When behavioral portfolio theory meets Markowitz theory

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Abstract
The Behavioral Portfolio Theory (BPT) developed by Shefrin and Statman is often confronted to the Markowitz’s Mean Variance Theory (MVT). Although the BPT optimal portfolio is theoretically not mean variance efficient, some recent studies show that under the assumption of normally distributed returns, MVT and some models incorporating features of BPT can generate similar asset allocations. In this paper, we compare the asset allocations generated by BPT and MVT without restrictions. Using US stock prices from the CRSP database for the 1995-2011 period, we empirically determine the BPT optimal portfolio. We show that the Shefrin and Statman’s optimal portfolio is MV efficient in more than 70% of cases. However, our results also indicates that MV investors will typically not select the BPT portfolio as this portfolio is always associated with a high return and an important level of risk. We show that the risk aversion coefficient of the BPT portfolio is up to 60 times smaller than the risk aversion degree of usual MV investors.

Keywords: Behavioral Portfolio Theory, Mean Variance Theory

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1. Introduction

The literature often sets Markowitz’s (1952) Mean Variance Theory (MVT hereafter) against the Behavioral Portfolio Theory (BPT hereafter) developed by Shefrin and Statman (2000). The theoretical prediction is that MVT and BPT should lead to different asset allocations. However, some recent studies (Levy and Levy, 2004; Das, Markowitz and Statman, 2004) indicate that when the asset returns are Gaussian, the asset allocations generated by MVT and BPT-like models show some similarities. These studies have in common that they assume normally distributed asset returns and that they only consider some appealing features of BPT. For instance the Das, Markowitz and Statman’s model use the global framework of BPT but do not integrate the transformation of objective probabilities into decision weights as suggested by Shefrin and Statman. In this paper, we compare the asset allocations generated by BPT and MVT without restrictions. To this end, we lift any assumption concerning the distribution of returns, we take into account all the features of BPT and we allow for short sales. To empirically determine the BPT optimal portfolio we develop a methodology to generate random portfolios of up to 80 stocks. Our algorithm, based on Rademacher’s (1937) integer partition, allows us to obtain a great diversity of portfolios with a significant level of diversification. Our results suggest that returns do not necessary need to be normally distributed in order for the asset allocations to coincide. We provide empirical evidence that the BPT optimal portfolio is mean-variance efficient in more than 70% of cases. However, our results also indicate that the BPT optimal portfolio belongs to a very specific part of the efficient frontier; it always lies on the upper right part of the frontier.

In MVT, the investor’s asset allocation results from a trade-off between expected return and variance. For a given level of expected return, the investor aims at minimizing the variance of her portfolio. In contrast with MVT, risk in BPT relates to the downside risk rather than to the variance of returns. The three core features of BPT are: (1) investors seek to secure a minimal level of final wealth; (2) investors consider their portfolio as a collection of subportfolios, each of them being optimal for a given mental account; (3) investors do not behave rationally as they exhibit optimistic and pessimistic behaviors. The first feature relates to Roy’s (1952) safety first criterion. Investors seek to avoid their final wealth to fall below a threshold. Therefore, they set an acceptable probability for their wealth to reach
this threshold. In BPT, this threshold does not relate to a subsistence level as in Roy (1952) but to a target value called the aspiration level (Lopes, 1987). This aspiration level is not unique. Investors’ portfolio can be seen as a collection of mental accounting subportfolios characterized by different aspiration levels. Finally, in BPT, investors do not behave rationally. Fear and hope drive their decisions (Lopes, 1987). Investors distort the objective probabilities and subjectively transform the cumulative distribution of outcomes in a systematic way (Tversky and Kahneman, 1992). Therefore, the BPT investor replaces objective probabilities by decision weights when she evaluates her portfolio expected return.

The question we address in this paper is whether BPT and MVT lead to differences in individual portfolio choices. More specifically, we want to locate the BPT optimal portfolio with respect to the MV frontier. In their seminal paper, Shefrin and Statman (2000) show that the BPT optimal portfolio is typically not MV efficient. However, when returns are normally distributed and when no short sales are allowed, the BPT optimal portfolio can be on the MV frontier. Recently, several papers attempt to demonstrate that some features of the BPT framework are consistent with MV framework. For instance, Levy and Levy (2004) show that while prospect theory findings are in contradiction with the foundations of MVT, the prospect theory and MV efficient sets can coincide. Das, Markowitz and Statman (2004) integrate appealing features of MVT and BPT into a new mental accounting (MA) framework. The authors assume a rational investor who divides her wealth among several mental accounts and who seeks to reach a threshold in each mental account. They demonstrate that the MA optimal portfolio always lies on the MV efficient frontier. Levy, De Giorgi and Hens (2012) show that the Security Market Line Theorem of the Capital Asset Pricing Model (CAPM) is intact in the Cumulative Prospect Theory (CPT) framework (Tversky and Kahneman, 1992). With regards to these studies, some features of BPT and MVT make the asset allocations to almost coincide. However, none of them take into consideration the whole framework of BPT. Levy and Levy (2004) and Levy, De Giorgi and Hens (2012) compare MVT with prospect theory and with cumulative prospect theory. They integrate, in their study, the behavioral aspects of BPT but do not take into consideration the safety first criteria which is a key attribute of BPT. In contrast, Das Markowitz and Statman (2004) take into consideration the mental accounting framework and the safety first criteria of BPT but they do not integrate the behavioral...
features of BPT where investors transform objective probabilities into decision weights. Moreover, all these studies lie on the same strong assumption of normally distributed stock returns. This assumption has been shown to be no realistic (Mandelbrot, 1963; Mantegna and Stanley, 1995). To the best of our knowledge, the only comparative analysis that has been carried out without any prior assumption concerning the return distribution is the empirical study realized by Hens and Mayor (2012). They show that when the asset returns are not normally distributed the asset allocation derived for CPT differs substantially from MV analysis. The Hens and Mayor’s empirical study has the advantage of relaxing the assumption of normally distributed returns but also lies on a major difficulty. Their data set is based on an empirical distribution realized with only 8 assets and 15 realizations. This small number of stocks in the portfolio does not permit a good level of diversification. The difference in the asset allocations between CPT and MVT could be explained by this limitation.

The aim of this paper is to compare BPT and MVT without restrictions and with a sufficient number of stocks in the portfolio. Our approach is to compute empirically the asset allocations generated by BPT and by MVT. We then compare whether these different allocations coincide. To this end, we use U.S. stock prices contained in the CRSP database to generate, by a bootstrap simulation, a universe of 100,000 possible asset allocations. To obtain a good level of portfolios diversification, we generate portfolios of up to 80 stocks. We consider two samples of portfolios: one sample with short sales constraints and one sample where short sales are allowed. We solve the Shefrin and Statman optimization program and determine, among these 100,000 portfolios, the BPT optimal portfolio. We establish that, in more than 70% of cases, the Shefrin and Statman’s portfolio is MV efficient.

BPT is developed on the foundations of CPT, but it does not incorporate all the features of this model. CPT is also characterized by the facts that: (1) Individuals maximize a value function based upon gains and losses rather than final wealth; (2) The aforementioned gains and losses are defined with respect to a given reference point; (3) The value function exhibits diminishing sensitivity and loss aversion. Our second analysis is to test whether the inclusion of these additional features when computing the BPT portfolio modifies the MV efficiency of the optimal portfolio. We transform the monetary outcomes of our sample of portfolios via the CPT value function and
determine a new optimal portfolio namely the $BPT_{CPT}$ optimal portfolio. We show that this new optimal portfolio leads to similar results in terms of MV efficiency.

Despite different foundations, we show the MVT and BPT lead to similar portfolios in the MV space. However, a closer look to the location of BPT and $BPT_{CPT}$ portfolios on the MV efficient frontier indicates that these portfolios always lie on the extreme North East part of the frontier. The BPT and $BPT_{CPT}$ portfolios are always characterized by an important risk and high return. We show that the risk aversion levels induced by BPT are incompatible with empirical observations. Therefore, even if the BPT optimal portfolio is located on the MV frontier, it will not be chosen by usual MVT investors.

The paper is structured as follows. Section 2 reviews the main characteristics of the Shefrin and Statman model. In section 3, we present the data and the methodology we use. Section 4 presents our empirical study. In section 5, we discuss the location the BPT and $BPT_{CPT}$ portfolios on the mean variance frontier. Section 6 concludes with a summary of our findings.

2. The model

The Behavioral Portfolio Theory (BPT) developed by Shefrin and Statman (2000) is drawn on Roy’s (1952) concept of safety first approach. This approach implies the investor’s portfolio risk not to be measured by the variance but rather by the probability of ruin. We say that ruin occurs when the investor’s final wealth $W$ falls under the subsistence level. The idea underlying Roy’s concept is that investors aim at minimizing this probability of ruin. Telser (1955) extends Roy’s concept by introducing the idea of an acceptable level for this probability of ruin. A portfolio is considered safe when the probability of ruin does not exceed a given level $\alpha$. In Telser’s (1955) model, investors care both about the expected return of the portfolio and about the probability of failure to reach the given subsistence level $s$. It

\begin{itemize}
\item[4] The concept of ruin corresponds to the failure to reach a given subsistence level.
\item[5] We note that in Telser, investors act as MVT investors since they care about the expected return of the portfolio and its risk. However, contrary to MVT, the risk of
follows that investors aim at maximizing their expected return while keeping the probability of ruin below a given level $\alpha$. Formally, the model writes

$$\max E(R) \ u.c. \ P(W < s) < \alpha,$$

where $R$ is the portfolio return, $s$ is the subsistence level, $\alpha$ is the acceptable probability of ruin and $W$ is the final wealth distribution.

BPT is drawn on Roy’s (1952) safety first approach but integrates also some features of behavioral economics and finance. It combines Lopes’ (1987) Security Potential and Aspiration theory (commonly denoted SP/A) with the mental accounting structure from Kahneman and Tversky’s (1979, 1992) prospect theory. In SP/A theory, the investor’s choice is driven by three factors, namely security ($S$), potential ($P$) and aspiration ($A$). The security factor and the potential factor relate to two emotional drivers, namely fear and hope. On the one hand, investors are driven by fear and wish to secure their wealth. On the other hand, they are willing to take risk to increase the potential gains. These two emotional drivers are expressed in the model through a modified probability distribution of outcomes. Fear (respectively hope) operates through an overweighting of the small probabilities associated to the worst (respectively best) outcomes. Thus, investors compute their expected wealth by applying an inverse S-Shape weighting function to the decumulative probability distribution of outcomes, and by substituting $E_\pi(W)$ to $E(W)$.

Lopes’ (1987) concept of aspiration generalizes the concept of subsistence level described above. Investors aim at reaching a specific level for their final wealth, denoted aspiration level or target value. The risk of the portfolio is therefore the probability of ending below this target value $A$. BPT incorporates the mental accounting structure of Kahneman and Tversky’s (1979) prospect theory. Investors have distinct mental accounts (e.g. education, retirement, bequest...) with different levels of aspiration. They do not consider their portfolio as a whole but rather as a collection of mental accounting subportfolios with distinct aspiration levels.

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the portfolio is not defined by the variance of the portfolio returns but is defined by the downside risk.
In BPT, investors maximize their expected wealth (calculated with decision weights) subject to the constraint that the probability of failure to reach the threshold level $A$ does not exceed a given level $\alpha$. This optimization program writes

$$\max E_{\pi}(W) \ u.c. \ p(W < A) < \alpha,$$

where $A$ is the aspiration level, $\alpha$ the maximum probability of ruin, $W$ is the final wealth distribution and $\pi$ a transformation function of probabilities. The literature propose several ways to transform probabilities. We choose to use the specification proposed by Tversky and Kahneman (1992) in cumulative prospect theory. This model is detailed in Appendix A.

In theory, the payoff of the BPT portfolio can be seen as the payoff of a portfolio combining bonds and a lottery ticket. This particular payoff results from the decision process of BPT investors; they do not allocate their wealth simply by solving a mean-variance optimization problem. Their portfolio can be viewed as a pyramid of assets where the riskless instruments are at the bottom and the riskier assets are on the top. BPT investors proceed in two steps to set their portfolio. First, they satisfy the safety first criteria at the cheapest price (concept of security) and then invest the remaining wealth in one Arrow Debreu security characterized by a high potential payoff (concept of potential).

3. Data and Methodology

3.1. Data

The primary dataset used in this article is the daily stock prices of 1,452 U.S. stocks contained in the CRSP database with a complete price history for the 1995-2011 period. We compute, using the stocks prices and the dividends paid by the firms, the monthly stocks returns\(^6\) on a daily frequency (rolling windows). The monthly return $R_{t,i}$ on stock $i$ for day $t$ is calculated as

$$R_{t,i} = \log(P_{t,i} + D_{t,i}) - \log(P_{t-20,i}),$$

\(^6\)We obtain 4,262 monthly returns.
where $P_{i,t}$ is the price of stock $i$ for day $t$ and $D_{i,t}$ is the dividend paid by the firm.

3.2. The bootstrap method

In our empirical analysis we consider that our investor can hold a portfolio composed of up to 80 stocks. For each simulation, we randomly select 80 stocks among the 1,452 in the database. We assume a single period economy and proceed in two steps.

We consider the matrix $R$ which contains the 4,262 monthly returns of the 80 randomly chosen stocks over the 1995-2011 period. $R$ is given by

$$R = \begin{bmatrix}
R_{1,1} & R_{1,2} & \ldots & R_{1,80} \\
R_{2,1} & \ldots & \ldots & \ldots \\
\vdots & \vdots & \ddots & \vdots \\
R_{4262,1} & \ldots & \ldots & R_{4262,80}
\end{bmatrix}, \quad (4)$$

where $R_{1,1}$ is the return on the first stock over the period starting the 3rd of January 1995 and ending the 31st of January 1995, $R_{2,2}$ is the monthly return of the second stock over the period starting the 4th of January 1995 and ending the 1st of February 1995, and so on...

We aim to generate a series of expected returns from the historical monthly returns. We use the bootstrap historical simulation method (Hull and White, 1998) to generate the possible scenarios. Our approach is the following. We randomly select a date $t$ (i.e. a row of $R$) among the 4,262 rows of the matrix $R_1$ which we call “date 0”. We then select the 250 monthly returns prior to date 0. We obtain a matrix $R^*$ defined as

$$R^* = \begin{bmatrix}
R_{t-250,1}^* & R_{t-250,2}^* & \ldots & R_{t-250,80}^* \\
R_{t-249,1}^* & \ldots & \ldots & \ldots \\
\vdots & \vdots & \ddots & \vdots \\
R_{t-1,1}^* & \ldots & \ldots & R_{t-1,80}^*
\end{bmatrix}. \quad (5)$$

7The 80 stocks considered vary for each simulation.
To model the first state of nature, i.e. the return obtained at the end of our single period, we randomly select a row of the matrix $R^*$. We repeat this process 1,000 times in order to obtain the 1,000 states of nature for our 80 stocks. Note that, by randomly selecting a row of the matrix $R^*$, we do not alter the structure of correlations between the different stocks. The matrix $\theta$ containing the 1,000 states of nature for the 80 stocks writes

$$\theta = \begin{bmatrix}
\theta_{1,1} & \theta_{1,2} & \ldots & \theta_{1,80} \\
\theta_{2,1} & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\theta_{1000,1} & \ldots & \ldots & \theta_{1000,80}
\end{bmatrix}, \quad (6)$$

where $\theta_{i,j}$ is the monthly return of stock $j$ if state of nature $i$ occurs.

### 3.3. Generation of portfolios

In practice, there is an infinity of different portfolios that the investor can choose to hold. However, for our study, we build a set of 100,000 portfolios. The investor will then choose one portfolio among the 100,000 possible choices. We also limit to 80 the maximum number of stocks that the investor can hold. This significant number of stocks in the portfolio permit a good level of portfolio diversification. We present below our methodology to generate the sample of 100,000 portfolios so that it results in the best possible approximation of the choice faced by the investor in reality. We consider two different situations. In the first situation, the investor cannot short stocks. In the second situation, short sales are possible.

#### 3.3.1. Portfolios without short sales

In order to obtain a good diversity between the different portfolios, we need to generate portfolio with different numbers of stocks and different weight distributions. Our approach is the following. We consider a portfolio of up to $n$ stocks. We assume that the weight associated to a given stock is equal to $k/n$ with $k = 0, 1, \ldots, n$. It follows that the most diversified portfolio is the $1/n$ portfolio while the least diversified portfolio is the one where the investor invests all her wealth in only one stock. Given the assumption that the weight associated to the different stocks is equal to $k/n$, we can model all the different possible portfolio compositions by considering all the possible
integer decompositions (without ranking) of the number \( n \). For example, for \( n = 4 \) there are 5 possible integer decompositions, which are

\[
\begin{align*}
I_1 &= \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}, & I_2 &= \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, & I_3 &= \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, & I_4 &= \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, & I_5 &= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.
\end{align*}
\]

(7)

If we want to randomly select a portfolio of up to four stocks, we select one combination among the five possible (see above) and then randomly reorder the vector of weights. In other words, we first select the structure of weights and then decide on which stocks to attribute the positive weights.

The total number of integer decompositions for \( n = 80 \) is equal to 15,796,476\(^8\) (see Rademacher, 1937). We then randomly select 100,000 decompositions among the 15,796,476 possible integer decompositions. We randomly reorder these vectors and transform them into weights by dividing each element by \( n = 80 \). For each simulation, we use the same 100,000 weight vectors (remember that the 80 stocks are different for each simulation). The average number of stocks in the portfolios is equal to 18.81 (median is 18) with a maximum of 70 stocks and a minimum of 3. We calculate the diversification of each portfolio by means of the Herfindahl index\(^9\). The mean Herfindahl index is equal to 0.1271 (median is 0.1163). The most (respectively least) diversified portfolio has a Herfindahl index of 0.0178 (respectively 0.7037).

3.3.2. Portfolios with short sales

We now consider the situation where our investor is allowed to short stocks. Our methodology to compute a portfolio when short sales are allowed is the following. We select at random three portfolios \( A \), \( B \) and \( C \)

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\(^8\)The algorithm used to generate the integer decompositions is available upon request.

\(^9\)The Herfindahl index of diversification is equal to

\[
HI = \sum_{i=1}^{n} (\omega_i)^2,
\]

where \( n \) is the number of stocks in the portfolio and \( \omega_i \) is the weight of stock \( i \).
following the methodology presented above. We obtain a portfolio with negative weights simply by considering the linear combination $A + B - C$. Indeed, the sum of the weights of $B - C$ is equal to 0. It follows that the sum of weights of the combination $A + B - C$ is equal to one. When allowing for short sales, the algorithm generates portfolios that contains more stocks than when short sales are constrained. The average number of stocks in the portfolios is equal to 42.87 (median is 43) with a maximum of 73 stocks and a minimum of 19.

3.4. The BPT optimal portfolio and the efficient frontier

Our aim in this section is to empirically locate the BPT optimal portfolio in the MV space. We estimate empirically the efficient frontier using the 100,000 portfolios we generated. The approach is the following. For each portfolio, we check if there exists another portfolio with a higher expected return and a lower variance. A portfolio is considered to be located on the efficient frontier if no portfolio exists in the sample with both a higher expected return and a lower variance. We denote by $S_{ef}$ the set of portfolios that are located on the efficient frontier.

The BPT optimal portfolio satisfies

$$\text{max } E_\pi(W) \text{ u.c. } p(W < A) < \alpha,$$

where $W$ is the final wealth distribution of the investor, $A$ is the aspiration level and $\alpha$ the acceptable probability of ruin.

We first determine the portfolios that satisfy the safety first constraint. A portfolio satisfies this constraint if its value at the end of the period is at least equal to the aspiration level $A$ in $1 - \alpha$ % of the states of nature. We denote $S^*$ the set of portfolios that satisfy the safety first constraint. By construction, $S^* \subset S$, $S$ being the set of 100,000 portfolios we generated. For illustration’s purposes, we plot, in Figure 1, the set $S$ composed of the 100,000 portfolios that we generated (black crosses) and the set $S^*$ composed of the portfolios that satisfy the safety first constraint (gray stars).

The BPT optimal portfolio is the portfolio of $S^*$ that maximizes $E_\pi(W)$. In order to compute the expected return with subjective decision weights,
we need to transform the objective probabilities (denoted \( p_i \)) into decision weights. The methodology for transforming the probabilities is the following. First, we rank, for each portfolio, the vector of states of nature \( \theta \) from the worst outcome to the best outcome (see equation 6). For portfolio \( i \), the vector of states of nature ordered from the worst outcome to the best outcome writes

\[
y_i = \begin{pmatrix} y_{i,1} \\ \vdots \\ \vdots \\ y_{i,1000} \end{pmatrix},
\]

(9)

where \( y_{i,k} \) is the value of portfolio \( i \) if the \( k^{th} \) state of nature is realized.

Second, we transform the objective probabilities into decision weights. We accomplish this transformation by applying the weighting function proposed by Tversky and Kahneman in CPT (1992). We assumed, in our bootstrap simulation, that each state of nature is equally likely. Thus, the vector \( p \) of objective probabilities writes

\[
p = \begin{pmatrix} p_1 = \frac{1}{1000} \\ \vdots \\ \vdots \\ p_{1000} = \frac{1}{1000} \end{pmatrix}.
\]

(10)

We define the vector of decision weights as

\[
\pi = \begin{pmatrix} \pi_1 \\ \vdots \\ \pi_k \\ \vdots \\ \pi_{1000} \end{pmatrix} = \left( \begin{array}{c} \pi_1 = w(\sum_{j=1}^{1000} p_j) - w(\sum_{j=2}^{1000} p_j) \\ \vdots \\ \pi_k = w(\sum_{j=i}^{1000} p_j) - w(\sum_{j=k+1}^{1000} p_j) \\ \vdots \\ \pi_{1000} = w(p_{1000}) \end{array} \right),
\]

(11)
where \( w \) is the weighting function\(^{10}\) proposed by Tversky and Kahneman (1992).

The BPT optimal portfolio is the portfolio of \( S^* \) that maximizes the inner product \( E_\pi(W) = y'\pi \). The final step is to check whether this portfolio is part of the set \( S^{ef} \) (the set that contains the portfolios located on the efficient frontier). We repeat this process 1,000 times in order to obtain a significant number of optimal portfolios.

3.5. Transformation of the monetary outcomes: the BPT\(_{CPT}\) portfolio

In BPT, Shefrin and Statman assume an investor that transforms probabilities into decision weights. But behavioral studies as CPT show that the distortion of objective probabilities is not the only irrational feature of investors\(^{11}\). The latter also make decision based on change of wealth rather than on total wealth and can exhibit risk seeking behavior regarding losses. Therefore, individuals determine the subjective value of each monetary outcome via a value function. Our aim is to check if the results would be the same if we assume a CPT investor who subjectively transforms monetary outcomes and objectives probabilities. We call BPT\(_{CPT}\) optimal portfolio the portfolio that satisfies

\[
\max E_\pi[v(W)] \text{ u.c. } p(W < A) < \alpha, \quad (12)
\]

where \( v \) is the CPT value function that transforms the monetary outcomes into utility\(^{12}\).

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\(^{10}\)The weighting function \( w \) writes

\[
w(p) = \frac{p^\gamma}{[p^\gamma + (1 - p)^\gamma]^{1/\gamma}}
\]

with \( \gamma = 0.61 \).

\(^{11}\)All the features of CPT are detailed in Appendix A.

\(^{12}\)The CPT value function is defined by

\[
v(y) = \begin{cases} (y - \kappa)^{0.88} & \text{if } y \geq \kappa \\ -2.25(-(y - \kappa))^{0.88} & \text{if } y < \kappa \end{cases}
\]
The function $v$ is defined relatively to a reference point denoted $\kappa$ that distinguishes between gains and losses. In this study, we assimilate the reference point $\kappa$ to the long-term risk-free rate (i.e., 10-year U.S. Treasury Bond). A stock return greater than the long-term risk-free rate is considered as a gain whereas a stock return inferior to the long-term risk-free rate is considered as a loss.

The methodology is the same as in the previous section to the difference that the \( \text{BPT}_{CPT} \) optimal portfolio is the portfolio of $S^*$ that maximizes the inner product $E_v[v(W)] = v'\pi$. For portfolio $i$, the vector of modified (ranked) outcomes is defined as

$$v_i = \begin{pmatrix} v(y_{i,1}) \\ \vdots \\ v(y_{i,n}) \end{pmatrix}. \quad (13)$$

We treat differently gains and losses in the value function. Therefore, we need to do the same in the weighting function. We define the modified vector of decision weights $\pi$ as

$$\pi = \begin{pmatrix} \pi_i^- \\ \pi_i^+ \\ \vdots \\ \pi_i^- \\ \pi_i^+ \\ \vdots \\ \pi_n^- \\ \pi_n^+ \end{pmatrix} = \begin{pmatrix} \pi_1^- = \pi^-(p_1) \\ \vdots \\ \pi_i^- = w^-(\sum_{j=1}^{i} p_j) - w^-(\sum_{j=2}^{i-1} p_j) \\ \vdots \\ \pi_k^+ = w^+(\sum_{j=k}^{n} p_j) - w^+(\sum_{j=k+1}^{n} p_j) \\ \vdots \\ \pi_n^+ = w^+(p_n) \end{pmatrix}, \quad (14)$$
where $w$ is the weighting function proposed by Tversky and Kahneman (1992).

The optimal portfolio is the one that maximizes the inner product $E_w[v(W)] = v\pi$. As in section 3.4, we realize this simulation 1,000 times in order to obtain a significant number of optimal portfolios.

4. Empirical analysis

We consider here two samples of 100,000 portfolios, one without short sales and one where short sales are allowed. Each investor sets a safety first constraint by specifying the return on a portfolio which should not fall below a level $A$ with more than $\alpha$ probability. Because the safety first constraint is not necessarily the same for each investor or each mental account, we consider several configurations for $\alpha$ and $A$. The aspiration level $A$ is given by the initial wealth capitalized at a rate $r$. We consider five different specifications for $r$: (1) the short-term risk free rate (i.e. 3-month Treasury Bill); (2) the long-term risk-free rate (i.e. 10-year U.S. Treasury Bond); (3) an annualized rate of 1%; (4) an annualized rate of 5%; and, (5) an annualized rate of 10%. The different values for alpha are 0.1, 0.2 and 0.3. For instance, when we set $r = 1\%$ and $\alpha = 0.1$, that means that the investor stipulates that she does not want the probability of failing to reach $A = W_0e^{1+r}$ to exceed $\alpha = 0.1^{14}$. We have a total of fifteen different specifications.

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13The weighting function $w$ writes

$$w^+(p) = \frac{p^{\gamma^+}}{[p^{\gamma^+} + (1 - p)^{\gamma^+}]^{1/\gamma^+}}$$

$$w^-(p) = \frac{p^{\gamma^-}}{[p^{\gamma^-} + (1 - p)^{\gamma^-}]^{1/\gamma^-}}$$

where $\gamma^+ = 0.61$ and $\gamma^- = 0.69$.

14For simplicity’s sake, we will write, in the rest of the paper, $A = 1\%$ when we set the rate $r$ equal to 1\%.
4.1. Non-gaussian portfolios returns

For each simulation, we run both a Jarque-Bera test and a Kolmogorov–Smirnov test to check whether the portfolio returns are Gaussian. The Jarque-Bera test indicates that, for each simulation, about 90,000 portfolios out of 100,000 present non-gaussian returns (at a 5% significance level). When we use the Kolmogorov–Smirnov test, all the portfolios have non-gaussian returns.

4.2. Safety first constraint

For each simulation, we check the number of portfolios (among the 100,000 portfolios) that meet the safety first constraint. Figure 2 indicates, for different parameterizations of alpha and different aspiration levels, the number of simulations with at least one portfolio meeting the safety first constraint. The number of simulations with at least one portfolio meeting the safety first constraint increases with alpha and decreases with the level of the aspiration level. With alpha equal to 0.1 and the aspiration level set as short term risk-free rate, at least one portfolio meets the constraint in 30.8% of the cases. When choosing alpha equal to 0.3 and the same aspiration level, this proportion increases to 91.7%. This result seems quite natural since the expectation of investors decreases with $\alpha$. When $\alpha$ decreases, the investor wants to secure more states of nature. Conversely, if $\alpha$ increases, the investor wants to secure less states of nature. Therefore, it is more likely for a portfolio to satisfy the safety first constraint when $\alpha$ is high. We consider, for our empirical analysis, only the simulations in which at least one portfolio meets the safety first constraint.

4.3. Optimal portfolios and the efficient frontier

Table 1 indicates the proportion of simulations for which the BPT optimal portfolio is located on the efficient frontier. We observe that the BPT optimal portfolio is located on the efficient frontier in nearly three out of four simulations. This proportion is relatively robust to changes in alpha or changes in the aspiration level. The introduction of short sales do not appear to modify the results.
Table 2 provides the proportion of simulations for which the BPT\textsubscript{CPT} optimal portfolio is located on the efficient frontier. The results are similar to the one of the previous subsection (when studying the BPT optimal portfolio). We observe that the BPT\textsubscript{CPT} optimal portfolio is located on the efficient frontier in three out of four simulations. This proportion is relatively robust to changes in alpha or changes in the aspiration level. Here as well, the introduction of short selling does not modify the results.

4.4. Multi account version

In BPT, Shefrin and Statman assume that investors behave in a compartmentalized manner where their portfolios are viewed as a collection of mental accounting subportfolios. Investors act as if they had different risk preferences in each mental account. Each subportfolio is associated with a goal and each goal is represented by a threshold level. Similarly, the probability of failing to reach the threshold level varies from an account to another. In this section, we consider that investors divide their aggregate portfolio into three subportfolios. The first one is devoted to retirement, the second one to education and the last one to bequest. Investors are risk averse in the retirement subportfolio, moderately risk averse in the education subportfolio and very little risk averse in the bequest subportfolio. We therefore set different aspiration levels and different levels of alpha in the three subportfolios. In the first one (respectively second and third one), we set $A = 1\%$ (respectively $A = $long-term risk-free rate and $A = 10\%$) and $\alpha = 0.1$ (respectively $\alpha = 0.2$ and $\alpha = 0.3$). The aggregate portfolio is composed at 60\% of the retirement subportfolio, 20\% of the education subportfolio and 20\% of the bequest subportfolio. We obtain that, when short sales are constrained, the BPT (respectively BPT\textsubscript{CPT}) aggregate portfolio is located on the MV frontier in 67.8\% (respectively 70.9\%) of the cases. When short sales are allowed, the BPT (respectively BPT\textsubscript{CPT}) aggregate portfolio is MV efficient in 803 (respectively 825) out of 1,000 simulations.
5. Characteristics of the BPT portfolios

In this section, we investigate the location of the BPT and BPT\textsubscript{CPT} optimal portfolios on the MV space. First, we locate the BPT optimal portfolio relative to the BPT\textsubscript{CPT} optimal portfolio. We wonder whether these portfolios coincide or whether one of them is always riskier than the other. Second, we take a closer look at the location of the BPT and BPT\textsubscript{CPT} optimal portfolios on the MV efficient frontier.

5.1. Location of BPT portfolio relative to BPT\textsubscript{CPT} portfolio

Table 3 displays the proportion of simulations for which BPT and BPT\textsubscript{CPT} optimal portfolios coincide. For instance, when short sales are allowed, when the aspiration level corresponds to the long-term risk-free rate and when $\alpha = 0.2$, BPT and BPT\textsubscript{CPT} optimal portfolios coincide in 67% of the cases. In the remaining 33%, the BPT optimal portfolio has a higher variance than the BPT\textsubscript{CPT} optimal portfolio (in more than 99% of the cases).\footnote{This result is robust for all the other specifications. The BPT optimal portfolio is located at the right side (North East) of BPT\textsubscript{CPT} portfolio in more than 98% of cases for the 15 specifications.} The BPT optimal portfolio appears to be riskier than the BPT\textsubscript{CPT} optimal portfolio. An explanation for this result is that BPT\textsubscript{CPT} investors transform monetary outcomes into utility via a value function $v$. This function $v$ is defined to integrate the behavioral observations of Tversky and Kahneman (1992). First, $v$ is concave over gains and convex over losses. That means that BPT\textsubscript{CPT} investors are characterized by a risk adverse behavior for most gains (gains associated to moderate and high probabilities). Second, $v$ is steeper for losses than for gains in order to account for the fact that investors have an asymmetric perception of gains and losses. Experimental evidence (Erev, Ert and Yechiam, 2008; Tversky and Kahneman, 1991) shows that individuals are loss averse; the loss of a given amount of money creates a distress greater than the satisfaction generated by the gain of the same amount. Losses are weighted about twice as strongly as gains. Thus, BPT\textsubscript{CPT} investors are characterized by a risk adverse behavior for most gains and a strong loss aversion. It seems natural, therefore, that these agents select less risky portfolios than BPT investors.
We observe, in Table 3, that the percentage of cases for which BPT and BPT\textsubscript{CPT} optimal portfolios coincide, decreases with the value of $\alpha$. When the aspiration level corresponds to the long-term risk-free rate and when $\alpha$ is equal to 0.3, BPT and BPT\textsubscript{CPT} portfolios coincide in only 480 simulations out of 1,000. This result is not surprising as, when $\alpha$ is small, less portfolios meet the safety first constraint.

5.2. Location of the portfolios on the efficient frontier

For all the different 15 specifications, the BPT and BPT\textsubscript{CPT} portfolios always exhibit a high expected return and an important level of risk. On average, the expected return of the BPT portfolio is on average 10 times greater than the return of the S&P 500 when short sales are allowed, and 5 times greater without short sales\textsuperscript{16}. We notice that the couple expected return / risk of the BPT portfolio is significantly higher when there are no short selling constraints. This difference is due to the way BPT investors select their portfolio. First, they secure their wealth with respect to their aspiration level. Second, they bet in few states of nature in order to meet their expectation of growing rich in a sizable way. In this second phase, BPT investors are willing to take more risks in order to have the chance to win a significant amount of money. The possibility of short-selling enhances this risk taking behavior.

Most of BPT and BPT\textsubscript{CPT} optimal portfolios are MV efficient but they are also very risky in terms of variance. Therefore, they are always located on the extreme upper right part of the efficient frontier. However, even if they belong to a very specific part of the frontier, they are still MV efficient. Thus, they coincide with portfolios that can theoretically be chosen by some Markowitz investors. In the canonical MVT problem, investors minimize the objective function $(1/2)\text{Var}(R)$ subject to the constraint $E[R] = e$ where $e$ stands for the expected return set by the agent. Each level of $e$ corresponds to a portfolio on the MV frontier. The greater $e$, the riskier is the portfolio.

\textsuperscript{16}This result for the BPT and BPT\textsubscript{CPT} optimal portfolios is robust to changes in the aspiration level and to changes in the level of alpha.
As the BPT optimal portfolio is located on the upper right part of the MV frontier, the MV investor whose optimal portfolio coincides with the BPT portfolio would deliberately choose to set a high level for \( e \) in her optimization program.

Another way to solve the MVT problem is to maximize \( E(R) - (\gamma/2)V(R) \) with different level of \( \gamma \), where \( \gamma \) stands for the risk aversion coefficient. Each level of \( \gamma \) corresponds to a portfolio on the MV frontier. The less risk averse the investor, the smaller is her risk aversion coefficient, and the greater is the expected return of her optimal portfolio. As the BPT optimal portfolio is associated with a significant couple expected return/risk, only a MV investor characterized by a small risk aversion coefficient will choose this kind of portfolios. In this section, we investigate the level of the risk aversion coefficient that corresponds to the location of the BPT optimal portfolio on the MV efficient frontier. We remind that a MV investor maximizes

\[
X'\overline{R} - (\gamma/2)X'VX \text{ u.c. } X'1 = 1,
\]

where \( X \) is the vector of portfolio weights of the 80 stocks, \( \overline{R} \) is the vector of the 80 expected returns, \( V \) is the return covariance, and \( 1 = [1, 1, \ldots, 1]' \).

The closed form solution of this optimization problem is

\[
X^* = \frac{1}{\gamma}V^{-1} \left[ \overline{R} - \left( \frac{1'V^{-1}\overline{R} - \gamma}{1'V^{-1}1} \right) 1 \right]. \tag{16}
\]

\( X^* \) gives the composition of the efficient portfolio for a given level of \( \gamma \). Furthermore, we know that \( X^* \) is also the well-known solution to the canonical MVT problem where investors minimize the objective function \((1/2)X'VX\) under the constraints \( X'\overline{R} = e \) and \( X'1 = 1 \). Therefore, \( X^* \) can also be written as (see Huang and Litzenberger (1988))

\[
X^* = X_1 + e \times X_2, \tag{17}
\]

\( X^* \) can also be written as (see Huang and Litzenberger (1988))

\[
X^* = X_1 + e \times X_2, \tag{17}
\]

\[
A = 1'V^{-1}\overline{R}, \quad B = 1'V^{-1}\overline{R}
\]
\[
C = 1'V^{-1}1, \quad D = BC - A^2
\]
\[
X_1 = \frac{1}{D} \left[ BV^{-1} \mathbf{1} - AV^{-1} \mathbf{R} \right]
\]
\[
X_2 = \frac{1}{D} \left[ CV^{-1} \mathbf{R} - AV^{-1} \mathbf{1} \right]
\]

(18)

Using equation 17, we have
\[
X_1 + e \times X_2 = \frac{1}{\gamma} \left[ V^{-1} \mathbf{R} - \frac{A}{C} \times V^{-1} \mathbf{1} \right] + \frac{V^{-1} \mathbf{1}}{C}.
\]
\[\Rightarrow e \times X_2 = \frac{1}{\gamma} \left[ V^{-1} \mathbf{R} - \frac{A}{C} \times V^{-1} \mathbf{1} \right] + \frac{V^{-1} \mathbf{1}}{C} - X_1.
\]

(19)

It is then possible to show (see proof in Appendix B) that
\[
e \times X_2 = \left( \frac{1}{\gamma} + \frac{A}{D} \right) \left( V^{-1} \mathbf{R} - \frac{A}{C} \times V^{-1} \mathbf{1} \right).
\]

(20)

Therefore, the expected return of the portfolio \( e \) and the risk aversion coefficient \( \gamma \) are linked by the following hyperbolic relation
\[
e = \frac{D}{C} \left( \frac{1}{\gamma} + \frac{A}{D} \right) \Leftrightarrow \gamma = \frac{D}{e \times C - A}.
\]

(21)

Our aim here is to discuss the value of the risk aversion coefficient associated with the expected return of the BPT and BPT\(_{CPT} \) portfolios (\( \gamma_{BPT} \) and \( \gamma_{BPTCPT} \)). We compute the relative value of \( \gamma_{BPT} \) and \( \gamma_{BPTCPT} \) by comparing \( \gamma_{BPT} \) and \( \gamma_{BPTCPT} \) to the risk aversion coefficient of usual MV portfolios. Our goal is to determine how much less risk averse are the BPT investors relative to usual MV investors. To this end, we consider three categories of MV portfolios: (1) the minimum-variance (min-v) portfolio (i.e.
the portfolio that minimizes the variance of returns); (2) a set of 4 efficient portfolios with expected return equal to the expected return of the min-v portfolio +2%,+5%,+10% and +20%; and, (3) the optimal portfolio of an investor that seeks to reach the return on the S&P 500$^{18}$, and compute their coefficient $\gamma$ ($\gamma_{\text{minv}}$). Table 4 and 5 displays the six ratios $\gamma_{\text{MV}}$ on $\gamma_{\text{BPT}}$ and $\gamma_{\text{CPT}}$. These ratios indicate how many times less risk averse is the MV investor who chooses the BPT portfolio relative usual MV investors. For instance when short-sales are allowed and alpha = 0.2, the MV investor who choose to invest in the BPT portfolio is 60 times less risk averse than a MV investor who choose an efficient portfolio associated with the S&P 500 expected return.

[INSERT TABLE 4 ABOUT HERE]

[INSERT TABLE 5 ABOUT HERE]

For all the specifications tested, the ratio $\gamma$ is significant. The MVT investor who decides to invest in the BPT portfolio is less risk averse than usual MV investors. This result is increased when short-sales are allowed. Thus, even if the BPT optimal portfolio is mean variance efficient, it will not be chosen by usual MV investors. Therefore, the coincidence of the BPT optimal portfolio and the MVT efficient set does not mean that these two theories lead to the same asset allocation.

6. Conclusion

This study aims at empirically selecting the optimal portfolio of the Behavioral Portfolio Theory (BPT) developed by Shefrin and Statman (2000). We compare the BPT portfolio to the portfolio chosen by a Markowitz investor. To run the simulations, we use U.S. stock prices contained in the CRSP database for the 1995-2011 period. We show that in 70% cases the BPT optimal portfolio belongs to the MV efficient frontier. Some recent studies (Levy and Levy (2004); Das, Markowitz and Statman (2004); Levy, De

$^{18}$For several dates of our sample, the expected return of the S&P 500 is too small to obtain a portfolio located on the efficient frontier. We exclude these dates from the calculation of $\gamma_{\text{S&P}}$.
Giorgi and Hens (2012)) also underline the coincidence of MVT and BPT-like models. The new contribution of our study lies on the fact that we compare the asset allocations generated by BPT and MVT without restrictions. We do not set any assumption on the distribution of returns, we allow short sales and we take into consideration all the features of BPT.

However, we provide empirical evidence that the BPT portfolio is always located on the upper right part of the mean variance frontier. It is thus characterized by a significant level of risk. Therefore, even if the BPT optimal portfolio belongs to the MV frontier, it will not be chosen by usual MV investors since it is associated with an extremely low degree of risk aversion. The coincidence of these two models does not imply that it does not matter for portfolio choice.

References


Appendix A. Cumulative Prospect Theory

The cumulative prospect theory (CPT) is based on four main observations.
(1) Investors use decision weights instead of probabilities and they overweight probabilities of extreme events.
(2) As in expected utility theory, investors determine the subjective value of each outcome via a value function. But, under CPT, utility is derived from changes in wealth, relative to a reference point with respect to which gains and losses are defined.
(3) The sensitivity with respect to the reference point mentioned in (2) is decreasing and individuals are loss averse.
(4) Experimental evidence has established a fourfold pattern of risk attitudes: risk aversion for most gains and low probability losses, and risk seeking for most losses and low probability gains.

Under CPT, a prospect $X = ((x_i, p_i), i = -m, \ldots, n)$ is evaluated through a valuation function. This function is defined as follows

$$V(X) = V(X^+) + V(X^-),$$

where $X^+ = \max(X; 0)$ and $X^- = \min(X; 0)$.

We set

$$V(X^+) \sum_{i=0}^{n-1} \left( w^+ \left( \sum_{j=i}^{n} p_j \right) - w^+ \left( \sum_{j=i+1}^{n} p_j \right) \right) v(x_i) + w^+(p_n)v(x_n)$$

$$\ldots$$

$$V(X^-) \sum_{i=-m+1}^{0} \left( w^- \left( \sum_{j=-m}^{i} p_j \right) - w^- \left( \sum_{j=-m}^{i-1} p_j \right) \right) v(x_i) + w^-(p_m)v(x_m)$$

where $v$ is a strictly increasing value function defined with respect to a reference point $x_0$ satisfying $v(x_0) = v(0) = 0$, and with $w^+(0) = 0 = w^-(0)$ and $w^+(1) = 1 = w^-(1)$. 

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The functional form for the value function $v$, proposed by Tversky and Kahneman (1992), is given by

$$v(x) = \begin{cases} x^\alpha & \text{if } x > 0 \\ -\lambda(-x)^\beta & \text{if } x < 0 \end{cases}. \quad (A.3)$$

For $0 < \alpha < 1$ and $0 < \beta < 1$ the value function $v$ is concave over gains and convex over losses. The parameter $\lambda$ determines the degree of loss aversion (Köbberling and Wakker, 2005). Based on experimental evidence, Tversky and Kahneman (1992) estimated the values of the parameters $\alpha$, $\beta$ and $\lambda$. They found $\alpha = \beta = 0.88$ and $\lambda = 2.25$.

Tversky and Kahneman proposed the following functional form for the weighting function $w$

$$w^+(p) = \frac{p^{\gamma+}}{[p^{\gamma+} + (1 - p)^{\gamma+}]^{1/\gamma+}} \quad (A.4)$$

$$w^-(p) = \frac{p^{\gamma-}}{[p^{\gamma-} + (1 - p)^{\gamma-}]^{1/\gamma-}}. \quad (A.5)$$

For $\gamma < 1$, this form integrates the overweighting of low probabilities and the greater sensitivity to changes in probabilities for extremely low and extremely high probabilities. The weighting function is concave near 0 and convex near 1. Tversky and Kahneman (1992) estimated the parameters $\gamma^+$ and $\gamma^-$ as 0.61 and 0.69 respectively.

**Appendix B. Proof of equation 20**

Equation 20 gives

$$e \times X_2 = \frac{1}{\gamma} \left[ V^{-1} \bar{R} - \frac{A}{C} \times V^{-1} \mathbf{1} \right] + \frac{V^{-1} \mathbf{1}}{C} - X_1. $$

Since $\frac{V^{-1} \mathbf{1}}{C}$ is independent from $e$ and $\gamma$, $\frac{V^{-1} \mathbf{1}}{C} - X_1$ is proportional to $V^{-1} \bar{R} - \frac{A}{C} \times V^{-1} \mathbf{1}$. 

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Proof. We have

\[
\frac{V^{-1}1}{C} - X_1 = \frac{V^{-1}1}{C} - \frac{1}{D} [BV^{-1}1 - AV^{-1}R]
\]

\[
= V^{-1}1 \left[ \frac{1}{C} - \frac{B}{D} \right] + \frac{A}{D} V^{-1}R
\]

\[
= V^{-1}1 \left[ \frac{D - BC}{CD} \right] + \frac{A}{D} V^{-1}R
\]

\[
= AV^{-1}1 \frac{A^2}{CD} + \frac{A}{D} V^{-1}R
\]

\[
= AV^{-1}1 \left[ V^{-1}R - \frac{A}{C} V^{-1}1 \right]
\]

It follows that equation 20 can be written as

\[
e \times X_2 = \left( \frac{1}{\gamma} + \frac{A}{D} \right) \left( V^{-1}R - \frac{A}{C} \times V^{-1}1 \right)
\].
Figure 1
Illustration: portfolios satisfying the safety first constraint

Figure 2
Proportion of simulations with at least one portfolio meeting the safety first constraint
### Table 1
Proportion of simulations for which the BPT optimal portfolio is MV efficient

<table>
<thead>
<tr>
<th>Aspiration level</th>
<th>Panel A: Short sales forbidden</th>
<th>Panel B: Short sales allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_{ST}$</td>
<td>$r_{LT}$</td>
</tr>
<tr>
<td>$\alpha = 0.1$</td>
<td>0.70</td>
<td>0.73</td>
</tr>
<tr>
<td>$\alpha = 0.2$</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>$\alpha = 0.3$</td>
<td>0.70</td>
<td>0.70</td>
</tr>
</tbody>
</table>

*Notes.* This table provides the proportion of simulations for which the BPT optimal portfolio is MV efficient. The probability of failure to reach the aspiration level (*i.e.* $\alpha$) takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate ($r_{ST}$); (2) the long-term risk-free rate ($r_{LT}$); (3) an annualized rate of 1% ($r_{1\%}$); (4) an annualized rate of 5% ($r_{5\%}$); and, (5) an annualized rate of 10% ($r_{10\%}$). The total number of simulations is 1,000. For each simulations, the BPT optimal portfolio is selected among a sample of 100,000 portfolios.

### Table 2
Proportion of simulations for which the $BPT_{CPT}$ optimal portfolio is MV efficient

<table>
<thead>
<tr>
<th>Aspiration level</th>
<th>Panel A: Short sales forbidden</th>
<th>Panel B: Short sales allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_{ST}$</td>
<td>$r_{LT}$</td>
</tr>
<tr>
<td>$\alpha = 0.1$</td>
<td>0.74</td>
<td>0.74</td>
</tr>
<tr>
<td>$\alpha = 0.2$</td>
<td>0.78</td>
<td>0.77</td>
</tr>
<tr>
<td>$\alpha = 0.3$</td>
<td>0.76</td>
<td>0.76</td>
</tr>
</tbody>
</table>

*Notes.* This table provides the proportion of simulations for which the $BPT_{CPT}$ optimal portfolio is MV efficient. The probability of failure to reach the aspiration level (*i.e.* $\alpha$) takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate ($r_{ST}$); (2) the long-term risk-free rate ($r_{LT}$); (3) an annualized rate of 1% ($r_{1\%}$); (4) an annualized rate of 5% ($r_{5\%}$); and, (5) an annualized rate of 10% ($r_{10\%}$). The total number of simulations is 1,000. For each simulations, the $BPT_{CPT}$ optimal portfolio is selected among a sample of 100,000 portfolios.
Table 3
Proportion of simulations for which the BPT optimal portfolio is the same portfolio as the BPT_{CPT} optimal portfolio

<table>
<thead>
<tr>
<th>Aspiration level</th>
<th>Panel A: Short sales forbidden</th>
<th>Panel B: Short sales allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r_{ST} )</td>
<td>( r_{LT} )</td>
</tr>
<tr>
<td>( \alpha = 0.1 )</td>
<td>0.87</td>
<td>0.89</td>
</tr>
<tr>
<td>( \alpha = 0.2 )</td>
<td>0.68</td>
<td>0.70</td>
</tr>
<tr>
<td>( \alpha = 0.3 )</td>
<td>0.49</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Notes. This table provides for which the BPT optimal portfolio is the same portfolio as the BPT_{CPT} optimal portfolio. The probability of failure to reach the aspiration level (i.e. alpha) takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (\( r_{ST} \)); (2) the long-term risk-free rate (\( r_{LT} \)); (3) an annualized rate of 1% (\( r_{1\%} \)); (4) an annualized rate of 5% (\( r_{5\%} \)); and, (5) an annualized rate of 10% (\( r_{10\%} \)). The total number of simulations is 1,000. For each simulations, the BPT and the BPT_{CPT} optimal portfolios are selected among a sample of 100,000 portfolios.
Table 4

**BPT optimal portfolio: risk aversion coefficient**

<table>
<thead>
<tr>
<th>Panel A: Short sales forbidden</th>
<th>Aspiration level = γ_{100}</th>
<th>Panel B: Short sales allowed</th>
<th>Aspiration level = γ_{100}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>γ_{1}</td>
<td>γ_{2}</td>
<td>γ_{3}</td>
</tr>
<tr>
<td>Aspiration level = γ_{100}</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>a = 0.1</td>
<td>70.02</td>
<td>28.24</td>
<td>14.91</td>
</tr>
<tr>
<td>a = 0.5</td>
<td>60.44</td>
<td>23.98</td>
<td>12.76</td>
</tr>
</tbody>
</table>

Notes: This table compares the risk aversion coefficient of the BPT optimal portfolio to the risk aversion coefficients of other portfolios such as the minimum variance portfolio or the S&P500 portfolio. The probability of failure to reach the aspiration level (i.e. alpha) takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) an annualized rate of 1% (r_{1%}); (4) an annualized rate of 5% (r_{5%}); and, (5) an annualized rate of 10% (r_{10%}). γ_{BPT} is the risk aversion coefficient of the BPT optimal portfolio. γ_{mv} is the risk aversion coefficient of the minimum variance portfolio. γ_{mv+x\%} corresponds to the risk aversion coefficient of the MV efficient portfolio which expected return is equal to the return of the minimum variance portfolio plus an annualized return of x%. γ_{S&P} represents the risk aversion coefficient of the MV efficient portfolio which expected return is equal to the return of the S&P500.
Table 5

**BPT\textsubscript{CPT} optimal portfolio: risk aversion coefficient**

<table>
<thead>
<tr>
<th>Panel &amp; short sale forbidden</th>
<th>Aspiration level</th>
<th>Panel &amp; short sale allowed</th>
<th>Aspiration level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>α</td>
<td></td>
<td>α</td>
</tr>
<tr>
<td>(1)</td>
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<td>3.68</td>
<td>3.65</td>
</tr>
<tr>
<td>(2)</td>
<td>3.21</td>
<td>3.23</td>
<td>3.21</td>
</tr>
<tr>
<td>(3)</td>
<td>2.98</td>
<td>3.00</td>
<td>2.98</td>
</tr>
</tbody>
</table>

Notes. This table compares the risk aversion coefficient of the BPT\textsubscript{CPT} optimal portfolio to the risk aversion coefficients of other portfolios such as the minimum variance portfolio or the S&P portfolio. The probability of failure to reach the aspiration level (i.e., alpha) takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (\(r_{ST}\)); (2) the long-term risk-free rate (\(r_{LT}\)); (3) an annualized rate of 1% (\(r_{1\%}\)); (4) an annualized rate of 5% (\(r_{5\%}\)); and, (5) an annualized rate of 10% (\(r_{10\%}\)). \(\gamma_{BPT\textsubscript{CPT}}\) is the risk aversion coefficient of the BPT\textsubscript{CPT} optimal portfolio. \(\gamma_{MV}\) is the risk aversion coefficient of the minimum variance portfolio. \(\gamma_{MV+x\%}\) corresponds to the risk aversion coefficient of the MV efficient portfolio which expected return is equal to the return of the minimum variance portfolio plus an annualized return of \(x\%\). \(\gamma_{SP500}\) represents the risk aversion coefficient of the MV efficient portfolio which expected return is equal to the return of the S&P500.