>0.757.76</td>
</tr>
<tr>
<td>Three-factor alpha (ann. %)</td>
<td>1.221.17</td>
<td>1.069.84</td>
<td>0.909.87</td>
<td>0.7610.08</td>
<td>0.667.20</td>
<td>0.627.79</td>
<td>0.657.89</td>
<td>0.526.82</td>
<td>0.636.83</td>
</tr>
<tr>
<td>Four-factor alpha (ann. %)</td>
<td>1.2110.96</td>
<td>1.069.80</td>
<td>0.9410.15</td>
<td>0.7710.02</td>
<td>0.647.76</td>
<td>0.607.38</td>
<td>0.707.45</td>
<td>0.567.31</td>
<td>0.667.13</td>
</tr>
<tr>
<td>CAPM beta (portfolio)</td>
<td>1.196.15</td>
<td>1.114.08</td>
<td>1.073.98</td>
<td>1.013.83</td>
<td>1.000.26</td>
<td>0.952.59</td>
<td>0.924.96</td>
<td>0.905.47</td>
<td>0.845.73</td>
</tr>
<tr>
<td>Volatility (ann. %)</td>
<td>21.56</td>
<td>19.87</td>
<td>17.86</td>
<td>16.48</td>
<td>16.35</td>
<td>15.94</td>
<td>15.44</td>
<td>15.40</td>
<td>15.16</td>
</tr>
<tr>
<td>Sharpe Ratio (ann.)</td>
<td>0.75</td>
<td>0.75</td>
<td>0.78</td>
<td>0.80</td>
<td>0.76</td>
<td>0.77</td>
<td>0.79</td>
<td>0.73</td>
<td>0.84</td>
</tr>
<tr>
<td>Months</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
</tr>
</tbody>
</table>

### Panel C

<table>
<thead>
<tr>
<th>Working capital ratio</th>
<th>High</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess Return (avg. mon. %)</td>
<td>1.364.56</td>
<td>1.325.25</td>
<td>1.265.53</td>
<td>1.195.63</td>
<td>1.095.08</td>
<td>1.004.65</td>
<td>0.944.40</td>
<td>1.045.28</td>
<td>0.944.81</td>
</tr>
<tr>
<td>CAPM alpha (ann. %)</td>
<td>0.845.48</td>
<td>0.937.77</td>
<td>0.869.12</td>
<td>0.829.74</td>
<td>0.715.89</td>
<td>0.617.82</td>
<td>0.585.84</td>
<td>0.735.88</td>
<td>0.625.78</td>
</tr>
<tr>
<td>Three-factor alpha (ann. %)</td>
<td>1.179.23</td>
<td>1.1410.94</td>
<td>0.9410.07</td>
<td>0.8810.50</td>
<td>0.775.75</td>
<td>0.678.66</td>
<td>0.534.80</td>
<td>0.647.85</td>
<td>0.526.02</td>
</tr>
<tr>
<td>Four-factor alpha (ann. %)</td>
<td>1.158.93</td>
<td>1.1710.96</td>
<td>0.9610.17</td>
<td>0.9010.05</td>
<td>0.8410.33</td>
<td>0.729.15</td>
<td>0.526.12</td>
<td>0.678.11</td>
<td>0.535.99</td>
</tr>
<tr>
<td>CAPM beta (portfolio)</td>
<td>1.315.99</td>
<td>1.145.08</td>
<td>1.083.56</td>
<td>1.013.50</td>
<td>1.033.58</td>
<td>1.042.12</td>
<td>0.943.24</td>
<td>0.914.92</td>
<td>0.876.49</td>
</tr>
<tr>
<td>Volatility (ann. %)</td>
<td>23.56</td>
<td>19.93</td>
<td>17.93</td>
<td>16.66</td>
<td>16.90</td>
<td>16.91</td>
<td>15.80</td>
<td>15.57</td>
<td>15.43</td>
</tr>
<tr>
<td>Sharpe Ratio (ann.)</td>
<td>0.69</td>
<td>0.80</td>
<td>0.84</td>
<td>0.86</td>
<td>0.77</td>
<td>0.71</td>
<td>0.71</td>
<td>0.80</td>
<td>0.73</td>
</tr>
<tr>
<td>Months</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
<td>520</td>
</tr>
</tbody>
</table>
Table 3. High cash minus low cash returns across credit ratings

This table shows returns, alphas, and related quantities for conditional-sorted portfolios formed on credit ratings and liquidity measures. At the beginning of each calendar month, I assign firms into 3 portfolios according to their credit rating (Aaa-A, Baa-B, and Caa-C) and then into three portfolios according to terciles of current ratio (Panel A), quick ratio (Panel B), or working capital ratio (Panel C). The portfolios are value-weighted using the previous month’s market equity values, refreshed every calendar month, and rebalanced every calendar month to maintain value-weighting. The highest and lowest realized return is excluded for each portfolio. Return is the time-series average of the monthly returns for the portfolio that is long the “high cash” portfolio and short the “low cash” portfolio. Volatility is the annualized standard deviation of the monthly return (in percentages). Sharpe ratio is the annualized average return divided by the annualized volatility. CAPM alpha and CAPM beta are the intercept and slope estimates from a time-series regression of monthly returns on the excess returns of the value-weighted CRSP “market” index (MKT). Three- and four-factor alphas are the intercepts from time-series regressions of monthly returns on the three Fama and French (1993) factors (MKT, SMB, and HML) or these three factors as well as the Carhart (1997) factor (UMD). Parentheses in subscript give \( t \)-statistics. For the returns and the alphas, the null value is zero, while it is one for the beta. Statistical significance at the 5% level is indicated in bold.

### Panel A: Current Ratio

<table>
<thead>
<tr>
<th></th>
<th>Aaa-A</th>
<th>Baa-B</th>
<th>Caa-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>0.01(0.05)</td>
<td>0.37(2.37)</td>
<td>1.16(2.21)</td>
</tr>
<tr>
<td>CAPM alpha (ann. %)</td>
<td>-0.05(-0.43)</td>
<td>0.29(1.92)</td>
<td>1.05(2.80)</td>
</tr>
<tr>
<td>Three-factor alpha (ann. %)</td>
<td>0.07(0.59)</td>
<td>0.40(2.99)</td>
<td>1.14(2.17)</td>
</tr>
<tr>
<td>Four-factor alpha (ann. %)</td>
<td>0.09(0.84)</td>
<td>0.37(2.74)</td>
<td>0.97(1.83)</td>
</tr>
<tr>
<td>CAPM beta (portfolio)</td>
<td>0.19(3.94)</td>
<td>0.26(2.28)</td>
<td>0.25(-0.53)</td>
</tr>
<tr>
<td>Volatility (ann. %)</td>
<td>9.58</td>
<td>12.19</td>
<td>36.39</td>
</tr>
<tr>
<td>Sharpe Ratio (ann.)</td>
<td>0.01</td>
<td>0.37</td>
<td>0.38</td>
</tr>
<tr>
<td>Months</td>
<td>502</td>
<td>505</td>
<td>404</td>
</tr>
</tbody>
</table>

### Panel B: Quick Ratio

<table>
<thead>
<tr>
<th></th>
<th>Aaa-A</th>
<th>Baa-B</th>
<th>Caa-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>-0.05(-0.41)</td>
<td>0.34(2.32)</td>
<td>1.24(2.33)</td>
</tr>
<tr>
<td>CAPM alpha (ann. %)</td>
<td>-0.09(-0.88)</td>
<td>0.26(1.87)</td>
<td>1.08(2.84)</td>
</tr>
<tr>
<td>Three-factor alpha (ann. %)</td>
<td>0.08(0.75)</td>
<td>0.48(3.94)</td>
<td>1.39(2.79)</td>
</tr>
<tr>
<td>Four-factor alpha (ann. %)</td>
<td>0.04(0.36)</td>
<td>0.43(3.48)</td>
<td>1.41(2.69)</td>
</tr>
<tr>
<td>CAPM beta (portfolio)</td>
<td>0.12(-3.94)</td>
<td>0.24(-2.49)</td>
<td>0.36(-5.43)</td>
</tr>
<tr>
<td>Volatility (ann. %)</td>
<td>9.17</td>
<td>11.32</td>
<td>37.08</td>
</tr>
<tr>
<td>Sharpe Ratio (ann.)</td>
<td>-0.06</td>
<td>0.36</td>
<td>0.40</td>
</tr>
<tr>
<td>Months</td>
<td>503</td>
<td>505</td>
<td>404</td>
</tr>
</tbody>
</table>

### Panel C: Working Capital Ratio

<table>
<thead>
<tr>
<th></th>
<th>Aaa-A</th>
<th>Baa-B</th>
<th>Caa-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>0.08(0.64)</td>
<td>0.43(2.81)</td>
<td>1.52(2.96)</td>
</tr>
<tr>
<td>CAPM alpha (ann. %)</td>
<td>0.02(0.13)</td>
<td>0.35(2.41)</td>
<td>1.46(2.84)</td>
</tr>
<tr>
<td>Three-factor alpha (ann. %)</td>
<td>0.17(1.49)</td>
<td>0.53(4.31)</td>
<td>1.52(2.94)</td>
</tr>
<tr>
<td>Four-factor alpha (ann. %)</td>
<td>0.22(1.92)</td>
<td>0.51(4.10)</td>
<td>1.35(2.58)</td>
</tr>
<tr>
<td>CAPM beta (portfolio)</td>
<td>0.18(-3.56)</td>
<td>0.27(-2.17)</td>
<td>0.14(-7.99)</td>
</tr>
<tr>
<td>Volatility (ann. %)</td>
<td>9.67</td>
<td>11.98</td>
<td>35.77</td>
</tr>
<tr>
<td>Sharpe Ratio (ann.)</td>
<td>0.10</td>
<td>0.43</td>
<td>0.51</td>
</tr>
<tr>
<td>Months</td>
<td>503</td>
<td>504</td>
<td>405</td>
</tr>
</tbody>
</table>
Table 4. High solvency minus low solvency returns using credit ratings across liquidity measures

This table shows returns, alphas, and related quantities for conditional-sorted portfolios formed on liquidity measures and credit ratings. At the beginning of each calendar month, I assign firms into 3 portfolios according to terciles of current ratio (Panel A), quick ratio (Panel B), or working capital ratio (Panel C), and then into three groups according to credit rating (Aaa-A, Baa-B, and Caa-C). The portfolios are value-weighted using the previous month’s market equity values, refreshed every calendar month, and rebalanced every calendar month to maintain value-weighting. The highest and lowest realized return is excluded for each portfolio. Return is the time-series average of the monthly returns for the portfolio that is long the “high solvency” (Aaa-A) portfolio and short the “low solvency” (Caa-C) portfolio. Volatility is the annualized standard deviation of the monthly return (in percentages). Sharpe ratio is the annualized average return divided by the annualized volatility. CAPM alpha and CAPM beta are the intercept and slope estimates from a time-series regression of monthly returns on the three Fama and French (1993) factors (MKT, SMB, and HML) or these three factors as well as the Carhart (1997) factor (UMD). Parentheses in subscript give t-statistics. For the returns and the alphas, the null value is zero, while it is one for the beta. Statistical significance at the 5% level is indicated in bold.

<table>
<thead>
<tr>
<th>Panel A: Current Ratio</th>
<th>High cash</th>
<th>Medium cash</th>
<th>Low cash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aaa-A minus Caa-C</td>
<td>Aaa-A minus Caa-C</td>
<td>Aaa-A minus Caa-C</td>
</tr>
<tr>
<td>Return (avg. mon. %)</td>
<td>0.33(0.70)</td>
<td>0.67(1.50)</td>
<td>0.98(2.97)</td>
</tr>
<tr>
<td>CAPM alpha (ann. %)</td>
<td>0.55(1.20)</td>
<td>0.88(2.62)</td>
<td>1.20(2.59)</td>
</tr>
<tr>
<td>Three-factor alpha (ann. %)</td>
<td>0.63(1.38)</td>
<td>0.81(1.88)</td>
<td>1.18(2.74)</td>
</tr>
<tr>
<td>Four-factor alpha (ann. %)</td>
<td>0.46(1.00)</td>
<td>0.51(2.20)</td>
<td>0.81(1.91)</td>
</tr>
<tr>
<td>CAPM beta (portfolio)</td>
<td>-0.51(-14.96)</td>
<td>-0.49(-15.73)</td>
<td>-0.51(-14.75)</td>
</tr>
<tr>
<td>Volatility (ann. %)</td>
<td>32.63</td>
<td>30.63</td>
<td>33.55</td>
</tr>
<tr>
<td>Sharpe Ratio (ann.)</td>
<td>0.12</td>
<td>0.26</td>
<td>0.35</td>
</tr>
<tr>
<td>Months</td>
<td>397</td>
<td>393</td>
<td>416</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Quick Ratio</th>
<th>High cash</th>
<th>Medium cash</th>
<th>Low cash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aaa-A minus Caa-C</td>
<td>Aaa-A minus Caa-C</td>
<td>Aaa-A minus Caa-C</td>
</tr>
<tr>
<td>Return (avg. mon. %)</td>
<td>0.61(1.39)</td>
<td>1.20(2.47)</td>
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<tr>
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<td>Four-factor alpha (ann. %)</td>
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<td>0.51</td>
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<td>Months</td>
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References


Appendices

A Additional details and proofs

This appendix gives additional details of the model’s development that were omitted from the main text, as well as the proofs of the paper’s propositions and corollaries.

A.1 Change of measure

The firm’s cumulated earnings and the economy’s stochastic discount factor were given by the $\mathbb{P}$-dynamics in (1) and (3), i.e.

$$dX_t = \mu^p X_t dt + \sigma X_t dW_t^p$$

and

$$d\Lambda_t = -r \Lambda_t dt - \eta \Lambda_t dZ_t^\eta,$$

where $W_t^p$ and $Z_t^\eta$ are standard $\mathbb{P}$-Brownian motions with correlation $\rho$. There thus exists a standard $\mathbb{P}$-Brownian motion, $\tilde{Z}_t^\eta$, independent of $Z_t^\eta$, such that

$$W_t^p = \rho Z_t^\eta + \sqrt{1 - \rho^2} \tilde{Z}_t^\eta.$$  

Now, suppose $\Lambda_0 = 1$, let $L_t = e^{r_t} \Lambda_t$, and note that $L_t$ may be written as

$$L_t = \exp \left( -\frac{1}{2} \eta' \eta t - \eta' \tilde{Z}_t^\eta \right),$$

where $\eta = (\eta, 0)'$ and $Z_t^\eta = (Z_t^\eta, \tilde{Z}_t^\eta)'$. Fix $T$ and define a risk-neutral pricing measure $\mathbb{Q}$ by $d\mathbb{Q}/d\mathbb{P} = L_T$. Girsanov’s Theorem then gives that the process $Z_t^\eta = (Z_t^\eta, \tilde{Z}_t^\eta)$, defined by

$$Z_t^\eta = Z_t^\eta + \eta t,$$

is a standard $\mathbb{Q}$-Brownian motion.

Finally, let $W_t^{\mathbb{Q}} = \rho Z_t^\eta + \sqrt{1 - \rho^2} \tilde{Z}_t^\eta$, and note that $W_t^{\mathbb{Q}}$ is standard $\mathbb{Q}$-Brownian motion which may be written as

$$W_t^{\mathbb{Q}} = \rho (Z_t^\eta + \eta t) + \sqrt{1 - \rho^2} \tilde{Z}_t^\eta = W_t^p + \eta pt.$$  

It thus follows that the $\mathbb{Q}$-dynamics of the firm’s cumulated earnings process may be written as

$$dX_t = \mu^Q X_t dt + \sigma X_t dW_t^{\mathbb{Q}},$$

where $\mu^Q = \mu^p - \eta pt \sigma$, exactly as in (4).

A.2 Proof of Proposition 1

Suppose cash holdings are of the form $C_t = C(X_t)$ for some twice continuously differentiable function $C(X)$. It follows by Itô’s Lemma along with (2) and (4) that

$$\begin{align*}
\frac{dC_t}{C_t} &= \left[ r C(X_t) + (1 - \tau)(\mu^Q X_t - k) 
- \frac{\partial^2 C(X_t)}{\partial X_t^2} \sigma^2 X_t^2 \right] dt 
+ \left( (1 - \tau) \sigma X_t - \frac{\partial C(X_t)}{\partial X_t} \sigma X_t \right) dW_t^Q, 
\end{align*}$$  

(17)

where subscripts denote partial derivatives. Since the firm has no access to external financing beyond time 0, it can, in particular, not issue additional equity. This implies that the cumulated dividend process has to be non-decreasing for all $r > 0$, which is satisfied if and only if $i$) the drift-term in (17) is nonnegative and ii) the volatility-term in (17) is zero.

The requirement that the volatility-term of $dD_t$ has to be zero implies the simple differential equation $C(X) = (1 - \tau)$, which has the general solution

$$C(X) = (1 - \tau)X + L$$

for some constant $L$. Plugging this solution into the drift-term of $dD_t$, and imposing the requirement that the drift has to be nonnegative, it follows that the solution has to satisfy

$$C(X) \geq (1 - \tau)\frac{X}{2}$$

for all $X$. To solve for the constant $L$, note that since $C(X) = (1 - \tau) > 0$, the target cash level is increasing in $X$. As the insolvency trigger $X^\star$ is a lower bound for $X$, it follows that $C(X) \geq C(X^\star) \geq (1 - \tau)\frac{X^\star}{2}$ for all $X$. Combining this inequality with the general solution then gives that the constant $L$ must satisfy

$$L \geq -(1 - \tau)X^\star + (1 - \tau)\frac{X^\star}{2} = (1 - \tau)\left[ -X^\star + \frac{X^\star}{2} \right].$$

By choosing $L$ as low as possible (i.e. the value where the inequality is binding), the expression for the solution given in Proposition 1 follows.

The remaining part of the proof is to assert that the target cash level, $C(X)$, is twice continuously differentiable in current cumulated earnings, $X$. If $X^\prime \leq X^\star$ is a discontinuity point for $C(X)$, then $C(X)$ must jump downwards at $X^\prime$, or else it would not have been the
target cash level an (infinitesimal) instance before $X'$. However, even a downwards jump at $X'$ would imply that $C(X)$ is not the target cash level an instance before $X'$. Hence, $C(X)$ must be continuous for all $X$.

On the other hand, if $C(X)$ is continuous but non-differentiable at some $X \geq X^*$, then $C(X)$ would for all $X \neq X'$ satisfy the same differential equation as before. However, this differential equation and the associated inequalities imply a solution that is twice continuously differentiable for all $X$. □

### A.3 Proof of Proposition 2

To prove that the conjectured dividend policy in (7), $dD_t^*$, is optimal, it must first be shown that it attains the maximal equity value in (5), $E(X, C)$, and, second, that no other dividend policy implies a higher equity value—that is, it must be shown that

1) $E(X, C) = \mathbb{E}_{t,C}^Q \left[ \int_t^{\tau} e^{-r(u-t)} dD_u^* + e^{-r(\tau-t)}C_{\tau} \right]$, and that

2) $E(X, C) \geq \mathbb{E}_{t,X,C}^Q \left[ \int_t^{\tau} e^{-r(u-t)} dD_u + e^{-r(\tau-t)}C_{\tau} \right]$ for all nonnegative, non-decreasing, adapted dividend processes, $D_t$.

In both parts, the proof will make use of the following two general results.

First, note that a general dividend process, $D_t$, is not necessarily continuous, meaning that it, and the corresponding cash process, $C_t$, may have jumps. Still, a general $D_t$ may be decomposed into the sum of its purely continuous part and its jumps: $D_t = D^c_t + \sum_{s \leq t} (D_s - D_{s-})$. Using this along with the generalized Itô’s Lemma and the dynamics of $X_t$ and $C_t$ in (2) and (4), it follows that the discounted gains process of equity, as defined by $G_t = e^{-rt}E(X_t, C_t) + \int_0^t e^{-r\tau}dD_{\tau}$, has the dynamics

$$
dG_t = e^{-rt} [-rE(X_t, C_{t-}) + \mathcal{AE}(X_t, C_{t-})]dt + e^{-rt} \left[ 1 - E_C(X_t, C_{t-}) \right] dD_t + e^{-rt} \sigma X_t [E_X(X_t, C_{t-}) + (1 - \tau)E_{C}(X_t, C_{t-})] dW_t + e^{-rt} [E(X_t, C_t) - E(X_t, C_{t-})] - E_C(X_t, C_{t-})(C_t - C_{t-})].$$

(18)

Here, the infinitesimal operator, $\mathcal{AE}(X, C)$, is given by

$$
\mathcal{AE}(X, C) = \mu XEX(X, C) + \left[ rC + (1 - \tau)(\mu X - k) \right] E_C(X, C) + \frac{1}{2}\sigma^2 X^2 E_{XX}(X, C) + \frac{1}{2}(1 - \tau)^2 \sigma^2 X^2 E_{CC}(X, C) + (1 - \tau) \sigma^2 X^2 E_X(X, C),
$$

while the jumps of the cash process, by (2), are given by $C_t - C_{t-} = -(D_t - D_{t-})$ for all $t$. Since equity is a traded asset, $G_t$ is a (possibly non-continuous) $\mathcal{Q}$-martingale, which will impose restrictions on the first two terms in (18) depending on the specific form of $D_t$.

Second, it follows by definition of the equity value function in (5) that

$$
E(X_t, C_t) \rightarrow C_{\tau} \quad \text{for} \quad t \rightarrow \tau.
$$

(19)

### Proof of Part 1)

Assume that $dD_t = dD_t^*$ for all $t$, and consider, in turn, the three regions implied by the conjectured dividend policy in (7).

First, when $0 < C_t < C$, dividend payouts are given by $dD_t^* = 0$ for all $t$, which is continuous and implies that $C_t$ is also continuous. By (18), this implies that $G_t$ is a $\mathcal{Q}$-martingale if and only if $-rE + \mathcal{AE} = 0$. Combining this with the implied form of (18) then gives

$$
d(e^{-rt}E(X_t, C_t)) = e^{-rt} \sigma X_t [E_X(X_t, C_t) + (1 - \tau) E_C(X_t, C_t)] dW_t^Q.
$$

Assuming that the derivatives of the equity value function are sufficiently integrable, it follows by integrating this expression between $t$ and $v \wedge \tau$ for any $v \geq t$ and taking expectations that

$$
e^{-rv}E(X, C) = \mathbb{E}_{t, X, C}^Q \left[ e^{-r(v-t)}E(X_{v\wedge\tau}, C_{v\wedge\tau}) \right].
$$

Since $v$ was arbitrary, it follows by letting $v \rightarrow \infty$ and applying (19) that

$$
E(X, C) = \mathbb{E}_{t, X, C}^Q \left[ e^{-r(\tau-t)}C_{\tau} \right],
$$

which proves part 1) in the first region.

Next, when $C_t = C(X_t)$, dividend payouts are given by $dD_t^* = rC(X_t) - (1 - \tau)k dt = r(1 - \tau)(X_t - X^*) dt$,
which again is continuous. In this region, equity value is given by \( E(X, C(X)) = E(X) \) and satisfies the ODE in (9)—hence, \(-rE + \mathcal{A}E = 0\). Furthermore, Proposition 3 and its proof (see Section A.4 of this appendix) give that the solution to (9) is separable in \( X \) and \( C \) and linear in \( C \), which implies \( E_C(X, C(X)) = 1 \). Combining these results with the implied form of (18), integrating between \( t \) and \( v \wedge \bar{t} \), and taking expectations, one then arrives at

\[
E(X, C) = \mathbb{E}_{t,X,C}^{Q} \left[ \int_{t}^{v \wedge \bar{t}} e^{-r(u-t)} dD_{u}^{*} \right. \\
\left. + e^{-r(u-t)} E(X_{v \wedge \bar{t}}, C_{v \wedge \bar{t}}) \right].
\]

Letting \( v \to \infty \) and applying (19) thus proves part i) for the second region.

Finally, when \( C_{t} > C(X_{t}) \), dividend payouts are \( dD_{t}^{*} = C_{t} - C(X_{t}) \) and thus not continuous. However, since this by (7) occurs if and only if \( C_{t} = C(X_{t}) \), it holds that \( E(X_{t}, C_{t}) = E(X_{t}, C(X_{t})) \). Therefore, by Proposition 3 and its proof, equity value at time \( t \) is separable in \( X \) and \( C \) and linear in \( C \), which gives \( E_C(X_t, C_{t}) = E_C(X_t, C(X_t)) = 1 \). Combining these results with (18), \( G_{t} \) is a martingale if and only if

\[
rE(X_{t}, C_{t}) + \mathcal{A}E(X_{t}, C_{t}) = -rE(X_{t}, \mathbb{C}(X_{t})) + \mathcal{A}E(X_{t}, C(X_{t})) = 0,
\]

while the jump-term of (18) becomes

\[
E(X_{t}, C_{t}) - E(X_{t}, C_{t}) - E_C(X_t, C_{t}) (C_{t} - C_{t}) = C_t - C(X_t) - (C_t - C(X_t)) = 0.
\]

Using the implied form of (18) and repeating the steps of the proof in the second region thus also proves part i) in the third region.

**Proof of Part ii)**

From part i), the dividend policy in (7) attains the maximal equity value in (5), \( E(X, C) \). The task is now to show that no other nonnegative, non-decreasing, adapted dividend process implies a higher equity value. Let \( D_{t} = D_{t}^{*} + \sum_{s \leq t} (D_{s} - D_{s}) \) be such a dividend process and let \( C_{t} \) be its corresponding cash process with jumps \( C_{t} - C_{t} = -(D_{t} - D_{t}) \) for all \( t \). Plugging this form of \( D_{t} \) into (18), the discounted gains process under the dividend policy \( D_{t} \) has the dynamics

\[
dG_{t} = e^{-rt} \left[ -rE(X_{t}, C_{t}) + \mathcal{A}E(X_{t}, C_{t}) \right] dt \\
+ e^{-rt} dD_{t} - e^{-rt} E_C(X_{t}, C_{t}) dD_{t}^{*} \\
+ e^{-rt} \sigma X_{t} \left[ E(X_{t}, C_{t}) + (1 - \tau) E_C(X_{t}, C_{t}) \right] dW_{t}^{Q} \\
+ e^{-rt} \left[ E(X_{t}, C_{t}) - E(X_{t}, C_{t}) \right].
\]

By the proof of part i), the maximal equity value satisfies \(-rE(X, C) + \mathcal{A}E(X, C) = 0\) for all \( X \) and \( C \). Hence, once more assuming that the deviates of the maximal equity value function are sufficiently integrable, it follows by integrating the last expression between \( t \) and \( v \wedge \bar{t} \), taking expectations, and rearranging, that

\[
E(X_{t}, C_{t}) = \mathbb{E}_{t,X,C}^{Q} \left[ e^{-r(u-t)} E(X_{v \wedge \bar{t}}, C_{v \wedge \bar{t}}) \right] \\
+ \mathbb{E}_{t,X,C}^{Q} \left[ \int_{t}^{v \wedge \bar{t}} e^{-r(u-t)} E_C(X_{u}, C_{u}) dD_{u}^{*} \right] \\
- \mathbb{E}_{t,X,C}^{Q} \left[ \sum_{t \leq s \leq u \wedge \bar{t}} e^{-r(u-t)} \left[ E(X_{s}, C_{s}) - E(X_{s}, C_{s}) \right] \right].
\]

Since it in general holds that \( E_{t}(X, C) \geq 1 \) (see footnote 11), the middle-term in the above expression can be bounded from below. Furthermore, the same property and the relation \( C_{t} - C_{t} = -(D_{t} - D_{t}) \leq 0 \) (since the dividend process is non-decreasing) imply

\[
-[E(X_{t}, C_{t}) - E(X_{t}, C_{t})] \geq D_{t} - D_{t},
\]

which thus bounds the last term. Therefore,

\[
E(X_{t}, C_{t}) \geq \mathbb{E}_{t,X,C}^{Q} \left[ e^{-r(u-t)} E(X_{v \wedge \bar{t}}, C_{v \wedge \bar{t}}) \right] \\
+ \mathbb{E}_{t,X,C}^{Q} \left[ \int_{t}^{v \wedge \bar{t}} e^{-r(u-t)} dD_{u}^{*} \right] \\
+ \mathbb{E}_{t,X,C}^{Q} \left[ \sum_{t \leq s \leq u \wedge \bar{t}} e^{-r(u-t)} (D_{s} - D_{s}) \right] \\
= \mathbb{E}_{t,X,C}^{Q} \left[ e^{-r(u-t)} E(X_{v \wedge \bar{t}}, C_{v \wedge \bar{t}}) \right] \\
+ \mathbb{E}_{t,X,C}^{Q} \left[ \int_{t}^{v \wedge \bar{t}} e^{-r(u-t)} dD_{u}^{*} \right] ,
\]

where the last equality uses \( D_{t} = D_{t}^{*} + \sum_{s \leq t} (D_{s} - D_{s}) \). Letting \( v \to \infty \) and applying (19) thus proves part ii), which completes the proof. \( \square \)
A.4 Proof of Proposition 3

The ODE (9) has the general solution
\[ E(X) = (1 - \tau) r \left[ \frac{X}{r - \mu_2} - \frac{X^*}{r} \right] + M_1 X^{\phi^+} + M_2 X^{\phi^-} \]
\[ = C(X) + (1 - \tau) \left[ \frac{\mu X}{r - \mu_2} - \frac{k}{r} \right] + M_1 X^{\phi^+} + M_2 X^{\phi^-}, \]
where the second equality follows by the form of the target cash holdings, \( C(X) \), in (6). Here, \( M_1 \) and \( M_2 \) are real-valued constants to be determined by boundary conditions, while the exponents \( \phi^+ \) and \( \phi^- \) are given by
\[ \phi^+ = \sqrt{\sigma^2 - 2\mu \pm \sqrt{(\sigma^2 - 2\mu)^2 + 8r\sigma^2}}, \]
\[ \phi^- = \frac{\sigma^2 - 2\mu \mp \sqrt{(\sigma^2 - 2\mu)^2 + 8r\sigma^2}}{2r}. \]

Since \( \phi^+ > 1 \), the value matching condition (10) implies \( M_1 = 0 \). Given this, the limited liability condition (11) implies
\[ M_2 = (X^*)^{-\phi^+} (1 - \tau) \left[ \frac{k}{r} - \frac{\mu X^*}{r - \mu_2} \right]. \]

By plugging the constants into the general solution, and defining \( \pi^G(X) = (X/X^*)^{\phi^+} \), the expression for \( E(X) \) in the proposition follows. \( \square \)

A.5 Proof Corollary ??

By the chain rule and the expressions in (12) and (6) for the equilibrium equity value and the target cash level, the parametric derivative of \( E(X) \) with respect to \( C(X) \) is given by
\[ \frac{dE(X)}{dC(X)} = \frac{\partial E(X)}{\partial X} \left( \frac{\partial C(X)}{\partial X} \right)^{-1} \]
\[ = 1 + \frac{\mu}{r - \mu_2} + \left[ \frac{k}{r} - \frac{\mu X^*}{r - \mu_2} \right] \phi^- \pi^G(X). \]

Since \( \pi^G(X) \to 0 \) if and only if \( X \to X^* \), it follows that
\[ \lim_{\pi^G(X) \to 0} \frac{dE(X)}{dC(X)} = 1 + \frac{\mu}{r - \mu_2} > 1, \]
because \( r > \mu_2 > 0 \) by assumption, thus proving part i).

On the other hand, since \( \pi^G(X) \to 1 \) if and only if \( X \to X^* \), it follows that
\[ \lim_{\pi^G(X) \to 1} \frac{dE(X)}{dC(X)} = 1 + \frac{\mu}{r - \mu_2} + \left[ \frac{k}{r} - \frac{\mu X^*}{r - \mu_2} \right] \phi^- \]
\[ = 1 + \frac{\mu}{r - \mu_2} = 1, \]
using the expression for \( X^* \) in (13), proving part ii). \( \square \)

A.6 Proof of Proposition 4

Using the expression in (3) for the equilibrium equity value, it follows by differentiating that
\[ \psi^E_i = \frac{X_i E(X_i)}{E(X_i)} \]
\[ = \frac{X_i}{E(X_i)} \left[ (1 - \tau) + \frac{1}{r - \mu_2} \frac{\mu_i}{\mu} \right] \]
\[ + (1 - \tau) \left[ \frac{k}{r} - \frac{\mu_i X^*}{r - \mu_2} \right] \phi^- \pi^G(X_i) \]
\[ = \frac{1}{E(X_i)} \left[ E(X_i) - C(X_i) + (1 - \tau)X_i + (1 - \tau) \frac{k}{r} \right. \]
\[ - (1 - \tau)(1 - \phi^-) \left[ \frac{k}{r} - \frac{\mu_i X^*}{r - \mu_2} \right] \pi^G(X_i) \right], \]
where the last equality follows by the expressions for 
\( E(X) \) and \( C(X) \) in (12) and (6). The form of \( \psi^E_i \) given in the proposition then follows by dividing \( E(X_i) \) into each term in the parentheses. \( \square \)

A.7 Proof of Corollary 4.1

The proofs of parts i) and ii) of the corollary will make use of the following results about the earnings sensitivity, \( \psi^E_i \).

First, using the expression for \( C(X) \) in (6), the sensitivity can be compactly written as
\[ \psi^E_i = 1 + \frac{(1 - \tau)X^* - (1 - \phi^-)M \pi^G(X_i)}{E(X_i)}, \]
(20)
where \( M = (1 - \tau) \left[ \frac{k}{r} - \frac{\mu X^*}{r - \mu_2} \right]. \)
Second, by differentiating the form of $\psi_t^E$ in (20) with respect to $X_t$, it follows that

$$
\frac{\partial \psi_t^E}{\partial X_t} = \frac{(1 - \phi^-) M \frac{x_\pi^X(X_t)}{X_t}}{E(X_t)} - \frac{E_X(X_t) \left[(1 - \tau)X^* - (1 - \phi^-) M \pi^X(X_t)\right]}{E^2(X_t)}
$$

where the last equality uses the definition $\psi_t^E = X_t \frac{E_X(X_t)}{E(X_t)}$, as well as (20). Hence, by the chain rule, the parametric derivative of $\psi_t^E$ with respect to $\pi^X(X_t)$ is given by

$$
\frac{d\psi_t^E}{d\pi^X(X_t)} = \frac{\partial \psi_t^E}{\partial X_t} \left(\frac{\partial \pi^X(X_t)}{\partial X_t}\right)^{-1}
= \frac{(1 - \phi^-) M \frac{x_\pi^X(X_t)}{X_t}}{E(X_t)} - \frac{\psi_t^E(\psi_t^E - 1)}{E(X_t)}.
$$

**Proof of Part i)**

Suppose $\pi^\square(X_t) \to 0$. Then $X_t \to \infty$, so the expression in (6) for $C(X_t)$ gives that

$$
\lim_{\pi^\square(X_t)\to 0} C(X_t) = (1 - \tau)X_t < (1 - \tau)\frac{\mu^\square X^*}{\mu^\square},
$$

since $r > \mu^\square > \frac{1}{4}r$ by assumption.

Next, since $\phi^- < 0$, Proposition 3 gives that as $X \to \infty$, $\pi^\square(X)$ approaches zero faster than $E(X)$ approaches infinity. Combined with the form of $\psi_t^E$ in (20), it thus follows that

$$
\lim_{\pi^\square(X_t)\to 0} \psi_t^E > 1.
$$

Finally, consider the expression in (21). As $\pi^\square(X_t) \to 0$, $E(X_t) \to \infty$, implying that the first term vanishes. By (22), the numerator of the second term is asymptotically positive, while $\phi^- < 0$ implies that the denominator is negative. The second term is therefore as a whole asymptotically positive, implying in total that

$$
\lim_{\pi^\square(X_t)\to 0} \frac{d\psi_t^E}{d\pi^\square(X_t)} > 0,
$$

which proves part i).

**Proof of Part ii)**

Suppose now that $\pi^\square(X_t) \to 1$. Then $X_t \to X^*$, so

$$
\lim_{\pi^\square(X_t)\to 1} C(X_t) = (1 - \tau)X_t > (1 - \tau)\frac{\mu^\square X^*}{\mu^\square},
$$

by the expression in (13) for $X^*$.

Next, the limited liability condition (11) and the expression for $C(X)$ in (6) give that

$$
\lim_{\pi^\square(X_t)\to 1} \psi_t^E = 1 + \frac{(1 - \tau)X^* - (1 - \phi^-) M C(X^*)}{C(X^*)}
= \frac{r - \mu^\square \phi^-}{\mu^\square} \phi^- - 1 < 1,
$$

because $\mu^\square > \frac{1}{4}r$ by assumption and $\frac{\phi^-}{\mu^\square} \phi^- \in (0, 1)$. Note that $\mu^\square > \frac{1}{4}r$ is a sufficient but not necessary condition for (23) to hold.

Finally, consider again the expression in (21). As $\pi^\square(X_t) \to 1$, $E(X_t) \to C(X^*)$ by the limited liability condition (11), which implies that the first term is asymptotically negative. By (23), the numerator of the second term will be asymptotically negative, while $\phi^- < 0$ implies that the denominator is negative. The second term is therefore as a whole asymptotically negative, implying in total that

$$
\lim_{\pi^\square(X_t)\to 1} \frac{d\psi_t^E}{d\pi^\square(X_t)} < 0,
$$

thus proving part ii) and completing the proof. \(\Box\)