I document that financial crises coincide with the sudden reversal of a long period of low aggregate volatility. I argue that shocks to the volatility of total factor productivity are a source of financial instability, and account for both the building-up of risk and the burst of financial crises. I develop a DSGE model with an occasionally binding collateral constraint and a frictional housing market which determines the liquidity of the collateral. In this environment, volatility shocks affect the frequency of fire sales by changing collateral liquidity. In a quantitative exercise, I find that the interaction of time-varying volatility and search frictions increases both the frequency financial crises and the corresponding fall in GDP by 62% and 71%, respectively. Moreover, in the model the liquidity of the collateral endogenizes agents’ loan-to-value ratios. Volatility shocks generate sizable time variations in the loan-to-value ratios, providing a foundation of financial shocks.

Key Words: Collateral Liquidity, Housing Market, Search Frictions, Occasionally Binding Borrowing Constraint, Non-Linear Dynamics.
1 Introduction

Can the volatility of macroeconomic fluctuations affect the frequency of financial crises? The financial instability hypothesis of Minsky (1992) conjectures that long periods of low aggregate fluctuations can generate a crisis ex-post, a phenomenon that Brunnermeier and Sannikov (2013) refer to as the volatility paradox.

In this paper I document a new stylized fact which is consistent with the volatility paradox. I build a data set of 30 financial crises across 20 developed countries and find that financial crises are associated with the sudden reversal of a prolonged period of low volatility of approximately 4 years. I argue that shocks to the volatility of aggregate total factor productivity are a source of financial instability, and account for both the building up of risk and the burst of financial crises. I develop a dynamic general equilibrium model with a collateral constraint and search frictions in the housing market. The frictional market determines the liquidity of housing, which serves as the collateral assets. In this environment, volatility shocks affect the frequency of fire sales scenarios by changing the liquidity of the collateral. Periods of low fluctuations foster the investment in real estate, heating up the housing market. The high liquidity of housing relaxes households’ borrowing capacity, generating a credit and an investment boom which reinforce each other. This spiral builds up systemic risk because the economy becomes fragile to the realizations of adverse shocks at high levels of households’ leverage. Indeed, the dynamics of the model is inherently non-linear. At low levels of leverage, negative shocks generate only mild recessions in output and credit. Instead, when households are highly indebted, a sudden peak in volatility can dry up the liquidity of the housing market, turning the investment and credit boom into a financial crisis. In this perspective, the roots of the Great Recession have to be traced back to the period of Great Moderation that the U.S. have experienced from the mid 1980’s. In a quantitative exercise,
I feed into the model the stochastic volatility of the Solow residual of the U.S. economy as estimated by Bayesian techniques. I solve the model using global methods and find that the interaction of search friction and time varying volatility increases the frequency of financial crises and the corresponding fall in GDP by 62% and 71%, respectively. When agents can instantaneously trade houses and the liquidity channel is shut down, the role of time-varying volatility becomes negligible.

In the model, volatility shocks are propagated into the real economy through the collateral constraint and the frictional housing market. On one hand, households’ borrowing capacity is limited by an occasionally binding borrowing constraint. When the constraint does not bind, volatility shocks generate mild business cycle fluctuations. When the constraint binds, households are forced to deleverage to sharply reduce their debt. As a result, households fire sale and originate a financial crisis, that is, a severe contraction of both credit and output. On the other hand, the housing market is intermediated by real estate agencies, which act as brokers by purchasing houses on a Walrasian market and selling them on a frictional market. Search frictions in the housing market endogenously determine the liquidity of the housing stock. Houses can be quickly sold to a buyer only in hot markets, when many households are looking for a real estate property. The frictional housing market creates a direct link between the liquidity of the collateral and households’ borrowing capacity. Indeed, a house has a high collateral value if lenders can sell it both quickly and at a high price if they seize it. Changes in the liquidity of the housing market directly affect households’ borrowing capacity by altering the value of the collateral asset. More importantly, search frictions generate partial irreversibilities in housing investment because real estate agencies charge a bid-ask spread on housing transactions: the price at which households sell houses to the brokers is lower than the
price at which they buy real estate properties. In this environment, changes in the volatility of the macroeconomic environment affect the investment propensity of the households. As discussed in Bloom (2009), times of high uncertainty are detrimental for investment. In uncertain times, agents are more cautious in investing in housing, because search frictions make the investment expensive to reverse. When volatility is low, housing investment spikes and the housing market heats up. The higher liquidity of housing translates into relaxed credit conditions, and eventually the expansion in investment generates a credit boom. Then, an increase in volatility cools down the investment in housing, reducing the liquidity of housing and households’ collateral values, potentially turning the credit boom into a bust.

This paper introduces a novel mechanism in DSGE models with financial frictions in the form of a borrowing constraint. Standard models a la Kiyotaki and Moore (1997) usually assume that the degree of pledgeability of the collateral asset (or equivalently the loan-to-value ratio or borrowing margin) is exogenous. Instead, in this paper the degree of pledgeability of the collateral is endogenous, and depends on the liquidity of the housing market. In the model, the equilibrium loan-to-value ratio is the ratio between the reservation price of a house - the value of a house on sale that is expected to be sold in the future and does not yield any dividend or utility to its owner - and the fundamental value of housing. The wedge between these two prices widens in illiquid markets, in which vacant houses are expected to being on sale for a longer time. In the extreme case in which vacant houses cannot be sold at any time in the future, housing reservation value goes to zero although its fundamental value is positive. From this perspective, this paper follows the contribution of Fostel and Geanakoplos (2008) and Geanakoplos (2010), which underline the importance to endogenize

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1 The bid-ask spread depends on the fact that real estate agencies purchase housing on a Walrasian market and sell housing on a frictional market. The spread goes to zero only where there is no search friction.
households’ borrowing margin. The link between liquidity and collateral values follows closely the mechanism highlighted by Brunnermeier and Pedersen (2009), in which market liquidity directly determines households’ funding liquidity, that is, the ease at which households can access new loans. I incorporate the borrowing margins a l’a Brunnermeier and Pedersen (2009) into a DSGE model, and find that this channel is quantitatively relevant at a business cycle frequency. Moreover, as long as loan-to-value ratios are endogenous, movements in aggregate productivity generate time variation in households’ borrowing margin. In the quantitative exercise, I show that volatility shocks determine sizable changes in the loan-to-value ratios, and can provide a rationale to the financial shocks introduced in Jermann and Quadrini (2012).

The analysis of this paper grounds on Mendoza (2010) and Bianchi and Mendoza (2013) on both the modeling and the quantification of financial crises. Following these authors, I define a financial crisis in the model as the state in which the credit constraint binds and households’ credit falls down by more than one standard deviation. Yet, in Bianchi and Mendoza (2013) a financial crisis coincides with negative realizations of a productivity and a credit shock. Instead, in my framework financial crises are generated by shocks to the volatility of aggregate productivity. This paper shows that a time-varying uncertainty generates dynamics consistent with volatility paradox of Brunnermeier and Sannikov (2013), and quantitatively accounts for the observed frequency of financial crises.

The ultimate source of action in this paper is represented by shocks to the volatility of total factor productivity. While standard RBC models assume that the economy is hit by a shock to the level of aggregate productivity, I postulate that both the level and the volatility of aggregate productivity are stochastic. These changes in the *exogenous volatility* of productivity may or may not gen-

---

2 The relationship between occasionally binding constraint and financial crises is also studied in He and Krishnamurthy (2010, 2013), in which the constraint applies to firms’ equity capital issuance.
erate sharp movements in the *endogenous volatility* of output, investment and credit, depending on the level of households’ leverage. What is the interpretation of the volatility shocks? These shocks can be rationalized by low frequency movements in the structural transformation of the economy. For instance, Carvalho and Gabaix (2013) show that the changes from the manufacturing sector towards the service sector and the financial sector can account for the movements in the volatility of U.S. macroeconomic variables over the last decades. Alternatively, Bloom (2009) documents that time variations in aggregate volatility are correlated with changes in the cross-sectional dispersion of firms growth rates.\(^3\)

### 1.1 Related Literature

This paper is connected to four strands of literature. First, I complement the extensive empirical evidence provided by Mendoza and Terrones (2012), Schularick and Taylor (2012), and Jorda et al. (2013a,b) on the dynamics of macroeconomic variables around financial crises. These authors show that financial crises are actually credit booms gone bust. I build a panel data of 30 financial crises and 118 normal recessions on a cross section of 20 developed countries from 1980 on, and show that although aggregate volatility does not display strong comovements with normal recessions, it is characterized by large swings around financial crises. Second, this paper contributes to the debate on the source of the recent house price boom and bust. On one hand, the global savings glut is often referred to as one of the main causes of the house price boom. For instance, Justiniano et al. (2014) show that the global savings glut accounts for around one fourth of the increase in U.S. house prices in the early 2000’s. On the other hand, Favilukis et al. (2013) argue that the boom and bust in the housing market is explained

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\(^3\)Christiano et al. (2013) shows that shocks to the cross-sectional dispersion of productivity across agents are an important source of fluctuations of macroeconomic variables.
by financial development in the mortgage market. While there is a burgeoning anecdotal and empirical evidence supporting improvements in financial markets during the 1980’s and 2000’s, it is harder to understand the reversal of the process of financial development amidst the financial crisis. In my model movements in the borrowing margin due to changes in the liquidity of the housing market provide a rationale to both the process of financial development and its reversal. The relaxation of credit conditions in the mortgage market can be explained by the high liquidity of the housing market in the 2000’s, when the time on the market of a house on sale was around 3 months. Analogously, the reversal of the process of financial development can be accounted for by the sudden dry up of liquidity in the housing market around the crisis, when the expected time on the market peaked at around 12 months. Third, this paper is related to the literature studying the role of search frictions in the housing market, which follows the seminal contribution of Wheaton (1990). Diaz and Jerez (2013) show that a model with a frictional housing market can reproduce the cyclicality of house prices. I add to this literature by showing that changes in the liquidity of the housing market can account for the burst of financial crises. Finally, the paper contributes to the literature focusing on the real effects of volatility shocks, such as Justiniano and Primiceri (2008), Bloom (2009) and Fernandez-Villaverde et al. (2011). I show that volatility shocks not only can generate business cycles, but can even account for the build up of risk and burst of financial crises.

4 There is also an extensive literature that studies the role on search frictions in over-the-counter financial markets, as Duffie et al. (2005, 2007). See Rocheteau and Weill (2011) for a review of this literature.

5 The basic setting of the housing market structure in my paper borrows from Ungerer (2013), which shows how monetary policy affect the dynamics of the housing market by altering the liquidity of houses on sale, and eventually households’ borrowing capacity. Although my paper grounds on this contribution, I focus on the link between volatility, housing market liquidity and financial crises.
2 Evidence on Volatility and Financial Crises

In this Section I document a new stylized fact concerning the dynamics of aggregate volatility around financial crises on a panel of 20 developed countries over the last 30 years. I find that crises coincide with the sudden reversal of a long period of low aggregate fluctuations. To shed light on the link between volatility and financial crises, I run a structural vector autoregression to evaluate how volatility shocks are propagate into the real economy by booms and busts in the housing market.

2.1 Data on Volatility and Financial Crises

To understand how volatility can affect the frequency of financial crises, I build a panel of 20 developed countries from 1980 until 2013. The countries covered are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States. For any of these countries I consider the series of growth of real credit to the private sector, real GDP growth, real house price growth, and an indicator of aggregate volatility. The growth rates are computed taking the first difference of the logarithm of each series. All the series are aggregated at an annual frequency. Following Bloom (2009), I proxy aggregate volatility with the logarithm of the variance of countries’ stock market indexes. Basically, I take the series of daily returns year by year, and compute the variance. I demean any of the above series such as each observation can be interpreted as the percentage deviation from the long-run mean. Finally, I consider the dates of financial crises and normal recessions. I take the dates of financial crises from multiple sources, that is, Bordo et al. (2001), Caprio

6Extending the panel back to the 60’s or 70’s does not alter the results because in those years the 20 developed countries under investigation experienced almost no financial crisis.
and Klingebiel (2003), Reinhart and Rogoff (2009), Laeven and Valencia (2012), Schularick and Taylor (2012), Jorda et al. (2013b). Financial crises are defined as credit crunches in which the financial sector experiences large losses and bank runs, that eventually lead to a spike in bankruptcies, forced merged and government intervention. The dates of normal recessions are instead given by the OECD recession indicators. Overall, the panel covers 30 events of financial crises and 118 events of normal recessions. I report the dates of financial crises and normal recessions by country in Table 1 and I refer to the Appendix for further details and all the sources of the data.

Table 1: The Dates of Financial Crises and Normal Recessions

<table>
<thead>
<tr>
<th>Financial Crises</th>
<th>Normal Recessions</th>
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</table>

Note: The dates of financial crises come from Bordo et al. (2001), Caprio and Klingebiel (2003), Reinhart and Rogoff (2009), Laeven and Valencia (2012), Schularick and Taylor (2012), Jorda et al. (2013b). Financial crises are defined as credit crunches in which the financial sector experiences large losses and bank runs, that eventually lead to a spike in bankruptcies, forced merged and government intervention. The dates of normal recessions come from OECD recession indicators.
2.2 The Dynamics Around Crises and Recessions

This Section studies the dynamics of credit growth, GDP growth, house price growth and volatility around financial crises and recessions. For any country and for any financial crisis and recession, I consider all the series above in a time window of nine years around the event of interest, that is, from four years before either the financial crisis or the recession up to four years afterwards. Then, I consider as the typical pattern around financial crises and recessions the series defined by the median observations across events for any year of the window. For example, to define the typical level of GDP growth the year preceding a financial crisis, I consider the GDP growth rate one year before each of the 30 financial crises of my sample and then compute the median.

Figure 1 plots the typical dynamics around financial crises and recessions of credit growth, GDP growth, house price growth and volatility, respectively. On one hand, Panel (a) of Figure 1 shows that credit growth rate display much more volatility around financial crises than around recessions. Moreover, financial crises are preceded by a credit boom in which credit grows around 2% above its long-run mean. Then, the trend is reversed upon the burst of the financial crisis, after which credit growth becomes highly negative. Instead, the dynamics around recessions does not present sizable deviations from the long-run mean of credit growth. A very similar dynamics characterizes also the GDP and house price growth rate around financial crises and recessions, as depicted in Panel (b) and (c). Both series experience a dramatic upsurge before a financial crisis and a deep decline in its aftermath. These graphs support the view of Reinhart and Rogoff (2009), Mendoza and Terrones (2012), Schularick and Taylor (2012), and Jorda et al. (2013a,b) that financial crises are actually booms gone bust.

On the other hand, Panel (d) documents how volatility asymmetrically varies around crises and recessions. While there are negligible deviations from the
long-run mean during recessions, the behavior of volatility around crises is characterized by large swings. Crises tend to be preceded by years in which volatility is around 40% below the long-run mean, and the burst of the crisis reverses the trend by pushing volatility up to around 80% above the long-run mean. More importantly, during the year preceding the crisis the level of volatility almost doubles, peaking from around 0% up to 80% above average. Thus, Figure 4 shows a new stylized facts: financial crises coincide with the sudden reversal of a long period (around 4 years) of low aggregate fluctuations.

Figure 1: Dynamics around Crises and Recessions.

(a) Real Credit Growth

(b) Real GDP Growth

(c) Real House Price Growth

(d) Aggregate Volatility

Note: The figure plots the median values of cross-country annual growth rates of real credit to the private non-financial sector (a), real GDP growth rates (b), real house price growth (c) and the volatility of stock market indexes (d) - measured in log differences from the long-run mean - around recessions and financial crises (9 year window). The continuous line indicates the dynamics around financial crises, while the dynamics around normal recession is presented in a dashed line. Volatility equals the variance of daily stock returns within a year. The dates of financial crises are taken from Reinhart and Rogoff (2009). Normal recessions are derived from the OECD recession indicators.
Appendix B.1 reports a wide range of robustness checks on this new stylized fact on the dynamics of volatility around financial crises. Figure B.1 shows that the shape of aggregate volatility around crises does change in case it is measured as the interquantile range of daily returns. Henceforth, the dynamics of the volatility is not driven by extreme observations. I further consider HP-filtered data, quarterly measures of stock market volatility and also a measure of volatility of GDP growth, based on the variance of quarterly GDP growth rates within rolling windows, and I find similar results. Instead, in Appendix D I report the typical shape of the above macroeconomic variables as measured by the mean, and not by the median. Moreover, I provide panel data evidence in which I show that changes in volatility do predict the happening of financial crises, even when controlling for other macroeconomic variables, country characteristics and fixed effects, and time fixed effects.

\section*{2.3 Volatility Shocks and the Housing Market}

The previous analysis has highlighted that changes in the level of aggregate volatility tend to coincide with the building up of a financial crisis and its burst. To understand the mechanism behind the result, I study the relationship between volatility shocks and the housing market, with a particular emphasis on the level of liquidity of the market. What is the mechanism behind this result? In this Section I shed light on the relationship between volatility and housing markets. I run a structural VAR model, in which I identify volatility shocks and compute the response of house price, the quantity of house sold and a measure of liquidity of the housing market.

The VAR is estimated using with monthly data from January 1963 until December 2013 on the level of S&P 500 returns, an indicator of volatility, the Federal Funds Rate, the consumer price index, industrial production and three
variables on the housing markets related to price, quantity and liquidity.

I borrow the volatility indicator from Bloom (2009). This variable identifies a number of large and arguably exogenous peaks of stock market volatility, and is defined such that it equals 1 in each of these dates and zero otherwise. These dates coincide with events like the assassination of Kennedy, the Arab-Israeli War, the Gulf War and the 9/11 attack. The identification restriction posits that within a month the volatility indicator reacts only to the level of the S&P stock returns, but not to any of the aforementioned macroeconomic variable. The presence of the stock returns allow me to disentangle volatility shocks from any change in the level of stock market data. As housing market variables, I consider the median sales price of new one family homes sold, the number of new one family homes sold and the months supply provided by the Census Bureau. The latter is the ratio of houses for sale to houses sold and measures the number of months a house for sale is expected to last on the market. Hereafter I refer to this variable as the time on the market.\footnote{As a measure of house price, I also consider the Conventional Mortgage Home Price Index and report the results in Appendix B.2.}

The benchmark ordering of the VAR considers the level of S&P 500 returns and the indicator of volatility first, then the series of prices, that is the interest rate, the consumer price index and the house price index, and finally the quantities with the industrial production, the level of sold houses and the time of the market. Figure 2 reports the response (together with one standard deviation bands) of house prices, the number of houses sold and the time on the market to a positive one standard deviation shock to volatility. Panel (b) shows that house prices respond very sluggishly to an increase in volatility, and start declining only around 10 months after the realization of the shock. At the peak, the response is around $-0.001\%$ below the baseline, which gives an annualized rate of $-1.21\%$. Instead, an increase in volatility reduces the number of houses sold
Figure 2: Volatility Shocks and the Housing Market.

(a) Volatility

(b) House Price

(c) House Sales

(d) Time on the Market

Note: VAR estimated from January 1963 to December 2013. The dashed lines are 1 standard-error bands around the response to a volatility shock. The coordinates indicate percent deviations from the baseline. The time on the market is measured by the monthly supply of homes, that is the ratio of houses for sale to houses sold. This statistic provides an indication of how many months the current inventory are expected to keep on sales if no additional new houses were built.

at peak by around $-0.0065\%$ on a monthly basis, which gives an annualized rate of $-8.08\%$. Finally, a volatility shock raises the expected time on the market of a house in sale by $0.005\%$ on a monthly basis, which corresponds to an annualized rate of $6.17\%$. This evidence suggests that volatility shocks do affect the housing market, mostly through changes in the number of houses sold and the time on the market of a house on sale, rather than by modifying the price of real estate.

This evidence is consistent with the dynamics of the volatility of GDP growth
and the liquidity of the housing market over the last decades. For instance, Figure 3 shows that the volatility of GDP growth rates has decreases starting from the 1980’s, a phenomenon which is known as the Great Moderation. Stock and Watson (2002) document that in those years the standard deviations of GDP, consumption and investment decreased by 41%, 38% and 22%, respectively. This trend has been partially reversed in the 2008 during the Great Recession. Figure 4 display that the behavior of the liquidity of the housing market comoves with the volatility of the macroeconomic environment. Periods of low fluctuations are characterized by a low time on the market while turbulent periods, such as the Oil Crises in the 1970’s and the Great Recession, have a much lower level of liquidity. More interestingly, the last period of the Great Moderation coincides with a historical low in the time of the market of a house of sale, which reached 3.5 months while the mean level is above 6 months. Amidst the Great Recession, the time of the market peaked up to around 12 months.

Figure 3: U.S. GDP Growth Rate

Note: The figure plots the quarterly series of US GDP growth rate from 1970Q1 until 2013Q4. The series is computed as the first difference of the log real GDP.
3 The Model

3.1 Environment

In the economy there is a continuum of identical households-entrepreneurs and real estate agencies. Each household-entrepreneur consists of a continuum of members. Members live in different dwellings, and there is perfect risk-sharing within the family. Households access a production function which assembles labor and housing to produce consumption goods. The technology is subject to aggregate productivity shocks with stochastic volatility.

Real estate agencies intermediate in the housing market. Although they instantaneously purchase real estate from household members, agencies trade with buyers on a frictional market. Hence, each period some houses on sale are not

Figure 4: Time on the Market of Houses on Sales in the United States

Note: The figure plots the quarterly series of the time on the market of houses on sale from 1963Q1 until 2013Q4. The time on the market series is given by the month supply of new one family houses from the Census Bureau.
matched with household members and remain vacant.

Households borrow from foreign lenders, and lack of commitment to repay debt. If households renege on debt, lenders seize their housing stock and sell it to real estate agencies. In order to avoid the repudiation of debt, lenders impose a constraint on households’ borrowing capacity. In equilibrium, households cannot borrow more than the collateral value of the housing stock, as priced by real estate agencies.

3.2 Timing

Every period is split into five different stages. In the first one, households observe the current realizations of the aggregate shocks. In the second one, households borrow from the foreign lenders. This stage serves as a rationale for having in equilibrium a borrowing constraint that depends on the current value of households’ collateral. In the third stage, production takes place. In the fourth stage, households homeowners who want to sell their houses trade with real estate agencies on a Walrasian market. I also assume that in this stage a mismatch shock hits a fraction of households members, which are forced to sell their houses to the agencies. In the fifth stage, households members buy houses from real estate agencies on a frictional market. Figure 5 summarizes the timing of the model.

Figure 5: Timing of the Model
3.3 Households

The economy is populated by a continuum of households-entrepreneurs \( i \in [0, 1] \). Each household consists of a continuum of ex-ante identical infinitely-lived members of measure one. Each member lives in a different dwelling and can own at most one house. Although members individually trade real estate on the housing market, the households pool members’ revenues. Following the family construct of Merz (1995), the households allocates total consumption across members, thereby there is perfect risk-sharing within the households.

The households maximize the sum of their members’ utilities. I assume that the instantaneous utility of a member of households \( i \) which owns a house equals

\[
c_{i,o}^{1-\xi} (1 + (1 + \xi))^{\xi} - \mu \frac{(1 - l_{i,o})^{1+\omega}}{1+\omega}
\]

where \( c_{i,o} \) denotes the consumption level of the member, and \( l_{i,o} \) is her leisure. Instead, if a member does not own a house, her instantaneous utility equals

\[
c_{i,n}^{1-\xi} - \mu \frac{(1 - l_{i,n})^{1+\omega}}{1+\omega}.
\]

The parameter \( \xi \) governs the substitutability between consumption and housing services, \( \mu \) denotes the relative disutility of work and \( \omega \) is the inverse of the Frisch elasticity. Given the beginning-of-period fraction of homeowners \( h_{i,t} \) within the household, and considering that perfect risk-sharing implies that all members share the same amount of leisure \( l_{i,t} \), in equilibrium the household behaves as if its lifetime utility equals\(^8\)

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \left[ c_{i,t}^{1-\xi} (1 + (1 + \xi) h_{i,t})^{\xi} - \mu \frac{(1 - l_{i,t})^{1+\omega}}{1+\omega} \right]
\]

\(^8\)As in Shimer (2010), the argument follows directly from the first-order condition of members’ maximization problem with respect \( c_{i,o} \) and \( c_{i,n} \). Appendix C.1 reports a formal derivation of the result.
where \( c_{i,t} = h_{i,t} c_{i,o,t} + (1 - h_{i,t}) c_{i,n,t} \) denotes the total consumption of the household, \( \beta \) is the time discount rate.

Households have a diversified stake in the real estate agencies \( j \in [0, 1] \), which pays out a profit \( \pi_{i,t} \). Furthermore, households access a decreasing return to scale technology that uses labor force \( n_{i,t} \), rented at the equilibrium wage \( w_t \), and housing services\(^9\) \( h_{i,t} \) to produce a homogeneous consumption good, as follows

\[
y_{i,t} = z_t F(n_{i,t}, h_{i,t}) .
\]  

(2)

The consumption good \( y_{i,t} \) is sold on a frictionless market, and is the numeraire of the economy. The production function is subject to an aggregate productivity shock \( z_t \). The process \( z_t \) follows an autoregressive motion with stochastic volatility

\[
z_t = \rho_z z_{t-1} + \epsilon_{z,t}
\]  

(3)

\[
\sigma_t = (1 - \rho_\sigma) \bar{\sigma} + \rho_\sigma \sigma_{t-1} + \eta \epsilon_{\sigma,t}
\]  

(4)

where \( \rho_z \) denotes the persistence of the level of productivity, \( \rho_\sigma \) is the persistence of the volatility of productivity, \( \bar{\sigma} \) is the long-run mean of volatility and \( \eta \) captures the degree of stochastic volatility of the process. When \( \eta = 0 \), the process reduces to a standard autoregression motion. Finally, \( \epsilon_{z,t} \) and \( \epsilon_{\sigma,t} \) denote the innovations to the level and volatility of productivity. I assume that they are i.i.d. following normal distributions \( N(0, \sigma_{\epsilon_z}) \) and \( N(0, \sigma_{\epsilon_\sigma}) \), respectively.

I appeal to this specification for aggregate productivity because the dynamics over time of the level and the volatility of aggregate productivity are pinned down by two different shocks, \( \epsilon_{z,t} \) and \( \epsilon_{\sigma,t} \), respectively. The fact that in a stochastic

\(^9\)In Iacoviello (2005), the economy is populated by impatient households which value housing services and impatient households which uses housing in a production function. Here, I consider a household-entrepreneur who both values housing services and uses housing as a production input. In the quantitative exercise I study the contribution of the dual role of housing to the main results of the paper.
volatility model there are two sources of uncertainty, one related to the level and the other one linked to volatility, allows me to disentangle the role of volatility shocks and their contribution to the quantitative results of the model.\footnote{E.g., in a GARCH model a unique shock drives the dynamics over time of both the level and the volatility of the process. I refer to Fernández-Villaverde et al. (2011) for further discussion on the topic.}

### 3.4 The Housing Market

In the model, houses are either occupied houses or vacant. On one hand, each household $i \in [0, 1]$ has a fraction of $h_{i,t}$ members which occupy a house. Following Iacoviello (2005), occupied houses can be used as an input in the production function. On the other hand, each real estate agency $j \in [0, 1]$ owns $a_{j,t}$ vacant houses, which instead do not enter in the production of the consumption good. I assume that houses are in a unit fixed supply, that is, $\int_0^1 h_{i,t} \, di + \int_0^1 a_{j,t} \, dj = 1.\footnote{Davis and Heathcote (2007) disentangle house prices in two components: structure and land. The authors show that the trend and volatility of house prices in the United States over the last decades are mostly driven by fluctuations in the price of land. Liu et al. (2013) show indeed that fluctuations in the price of land are a driving force of the business cycle. In this vein, the housing stock in fixed supply of my model can be interpreted as stock of land.}

The transactions of housing services are carried out in a sequential fashion. First, household homeowners sell their houses to real estate agencies on a Walrasian market at a price $q_{w,t}$. I assume that in this stage a fraction $\psi$ of household homeowners is hit by a mismatch shock and forced to sell its houses.\footnote{The presence of the mismatch shock is often assumed in the literature of search frictions in the housing market, and dates back to Wheaton (1990). The shock allows real estate agencies to always own some vacant houses. The mismatch shock can be interpreted by job mobility across locations, which forces homeowners to sell their real estate before relocating to a new city. E.g., Glower et al. (1998) find that homeowners who change jobs tend to sell their houses faster than average. Analogously, an increase in the number of people within a household could force homeowners to sell their house and buy a larger one. The same argument applies in case of divorce. Finally, both changes in agents’ taste for neighborhood amenities and sudden changes in expenditures (e.g., health) provide a rationale for such shock. The mismatch shock is analogous to the exogenous separation shock used in the search models of the labor market, see Pissarides (2000).}

When the Walrasian market closes, household members that do not own a house and want to buy one meet with real estate intermediaries in a frictional market.\footnote{In the model, real estate agencies can instantaneously match the households who are willing to sell their}
in the housing market is time consuming. Upon a match, a house is traded at an equilibrium price \( q_{f,t} \) which is determined in a Nash bargaining problem.

3.4.1 Walrasian Market

Household homeowners that want to sell their houses trade with real estate agencies on a Walrasian market. Every period there is a fraction of \( s_{i,t} \) members per household that decides to sell its houses. Furthermore, a mismatch shock hits a fraction \( \psi \) of homeowners, which is forced to leave its houses and sell them to real estate agencies. The mismatch shock is iid across agents and over time.

In this market real estate agencies trade also with each other their stock of vacant houses. Therefore, in the Walrasian market the total amount of houses traded equals

\[
\int_0^1 \chi_{i,t} h_{i,t} di + \int_0^1 a_{j,t} dj, \quad \text{where} \quad \chi_{i,t} = s_{i,t} + \psi.
\]

\(^{14}\) Houses on sale are instantaneously purchased by the real estate sector at the equilibrium price \( q_{w,t} \).

3.4.2 Frictional Market

When the Walrasian market closes, a frictional market opens in which household members that want to buy their houses meet with real estate agencies. Every period, for each household there is a fraction \( 1 - (1 - \chi_{i,t}) h_{i,t} \) of members which does not own a house and seeks to buy one from real estate agencies in a frictional market. Household members exercise a search effort \( e_{i,t} \) in order to match with an agency. I assume that every unit of search effort comes at a monetary cost \( \kappa e_{i,t}^2 \). On the other side of the market, agencies put up on sale their own entire stock of vacant house

\[
\int_0^1 a_{j,t} dj + \int_0^1 \chi_{i,t} h_{i,t} di,
\]

which consists of their beginning-of-period stock of houses and the real estate purchased on the Walrasian market.

\(^{14}\) The formulation of the total supply of houses implicitly assumes that the mismatch shock hits the fraction of household homeowners who is not willing to sell its house. This assumption is inconsequential because in equilibrium only the members hit by the mismatch shock sell their houses.
The ratio between the total amount of buyers (measured in efficiency units) to the total supply of houses on sale defines the tightness of the housing market

\[ \theta_t = \frac{\int_0^1 \left[ 1 - (1 - \chi_{i,t}) h_{i,t} \right] e_{i,t} \, di}{\int_0^1 a_{j,t} \, dj + \int_0^1 \chi_{i,t} h_{i,t} \, di}. \]  

(5)

A high market tightness \( \theta_t \) indicates that the housing market is hot, that is, there are more buyers than sellers. Vice versa, a cold housing market is characterized by a low level of tightness.

Following Wheaton (1990), the aggregate number of successful match \( m_t \) is defined by a constant return to scale Cobb-Douglas matching function

\[ m_t = \left( \int_0^1 \left[ 1 - (1 - \chi_{i,t}) h_{i,t} \right] e_{i,t} \, di \right)^{1-\gamma} \left( \int_0^1 a_{j,t} \, dj + \int_0^1 \chi_{i,t} h_{i,t} \, di \right)^{\gamma} \]  

(6)

where \( \gamma \in (0, 1) \). The matching function stipulates that not all the houses supplied to the market by real estate agencies are matched to a buyer. Indeed, the probability at which real estate agencies sell houses is

\[ P_{s,t} = \frac{m_t}{\int_0^1 a_{j,t} \, dj + \int_0^1 \chi_{i,t} h_{i,t} \, di} = \theta^{1-\gamma} \]

which is increasing to the market tightness \( \theta_t \). Intuitively, the probability at which real estate agencies meet with buyers is increasing in hot housing market, that is, market in which there is a disproportionately larger amount of buyers exerting a high effort. Instead, the probability that a household member exerting effort \( e_{i,t} \) meets with a real estate agency equals

\[ P_{b,t} = \frac{e_{i,t} \, m_t}{\int_0^1 \left[ 1 - (1 - \chi_{i,t}) h_{i,t} \right] e_{i,t} \, di} = e_{i,t} \theta^{-\gamma}. \]

Thus, the probability at which a household member meets real estate agencies
is increasing in its own level of search effort, but decreasing in the search effort of the other buyers on the market. Moreover, the probability of buying a house negatively depends on the tightness of the market. In a hot market, there are much more buyers than sellers, and any given household member is less likely to meet with a real estate agency. Given the probability of buying a house, I can characterize the law of motion of the stock of houses occupied by the household members as

\[ h_{i,t+1} = (1 - \chi_{i,t}) h_{i,t} + P_{b,t}e_{i,t}. \]

I assume that the transaction is instantaneously carried out upon a match. In such a case, the transaction price of the house \( q_{f,t} \) is defined by a Nash bargaining problem, which I describe in Section 3.7.

### 3.4.3 Real Estate Agencies

The housing market is intermediated by a real estate sector. Real estate agencies are owned by the households and operate over two periods, as follows. Each agency enters the economy during the Walrasian market, and purchase \( x_{j,t} \) houses at the price \( q_{w,t} \). Afterwards, the agency puts up on sale its own entire stock of houses on the frictional market. In this stage, the agency manages to sell \( P_{s,t}x_{j,t} \) houses at the price \( q_{f,t} \). In the following period, the remaining stock of housing \( (1 - P_{s,t})x_{j,t} \) is sold on the Walrasian market to new founded agencies at the price \( q_{w,t+1} \). The agency rebates the profits to the households and exits.

---

15The real estate sector represents an exposition device to compute the reservation value of vacant houses. Haughwout et al. (2011) document that at the peak of the last housing boom almost half of purchase mortgages was associated with real estate investors. The County Business Patterns of the Census Bureau reports that in 2011 there were 285,834 establishments operating in real estate sales. These agencies employed 1,397,087 workers, for a total payroll of $14.9 billion.

16This assumption avoids that the search frictions generate ex-post heterogeneity across agencies.

17The purchase of houses on the Walrasian market by real estate agencies is funded as follows. I assume that there is perfect commitment between agencies and foreign lenders, and between agencies and the owners (the households). Agencies get a loan from the foreign lenders to purchase the houses on sale from the different households. The loan is then repaid with the earnings that the households receive from the sale of the houses. The loan is repaid within the period and does not carry interest payments.
the market. Since there is perfect competition and free entry, in equilibrium real estate agencies make zero profit. The functioning of real estate agencies is summarized in Figure 6.

Figure 6: Housing Transactions of a Real Estate Agency

The problem of a real estate agency can be then written as

\[ \pi_{j,t} = \max_{x_{j,t}} \left\{ -q_{w,t}x_{j,t} + P_{s,t}q_{f,t} + (1 - P_{s,t})x_{j,t}E_t[\Lambda_{t,t+1}q_{w,t+1}] \right\} \]  

(8)

where \( \Lambda_{t,t+1} \) is the households’ stochastic discount factor. Free entry in the real estate sector pins down the equilibrium price (or reservation value) of vacant houses as

\[ q_{w,t} = P_{s,t}q_{f,t} + (1 - P_{s,t})E_t[\Lambda_{t,t+1}q_{w,t+1}] . \]  

(9)

Equation (9) stipulates that the price of houses purchased by the real estate sector depends on the liquidity and the house price of the frictional market, and the continuation value of vacant housing. On one hand, when the probability of selling a vacant house in any future period goes to zero, than vacant houses have no reservation value. On the other hand, when the current frictional market is perfectly liquid, that is \( P_{s,t} = 1 \), then the reservation value of vacant houses equals the fundamental value of houses as priced by the frictional market. As long as the frictional market is partially illiquid, then \( q_{w,t} \leq q_{f,t} \), and the price at which real estate purchase houses is lower than the price at which they sell houses. Hence, the structure of the housing market endogenously generate a
bid-ask spread $q_{f,t} - q_{w,t}$ which depends on the liquidity of the frictional market.

To close the characterization of real estate agencies, I derive the law of motion of the total stock of vacant houses owned by the real estate sector

$$
\int_0^1 a_{j,t+1} dj = (1 - P_{s,t})(\int_0^1 a_{j,t} dj + \int_0^1 (s_{i,t} + \psi) h_{i,t} di).
$$

(10)

The presence of the mismatch shock $\psi$ together with the search frictions on the housing market guarantee that the real estate sector always hold vacant houses.

### 3.5 Borrowing Constraint

At the beginning of each period, households observe the realizations of the aggregate shocks and then decide how much to borrow $d_{i,t+1}$. Households borrow from risk-neutral foreign investors, which inelastically supply funds at the gross interest rate $R^{18}$. Hereafter I denote with $d_{i,t+1} \geq 0$ the case in which households borrow from the foreign investors, and with $d_{i,t+1} < 0$ the case in which households save and effectively lend resources to the foreign investors. Households need also to purchase a fraction $\nu$ of the labor cost $w_t n_{i,t}$ in advance of production. Hence, they receive a working capital loan from the foreign investors. Working capital loans are repaid within the period and do not carry interest payments.

Agents lack full commitment and can immediately decide to renege on their debt. In such a case, the lenders seize the housing stock of the household $h_{i,t}$ and sell it to the real estate agencies on the Walrasian market. Under the further assumptions that financial contracts are not exclusive, agents can renege on their debt only at the beginning of each period and there is no additional penalty.

---

18 As in Bianchi and Mendoza (2013), I assume that the US economy is a price taker on the world market for bonds. This assumption is consistent with the analysis of Mendoza and Quadrini (2009) and Warnock and Warnock (2009) which show that foreign capital inflows in the US since mid 1980’s have had a downward pressure on the interest rates of Treasury Bills. In the quantitative analysis I relax this assumption as a robustness check.
in repudiating the debt, Appendix C.2 shows that in equilibrium the collateral constraint equals
\[
\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t} \leq q_{w,t} h_{i,t}. \tag{11}
\]
As in Iacoviello (2005), households’ borrowing capacity is determined by the collateral value of the housing. In Iacoviello (2005) the collateral value of housing equals its fundamental value. In my model the collateral value of households’ housing stock is always lower than its fundamental value, as long as the housing market is not perfectly liquid, and there is a bid-ask spread between the housing price on the frictional market \(q_{f,t}\) and the one on the Walrasian market \(q_{w,t}\).

Multiplying and dividing the right-hand side of the constraint by the price of occupied housing \(q_{f,t}\), the constraint becomes
\[
\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t} \leq \frac{q_{w,t}}{q_{f,t}} \times \frac{q_{f,t} h_{i,t}}{q_{w,t} q_{f,t}} \tag{12}
\]
Equation 12 shows that the collateral value of agents depends on the fundamental value of their housing stock, multiplied by a factor which I define as the liquidity margin. Note that standard models assume that the degree of pledge-ability of the collateral is an exogenous parameter. Instead, in this framework the margin is endogenous and depends on the liquidity of the housing market. When the housing market liquidity freezes out, the low probability of selling vacant houses raises the wedge between the price of occupied housing \(q_{f,t}\) and the one of vacant housing \(q_{w,t}\). As the liquidity margin shrinks, agents’ borrowing constraint gets tighten. Therefore, Equation 12 defines the direct link throughout which the liquidity of the housing market endogenously determines agents’ borrowing capacity. In this environment borrowing margins move over time because of changes in the liquidity of the housing market.
3.6 Decentralized Equilibrium

In the fourth and last stage of a period, production takes place. The households use output net of the labor cost $z_t F(n_i,t, h_{i,t}) - n_{i,t} w_t$, the revenues from supplying labor $(1-l_{i,t}) w_t$, the new amount of borrowing $\frac{d_{i,t+1}}{R}$ and the revenues from selling houses $q_{w,t} (s_{i,t} + \psi) h_{i,t}$, to finance consumption $c_{i,t}$, the searching cost $\kappa e_{i,t}^2$, the repayment of debt $d_{i,t}$, and the purchases of new occupied houses $q_{f,t} P_{b,t} e_{i,t}$. Therefore, households’ budget constraint reads

\[
c_{i,t} + \kappa e_{i,t}^2 + q_{f,t} P_{b,t} e_{i,t} + d_{i,t} = \left[ z_t F(n_i,t, h_{i,t}) - n_{i,t} w_t \right] + (1-l_{i,t}) w_t + q_{w,t} (s_{i,t} + \psi) h_{i,t} + \frac{d_{i,t+1}}{R} + \pi_{i,t}. \quad (13)
\]

Hereafter, I focus on a symmetric competitive equilibrium. Since households are all ex-ante identical and there is no source of idiosyncratic uncertainty, households face the same budget and borrowing constraint, and take identical optimal choices. The same applies to real estate agencies. Therefore, I drop the subscripts from all the variables of the model.

The states of the households’ problem are given by its stock of occupied houses $h_t$, the level of household debt $d_t$, aggregate bond holdings $D_t$, the aggregate stock of houses occupied by the households $H_t$ and finally the level and volatility shocks to productivity, $\epsilon_z,t$ and $\epsilon_{\sigma},t$. Since the stock of housing is in fixed supply, agents do not need to take in account real estate agencies’ stock of vacant houses $a_t$. Furthermore, as long as prices depends on the aggregate level of bond holdings, and optimal decisions depend on current and future prices, agents have to forecast also future aggregate bond holdings. I denote by $\Gamma_D (H, D, \epsilon_z, \epsilon_{\sigma})$ the law of motion of aggregate bond holding $D$ perceived by any household, and $\Gamma_H (H, D, \epsilon_z, \epsilon_{\sigma})$ is the law of motion of the aggregate stock of housing occupied
by the households $H$. Then, the individual maximization problem is

$$V(h, d; H, D, \varepsilon_z, \varepsilon_\sigma) = \max_{c,n,e,s,d'} \left[ c^{1-\xi} (1 + (\xi - 1) h)^\xi - \mu \frac{n^{1+\omega}}{1 + \omega} + \sum_{i} \varepsilon_i e^i \right]$$

s.t. $c + d + C_h = zF(n, h) + \frac{d'}{R} + \pi + G_h$

$C_h = \kappa e^2 + q_f(H, D, \varepsilon_z, \varepsilon_\sigma) P_b(H, D, \varepsilon_z, \varepsilon_\sigma) e$

$G_h = q_w(H, D, \varepsilon_z, \varepsilon_\sigma) (s + \psi) h$

$h' = (1 - s - \psi) h + P(H, D, \varepsilon_z, \varepsilon_\sigma) e$

$\frac{d'}{R} + \nu w(H, D, \varepsilon_z, \varepsilon_\sigma) n \leq q_w(H, D, \varepsilon_z, \varepsilon_\sigma) h$

$D' = \Gamma_D(H, D, \varepsilon_z, \varepsilon_\sigma)$

$H' = \Gamma_H(H, D, \varepsilon_z, \varepsilon_\sigma)$

where Equation (15) denotes the budget constraint, Equation (16) defines the total cost of trading housing $C_h$, Equation (17) is instead the total gain from trading housing $G_h$, Equation (18) denotes the law of motion of households’ occupying houses, Equation (19) is the borrowing constraint and Equations (20-21) stipulate the perceived laws of motion for total bond holdings and occupied housing. Note that in the symmetric equilibrium $n_t = 1 - l_t$.

Upon observing the states of the economy, agents decide the optimal policy on consumption $\hat{c}(h, d; H, D, \varepsilon_z, \varepsilon_\sigma)$, working hours $\hat{n}(h, d; H, D, \varepsilon_z, \varepsilon_\sigma)$, the level of search effort in the housing market $\hat{e}(h, d; H, D, \varepsilon_z, \varepsilon_\sigma)$, the fraction of houses to put up on sale $\hat{s}(h, d; H, D, \varepsilon_z, \varepsilon_\sigma)$, and the amount of resources to borrow from the foreign investors $\hat{d}'(h, d; H, D, \varepsilon_z, \varepsilon_\sigma)$. In equilibrium, the perceived level of aggregate bond holdings $\hat{\Gamma}_D(H, D, \varepsilon_z, \varepsilon_\sigma)$ has to coincide with the individual policy $\hat{d}'(h, d; H, D, \varepsilon_z, \varepsilon_\sigma)$, and the same applies for the law of motion of the
aggregate stock of occupied housing $\Gamma_H (H, D, \epsilon_z, \epsilon_{\sigma})$.

### 3.7 Nash Bargaining

The price $q_{f,t}$ of an occupied housing $h_{i,t}$ which is sold on the frictional housing market is determined through the following Nash bargaining problem

$$q_{f,t} \equiv \arg \max_{q_{f,t}} \left\{ S(q_{f,t})^\zeta B(q_{f,t})^{1-\zeta} \right\}$$  \hspace{1cm} (22)

subject to

$$S(q_{f,t}) = q_{f,t} - \mathbb{E}_t [\Lambda_{t,t+1} q_{w,t+1}] \geq 0$$  \hspace{1cm} (23)

$$B(q_{f,t}) = V^H_t - q_{f,t} \geq 0$$  \hspace{1cm} (24)

where $S(q_{f,t})$ is sellers’ surplus in case of a transaction, $B(q_{f,t})$ denotes buyers’ surplus, $\zeta$ is sellers’ bargaining power, and $V^H_t$ is the fundamental value that households attributes to a new unit of occupied housing. The expected future price of vacant houses is the outside opportunity for an agency in case it does not manage to sell its houses on the frictional market.

In the symmetric competitive equilibrium, each household has the same fundamental value of occupying a house and therefore the identity of the buyer does not matter on the specification of the house price. Indeed, in equilibrium the price of a occupied house is

$$q_{f,t} = \zeta V^H_t + (1 - \zeta) \mathbb{E}_t [\Lambda_{t,t+1} q_{w,t+1}] .$$  \hspace{1cm} (25)

Households’ fundamental value of housing can be derived using the envelope
condition on the optimal stock of occupied housing, which yields

$$V_t^H = \mathbb{E}_t \left[ A_{t,t+1} \left( \chi_{i,t+1} q_{w,t+1} + (1 - \chi_{i,t+1}) V_{t+1}^H + z_{t+1} F_{h_{t+1}} + \frac{U_{h_{t+1}}}{U_{c_{t+1}}} + \phi_{t+1} q_{w,t+1} + + \frac{\phi_{t+1}}{U_{c_{t+1}}} q_{w,t+1} \right) \right] \tag{26}$$

where $Y_{x_t}$ denote the derivatives of the function $Y(\cdot)$ with respect the term $x_t$ and $\phi_t$ is the Lagrange multiplier associated to the borrowing constraint of the households. The fundamental value of a marginal house bought by a household member can be interpreted as follows. First, with probability $\chi_{i,t+1}$ the new homeowner will sell its house to real estate agencies, either voluntarily or because of the mismatch shock. With the remaining probability $1 - \chi_{i,t+1}$, the house will be effectively owned by the household member over the following period. In this case, the household is able to use the house as an input in the production function and gain the marginal productivity $z_{t+1} F_{h_{t+1}}$, and enjoy the flow of utility from occupying the house $\frac{U_{h_{t+1}}}{U_{c_{t+1}}}$. Moreover, the household enjoys the continuation value of owning the house $V_{t+1}^H$. Finally, the ownership of an additional house increases the collateral value of households’ housing stock, relaxing its borrowing constraint. Thereby, the household can access a larger loan, increase consumption and raise its utility level by $\frac{\phi_{t+1}}{U_{c_{t+1}}} q_{w,t+1}$.

Hereafter, I set the sellers’ bargaining power $\zeta$ to be equal to the elasticity of the matching function with respect to sellers, that is

$$\zeta = \frac{\partial m_t}{\partial \left( \int_0^1 a_{j,t} dj + \int_0^1 (s_{i,t} + \psi) h_{i,t} di \right)} \frac{\int_0^1 a_{j,t} dj + \int_0^1 (s_{i,t} + \psi) h_{i,t} di}{m_t} = \gamma.$$  

The above equation is known as Hosios condition after Hosios (1990), and guarantees the efficiency of the random search model. In addition, the Hosios condition together with the assumption that the matching function is a Cobb-Douglas im-
ply that the elasticity of the matching function with respect to both sides of the market is constant and is pinned down by the elasticity of the matching technology (6). Therefore, in this framework the relative bargaining power of both sellers and buyers does not vary over time and across states.

3.8 Definition of Decentralized Equilibrium

In this environment, a recursive decentralized equilibrium is defined by the individual value function $V(h, d; H, D, \epsilon_z, \epsilon_\sigma)$ and optimal policy functions \(\{\hat{c}(h, d; H, D, \epsilon_z, \epsilon_\sigma), \hat{n}(h, d; H, D, \epsilon_z, \epsilon_\sigma), \hat{\epsilon}(h, d; H, D, \epsilon_z, \epsilon_\sigma), \hat{d}(h, d; H, D, \epsilon_z, \epsilon_\sigma)\}\), an optimal policy function on the purchase of vacant houses for the real estate agencies $\hat{a}'(H, D, \epsilon_z, \epsilon_\sigma)$, pricing functions for occupied housing $q_f(H, D, \epsilon_z, \epsilon_\sigma)$, vacant housing $q_w(D, \epsilon_z, \epsilon_\sigma)$ and labor $w(H, D, \epsilon_z, \epsilon_\sigma)$, probabilities of selling and buying a house $P_s(D, \epsilon_z, \epsilon_\sigma)$ and $P_b(D, \epsilon_z, \epsilon_\sigma)$, and a perceived law of motion for aggregate bond holdings $\Gamma_D(H, D, \epsilon_z, \epsilon_\sigma)$ and aggregate occupied housing $\Gamma_H(H, D, \epsilon_z, \epsilon_\sigma)$ such that:

1. Given the pricing functions $q_f(H, D, \epsilon_z, \epsilon_\sigma)$, $q_w(H, D, \epsilon_z, \epsilon_\sigma)$ and $w(H, D, \epsilon_z, \epsilon_\sigma)$, the probability of selling and buying a house, $P_s(H, D, \epsilon_z, \epsilon_\sigma)$ and $P_b(H, D, \epsilon_z, \epsilon_\sigma)$, and the law of motions of aggregate bond holdings $\Gamma_D(H, D, \epsilon_z, \epsilon_\sigma)$ and aggregate occupied housing $\Gamma_H(H, D, \epsilon_z, \epsilon_\sigma)$, the households’ problem is solved by $V(h, d; H, D, \epsilon_z, \epsilon_\sigma)$ and $\{\hat{c}(h, d; H, D, \epsilon_z, \epsilon_\sigma), \hat{n}(h, d; H, D, \epsilon_z, \epsilon_\sigma), \hat{\epsilon}(h, d; H, D, \epsilon_z, \epsilon_\sigma), \hat{d}(h, d; H, D, \epsilon_z, \epsilon_\sigma)\}$.

2. The housing markets clear, the probability of buying a house is

$$P_b(H, D, \epsilon_z, \epsilon_\sigma) = \frac{\hat{c}(h, d; D, \epsilon_z, \epsilon_\sigma) \left[ (1 - (1 - \chi) h) \hat{c}(h, d; D, \epsilon_z, \epsilon_\sigma) \right]^{1 - \gamma} (1 - (1 - \chi) \hat{c}(h, d; D, \epsilon_z, \epsilon_\sigma))}{(1 - (1 - \chi) h) \hat{c}(h, d; D, \epsilon_z, \epsilon_\sigma)}.$$
the probability of selling a home is
\[ P_s(H, D, \epsilon, \epsilon) = \frac{[(1 - (1 - \chi) h) \hat{c} (h, d; H, \epsilon, \epsilon)]^{1-\gamma} (1 - (1 - \chi) h)^{\gamma}}{1 - (1 - \chi) h}, \]

where \( \chi = \psi + \hat{s} (h, d; H, D, \epsilon, \epsilon) \) and the prices of occupied and vacant housing are determined by equation (25) and (9), respectively.

3. The labor market clears at the equilibrium wage \( w (H, D, \epsilon, \epsilon) \).

4. The real estate agencies choose \( \hat{\alpha}' (H, D, \epsilon, \epsilon) \) to maximize the profits defined in (8).

5. The perceived law of motion of aggregate bond holdings coincide with the actual one, that is, \( \Gamma_D (H, D, \epsilon, \epsilon) = \hat{d}' (h, d; H, D, \epsilon, \epsilon) \).

6. The perceived law of motion of the aggregate stock of occupied houses coincide with the actual one, that is, \( \Gamma_H (H, D, \epsilon, \epsilon) = (1 - \psi - \hat{s} (h, d; H, D, \epsilon, \epsilon)) h + P_b (H, D, \epsilon, \epsilon) \hat{c} (h, d; H, D, \epsilon, \epsilon) \).

7. The resource constraint holds: \( D + \hat{c} (h, d; H, D, \epsilon, \epsilon) = z F (h, \hat{n} (h, d; H, D, \epsilon, \epsilon)) + \hat{d}' (h, d; H, D, \epsilon, \epsilon) / R \).

3.9 Characterization of the Equilibrium

3.9.1 First Order Conditions

The first order conditions of the households’ problem yield the following optimal choices on consumption, supply of working hours, number of workers to hire,
housing (dis)investment and borrowing:

\[ w_t = \frac{U_t}{U_{ct}} \]  
\[ z_t F_{nt} = w_t \left[ 1 + \phi_t \nu \right] \]  
\[ q_{f,t} + \frac{2 \kappa e_t}{P_{b,t}} = \mathbb{E}_t \left[ \Lambda_{t,t+1} \left( (s_{t+1} + \psi) q_{w,t+1} + (1 - s_{t+1} - \psi) V_{t+1}^H + z_{t+1} F_{ht+1} + \right. \right. \]  
\[ \left. \left. + \frac{U_{ht+1}}{U_{ct+1}} + \frac{\phi_{t+1}}{U_{ct+1}} q_{w,t+1} \right) \right] \]  
\[ q_{w,t} = \mathbb{E}_t \left[ \Lambda_{t,t+1} \left( (s_{t+1} + \psi) q_{w,t+1} + (1 - s_{t+1} - \psi) V_{t+1}^H + z_{t+1} F_{ht+1} + \right. \right. \]  
\[ \left. \left. + \frac{U_{ht+1}}{U_{ct+1}} + \frac{\phi_{t+1}}{U_{ct+1}} q_{w,t+1} \right) \right] \]  
\[ U_{ct} = \beta R \mathbb{E}_t [U_{ct+1}] + \phi_t \]  

where households’ stochastic discount factor is \( \Lambda_{t,t+1} = \frac{\beta U_{ct+1}}{U_{ct}} \), \( Y_x \) denote the derivatives of the function \( Y(\cdot) \) with respect the term \( x_t \), and \( \phi_t \) is the Lagrange multiplier associated to the borrowing constraint of the households.

Equations (27) is the standard condition for the optimal labor supply. Instead, the optimal labor demand (28) is distorted by the presence of the Lagrange multiplier associated to the borrowing constraint \( \phi_t \). In the states in which the borrowing constraint binds, the multiplier \( \phi_t \) is positive, and the shadow price of the borrowing constraint defines a wedge above the marginal cost. Hence, when a household is borrowing constrained, the cost of hiring labor force de-facto increases, forcing the entrepreneurs to reduce the number of workers hired and the overall level of production.\(^{19}\)

Equations (29) and (30) represent the equilibrium condition for the search effort and the fraction of housing to put up on sale on the frictional market,

\(^{19}\)As shown in Jermann and Quadrini (2012), the link between the borrowing constraint and the labor demand comes from the working capital loans. Indeed households need to finance a share of the labor costs in advance of production. As a result, when the borrowing constraint binds, agents cannot access to large working capital loans and the overall labor force shrinks.
respectively. Equation (29) stipulates that in equilibrium the overall cost of searching for a house equal its marginal gain. The cost is the sum of the searching cost and the house price. The gain is the sum of the production dividends, the flow of utility derived from occupying a house, and the extra amounts of resources obtained by relaxing the borrowing constraint with an additional unit of collateral. Then, the households sell the newly purchased house on the Walrasian market with probability $s_t + \psi$. Otherwise, the households enjoy the continuation value of the house. Equation (30) is a similar condition and refers to the marginal gain and cost of selling a house.

Finally, equation (31) characterizes the optimal choices of bonds. Again, the borrowing constraint adds an extra-financing cost $\phi_t$ which increases the actual repayment cost. Therefore, in the states in which the borrowing constraint binds, households de-facto incur in an interest rate that is above the one charged by foreign investors.

3.9.2 Properties of the Decentralized Equilibrium

**Proposition 1.** In a steady-state equilibrium, the margin of households’ borrowing constraint $q_{w,t}/q_{f,t}$ positively depends on the liquidity of the frictional housing market.

*Proof. See Appendix C.3.*

The margin of the households’ borrowing constraint is endogenous and depends on the liquidity of the housing market. When the market heats up, the margin increases and therefore households’ borrowing capacity is relaxed. Analogously, a liquidity freeze tightens the borrowing margin, decreasing households’ borrowing capacity. This result implies that the observed movements in maximum loan-to-value ratios cannot be explained only by financial development or policy regulation, but also by changes in the liquidity of the housing market.
Proposition 2. The price of houses on the frictional market $q_{f,t}$ negatively depends on the current shadow value of households’ borrowing constraint.

Proof. See Appendix C.4.

When households become borrowing constrained, they decrease the level of search effort on the housing market generating a fire sales spiral which is detrimental for housing prices $q_{f,t}$. As a result, in this environment, fire sales negatively affect both households’ collateral value and their borrowing margin, and therefore the fire sales spiral gets propagated even further.

Proposition 3. Household members always exert a positive effort in the frictional housing market, that is, $e_t > 0$, $\forall t$.

Proof. See Appendix C.5.

If the households’ members decide not to exert any effort, $e_t = 0$, then no match is carried out. In this extreme case, the liquidity of the market goes to zero. As long as the price of vacant houses on the Walrasian market is increasing in the liquidity of the market, a liquidity freeze deflates agencies’ outside opportunity. As a result, the price on the frictional market $q_{f,t}$ shrinks up to the point that households’ decide to exert effort, equalizing the optimal condition (29). Thus, as long as real estate agencies sell housing stock on the frictional market, which is always the case because of the presence of the mismatch shock, households’ members will always exert a positive search effort, implying a strictly positive number of matches in each period.

Proposition 4. Household members never voluntarily sell houses in the Walrasian market, that is, $s_t = 0$, $\forall t$.

Proof. See Appendix C.6.

As long as household members exert costly search effort on the frictional housing market, it is not optimal for household homeowners to sell their houses at the same time. In equilibrium, house sales from household members are fully
accounted for by the mismatch shock.

**Proposition 5.** The tightness of the housing market equals households’ members search effort. Moreover, the probability of selling a house equals the probability of buying a house, and both probabilities positively depend on the level of effort exerted by households’ members.

In a symmetric competitive equilibrium, the dynamics of the frictional housing market is starkly simplified. Indeed, in such an equilibrium every household opts for the same level of search effort, implying that \( \int_0^1 (1 - (1 - s_{i,t} - \psi) h_{i,t}) e_{i,t} \, di = (1 - (1 - \psi) h_t) e_t \). As a result, the equilibrium market tightness becomes

\[
\theta_t = \frac{(1 - (1 - \psi) h_t) e_t}{a_t + \psi h_t} = e_t
\]

since the total housing stock is an unitary fixed supply. The tightness of the housing market entirely depends on the search effort exerted by buyers. Hence, the housing market is hot as long as the level of effort is high. This result further implies that in each period the probability of selling a house equalizes the probability of buying a house, that is

\[
P_{s,t} = P_{b,t} = \theta_t^{1-\gamma} = e_t^{1-\gamma}.
\]

Thereby, hot housing market are characterized by a high level of effort from buyers, a high probability of buying a house and also a high probability of selling a house. The opposite applies in cold markets.

### 3.9.3 Partial Irreversibilities and the Real Effect of Volatility Shocks

**Implication 1.** The frictional housing market generates partial irreversibilities in housing investment.

The sequential trading structure of the housing market implies that the price
at which households purchase housing, $q_{f,t}$, is higher than the price at which they sell houses $q_{w,t}$. Thus, the investment in housing is partial irreversible, and the degree of irreversibility endogenously depends on the level of liquidity of the market. The investment is less irreversible in hot housing market.

**Implication 2.** Partial irreversibilities in investment together with the presence of a decreasing to scale production function, allows volatility shocks to have real effects: an increase in volatility freezes housing investment.

Partial irreversibilities in investment coupled together with a decreasing return to scale production function makes changes in volatility to have real effects. When it is expensive to reverse investment, high levels of volatility make households’ members to be cautious and discourage search effort. As a result, a high volatility reduces the liquidity of the housing market. Instead, in a stable macroeconomic environment, agents increase their search effort and the housing market heats up. Decreasing returns to scale are key for this result. Caballero (1991) shows that a higher uncertainty decreases investment only in environment in which asymmetric adjustment costs interact with either imperfect competition or decreasing returns to scale technologies. Indeed, if profits are convex in demand or costs, then uncertainty actually rises expected profits leading to an investment boom.

### 4 Quantitative Analysis

#### 4.1 Calibration Strategy

I calibrate one period of the model to correspond to one quarter\(^{20}\). To understand the quantitative relevance of the link between volatility, liquidity and financial

---

\(^{20}\)Models featuring search frictions are usually calibrated to a monthly frequency to preserve the action in and out the housing market. Also empirically, the time on the market of a house on sale is measured in months. Yet, the calibration to a quarterly frequency allows me to discipline the role of the volatility shocks to total factor productivity using the quarterly series of Solow residual.
crises, I estimate the shocks to both the level and the volatility of the aggregate total factor productivity of the U.S. economy using quarterly data from 1947Q2 until 2013Q4. The level and volatility shocks are estimated using a Bayesian Sequential Monte Carlo methods.

Then, I calibrate most of the parameters of the model to the values either estimated or used in previous papers. The main parameters which I calibrate to an empirical targets is the cost of searching effort in the housing market. Indeed, in Section 3.9.2 I show that the probabilities of buying and selling a house in the frictional market are increasing the search effort exerted by the households. If the effort was costless, then the search frictions would be offset by an infinitely amount of search effort exerted by the households, and the liquidity of the housing market would be perfect. Therefore, I calibrate the cost of search effort to match the long-run mean of the time of the market of a house on sale.

The model is solved using global methods, and then does not rely on approximations based on Taylor expansions around the steady state. Although the algorithm is much more time intensive, it preserves the non-linear dynamics of the model. I refer to Appendix D.4 for all details on the algorithm.

4.1.1 Estimating the Volatility of Total Factor Productivity

In the model, the ultimate source of the building up of risk and burst of financial crises is given by shocks to the volatility of total factor productivity. To understand the quantitative relevance of this mechanism, I take the actual series of volatility shocks from the data. Namely, I take the series of total factor productivity provided by Fernald (2009). The series of total factor productivity is built as the Solow residual of the U.S. economy using quarterly data on output, capital and labor from 1947Q2 until 2013Q4.

\[21\]

I estimate the shocks to the level

\[21\] The series of Solow residual provided by Fernald (2009) has to be considered just a proxy of the concept of productivity implied by my model, see Appendix D.3.1 for further discussions on this issue.
and volatility of total factor productivity following Born and Pfeifer (2013).

First, I take the series of the Solow residual and apply a one-side HP filter, with a parameter that equals 1600. The one-side HP filter guarantees that the time ordering of the data is not altered while de-trending the data. In this way, I recover the series of total factor productivity \( z_t \). In the model, I postulate that \( z_t \) follows a AR(1) process with stochastic volatility, as follows

\[
z_t = \rho z_{t-1} + \sigma_t \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim N(0, 1)
\]

\[
\sigma_t = (1 - \rho) \bar{\sigma} + \rho \sigma_{t-1} + \eta \epsilon_{\sigma,t}, \quad \epsilon_{\sigma,t} \sim N(0, 1)
\]

where \( z_t \) is observable as the Solow residual, while both \( \varepsilon_{z,t} \) and \( \epsilon_{\sigma,t} \) are unknown to the econometrician. The shocks can be recovered by applying a filter to the data. In this framework the Kalman filter is not suitable because it applies only to linear series, while here the shocks to the volatility enter non-linearly through the exponential function on the levels of total factor productivity. Thus, I evaluate the likelihood of this process by appealing to the Sequential Importance Sampling (SIS) particle filter introduced in Fernandez-Villaverde and Rubio-Ramirez (2007) and Fernandez-Villaverde et al. (2011).

Since the short span of the time series of the Solow residual, I use Bayesian techniques to estimate the likelihood of the process of productivity. I elicit some unrestrictive priors, and after deriving the likelihood of the process for some given parameters with the SIS particle filter, I maximize the posterior likelihood using the Tailored Randomized Block Metropolis-Hastings (TaRB-MH) algorithm of Chib and Ramamurthy (2010). Finally, I recover the historical distribution of the volatility of total factor productivity by appealing to the backward-smoothing routine of Godsill et al. (2004).

I run the estimation of the shocks to total factor productivity under two sce-
narios. In the first one, the shocks to the level and volatility of productivity $\epsilon_{z,t}$ and $\epsilon_{\sigma,t}$ are independent to each other. In the second one, I relax this restriction and estimate the correlation between the shocks by assuming that $\epsilon_{z,t}$ and $\epsilon_{\sigma,t}$ are drawn from a multivariate normal distribution with some unknown correlation matrix. Each step of the estimation algorithm is carefully described in the Supplementary Appendix D.3.

4.1.2 Estimation Results

For the estimation of the AR(1) process with stochastic volatility for the series of TFP of the U.S. economy, I elicit priors following Born and Pfeifer (2013). For both the autocorrelation coefficients of the level and volatility equation, that is $\rho_z$ and $\rho_\sigma$, I consider a uniform distribution. In the case of the autocorrelation of the level equation $\rho_z$, the mean of the distribution equals 0.00 and the standard deviation is 0.58. In the case of the autocorrelation of the volatility equation $\rho_\sigma$, the mean is 0.90 and the standard deviation is 0.10. The implicit assumption is that the process of volatility is known to be highly persistent over time. I consider a Beta prior with mean 0.50 and standard deviation 0.10 for the degree of stochastic volatility $\eta$. Finally, I define a Gamma distribution with mean $-7.00$ and standard deviation $5.34$ for the long run mean of the level shocks.

Table 2 reports the results of the estimation exercise. I find strong evidence of persistence in both the level and volatility equation. Especially the latter case is important. Indeed, the mechanism of the model relies on the existence of a prolonged period of low volatility which fosters a boom in investment and credit, and makes the leverage of the economy to reach very high values. Since the autocorrelation of the volatility equation equals 0.65, the model is able to reproduce long periods of both high and low aggregate fluctuations. The process is also

---

22The parameter follows a Beta distribution once divided by 0.999.
characterized by a high degree of stochastic volatility. Indeed, an increase of one standard deviation in volatility raises the volatility of TFP by \((e^\eta - 1) \times 100\), that equals 37%.

Table 2: Estimation of the Stochastic Volatility of TFP

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_z)</td>
<td>Uniform</td>
<td>0.00</td>
</tr>
<tr>
<td>(\rho_\sigma)</td>
<td>Uniform</td>
<td>0.90</td>
</tr>
<tr>
<td>(\eta)</td>
<td>Beta</td>
<td>0.50</td>
</tr>
<tr>
<td>(\bar{\sigma})</td>
<td>Gamma</td>
<td>-7.00</td>
</tr>
</tbody>
</table>

Note: \(\rho_z\) denotes the autocorrelation parameter of the level equation, while \(\rho_\sigma\) is the autocorrelation of the volatility equation. \(\eta\) captures the degree of stochastic volatility in the process, and \(\bar{\sigma}\) is the steady-state volatility of the level shocks.

4.1.3 Calibration Exercise

Most of the parameters of the model are targeted to values estimated or used in previous papers. The calibration closely follows Bianchi and Mendoza (2013).

I stipulate a decreasing return to scale production function

\[ y_t = z_t n_t^{\alpha_n} h_t^{\alpha_h} \]

where \(\alpha_n + \alpha_h < 1\). The parameter \(\alpha_h\) is calibrated to match the ratio of housing stock value over the GDP. Using data from the Flow of Funds from 1952Q1 until 2013Q4, the ratio of the market value of the real estate of the private nonfinancial sector over GDP is 2.24. In the model, this average is matched
by a value of $\alpha_h = 0.11$. Instead, the labor share is set to the standard value of $\alpha_n = 0.64$. Overall, the returns to scale of the technology sum up to 0.75. Finally, the productivity process $z_t$ inherits the data generator process estimated in the previous Section.

With respect the instantaneous utility function of the household, I consider the following one household

$$
\left[ \frac{1^{-\delta}}{c_{i,t}} \left( 1 + (1 + \xi) h_{i,t} \right)^{\xi} - \mu \frac{(1-h_{i,t})^{1+\omega}}{1+\omega} \right]^{1-\delta} - 1
$$

where $\delta > 1$. This utility function is a monotone increasing function of the utility function introduced in the Equation (1). This utility function belongs to the class of preferences introduced in Greenwood et al. (1988), and does not have any wealth effect on the labor supply. Following Bianchi and Mendoza (2013), I choose this utility function because in the presence of a wealth effect on labor supply, labor would counter-factually increase during a crisis. Note that this class of utility function is widely used in the literature studying boom and bust in the housing market, as Justiniano et al. (2014). First, I set $\xi = 0.899$ following Justiniano et al. (2014). Second, the disutility of work $\mu = \alpha_n$ to have mean hours that equal 1. Third, the Frisch elasticity is set to $1/\omega = 1$. Fourth, I set the risk aversion $\delta = 2$ as in Bianchi and Mendoza (2013) and in line with the literature of DSGE models with open economies. The subjective time discount factor is set to the standard value at the quarterly frequency of $\beta = 0.99$.

I calibrate the gross real interest rate to $R = 1.0065$, that is the value that Bianchi and Mendoza (2013) estimate for the average of the ex-post real interest rate on three months Treasury Bills over the last three decades. Instead, the working capital coefficient is set to $\nu = 0.17$. To compute this value, I use firms’ M1 money holdings to proxy for their working capital. Since two-thirds of the
total M1 stock are held by firms, M1 accounts on average for 16% of annual GDP over the period 1959Q1-2013Q4, and the 0.64 labor share defined above, I set \( \nu = (2/3) \times 0.16/0.64 \).

Finally, I calibrate the parameters characterizing the dynamics of the housing market as follows. I define the mismatch shock to be equal to \( \psi = 0.0278 \) to match the average stay in a house of 9 years reported by Ngai and Tenreyro (2014). The parameter of the matching function which refers to the houses supplied to the market by the real estate sector is set to \( \gamma = 0.21 \) following the value reported in Genesove and Han (2012). Remind that the bargaining power of the seller is set such as \( \zeta = \gamma \) as Hosios’ condition posits. Finally, the monetary cost of exerting searching effort in the frictional market is calibrated to match the average time on the market of a house on sale using data from 1963Q1 until 2013, which is 6.21 months. In this way, I find a value of \( \kappa = 0.93 \).

### 4.2 Quantitative Results

I compare the quantitative performance of the model under the benchmark calibration with three alternative economies. In the first one, which I refer to as the “Search Frictions” economy, there is a frictional housing market but I shut down the stochastic volatility of the model. Thereby, in this environment TFP follows a standard AR(1) process and level shocks provide the only source of exogenous variation. In the second alternative, which I refer to as the “Stochastic Volatility” economy, I consider a stochastic volatility for TFP but I shut down the channel of liquidity. Namely, I consider an infinitely efficient matching function which turns the frictional housing market into a Walrasian one. In the third alternative, which I refer to as the “Level Competitive” economy, I shut down both the stochastic volatility of TFP and the search frictions. This environment
### Table 3: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree substitution consumption/housing</td>
<td>( \xi = 0.111 )</td>
<td>Justiniano et al. (2014)</td>
</tr>
<tr>
<td>Disutility from work</td>
<td>( \mu = \alpha_n )</td>
<td>Normalization</td>
</tr>
<tr>
<td>Inverse Frisch elasticity</td>
<td>( \omega = 1 )</td>
<td>Bianchi and Mendoza (2013)</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>( \delta = 1.5 )</td>
<td>Standard value</td>
</tr>
<tr>
<td>Time discount</td>
<td>( \beta = 0.99 )</td>
<td>Standard value</td>
</tr>
<tr>
<td>Share labor</td>
<td>( \alpha_n = 0.64 )</td>
<td>Standard value</td>
</tr>
<tr>
<td>Share housing</td>
<td>( \alpha_h = 0.11 )</td>
<td>Ratio real estate value over GDP</td>
</tr>
<tr>
<td>Gross real interest rate</td>
<td>( R = 1.0065 )</td>
<td>Average return Treasury Bills</td>
</tr>
<tr>
<td>Working capital parameter</td>
<td>( \nu = 0.17 )</td>
<td>Ratio M1 over GDP held by firms</td>
</tr>
<tr>
<td>Mismatch shock</td>
<td>( \psi = 0.0278 )</td>
<td>Ngai and Tenreyro (2014)</td>
</tr>
<tr>
<td>Sellers’ matching function parameter</td>
<td>( \gamma = 0.21 )</td>
<td>Genesove and Han (2012)</td>
</tr>
<tr>
<td>Sellers’ bargaining power</td>
<td>( \zeta = \gamma )</td>
<td>Hosios’ condition</td>
</tr>
<tr>
<td>Cost searching effort</td>
<td>( \kappa = 0.93 )</td>
<td>Average TOM house on sale</td>
</tr>
</tbody>
</table>

Note: The table reports the calibrated value of all the parameters of the model, except for the DGP of the technology shock. TOM refers to the expected time on the market.
has only level shocks to TFP and a Walrasian housing market. Note that the addition of stochastic volatility does not change the unconditional volatility, but it only implies a time varying dynamics of volatility around its conditional mean. Hence, the level of risk is the same in all the scenarios that I compare.

Table 4: Results

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Search Frictions</th>
<th>Stochastic Volatility</th>
<th>Level Competitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Crises</td>
<td>1.58%</td>
<td>1.12%</td>
<td>1.21%</td>
<td>0.98%</td>
</tr>
<tr>
<td>Magnitude Crises</td>
<td>−5.27%</td>
<td>−3.89%</td>
<td>−3.95%</td>
<td>−3.04%</td>
</tr>
<tr>
<td>Std Dev</td>
<td>3.6%</td>
<td>2.2%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Borrowing Margin</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The magnitude of financial crises corresponds to the annual fall in GDP growth, upon the realization of a financial crisis. The borrowing margin is the ratio of the Walrasian and frictional house prices, \( \frac{q_{w,t}}{q_{f,t}} \). In the model, a financial crisis correspond to the state in which the collateral constraint binds and aggregate credit falls down by more than one standard deviation.

Table 4 reports the results of the four different economies on the frequency and the magnitude of financial crises, and the standard deviation of the borrowing margin \( \frac{q_{w,t}}{q_{f,t}} \). To be consistent with the empirical literature on crises, I define a financial crisis in the model as the state in which the borrowing constraint binds and aggregate credit falls down by more than one standard deviation. The magnitude of the crisis is computed as the cumulative loss in GDP growth within the year following the start of the financial crisis. The interaction of a frictional housing market and time varying volatility generates a frequency of financial crises of 1.58%, and a severity of −5.27%. Over the last century, in the United States there have been 2 major financial crises (1932, 2007), and a minor one (the savings and loan crisis). Hence, the benchmark model accounts for between 53% and 80% of the observed frequency of crises. On the other hand, when
I consider the “Level Competitive” economy, in which the housing market is perfectly liquid and the volatility of TFP is constant over time, the severity of crises equals 0.98% with a magnitude of $-3.04\%$. Therefore, the interaction of volatility shocks and search frictions in the housing market raises the probability of experiencing a financial crisis by around 62%, from 0.98% to 1.58%. Volatility shocks and search frictions boost also the magnitude of the crises by around 71%, from $-3.04\%$ to $-5.27\%$.

The results of the “Search Frictions” and “Stochastic Volatility” disentangle the contribution of each of the two features on the performance of the benchmark model. Search frictions increase the frequency of crises by 14%, while the time varying volatility raises the frequency by 23%. Therefore, the interaction of these two features accounts for the remaining 25% of the difference in the probability of experiencing a crisis between the benchmark economy and the “Level Competitive” one. The same applies for the magnitude of crises. Search frictions raise the fall in GDP growth by 28%, while the time varying volatility amplifies the severity of crises by 30%. The interaction of the two features explain the remaining 24% of the decline in GDP growth experienced in the benchmark economy. These results point out that either search frictions or volatility shocks dramatically improves the performance of the model, although falling short in reproducing the characteristics of crises observed in the data. Especially when volatility shocks are shut down and level shocks are the only source of exogenous variation in the model, the model cannot generate a high frequency crises and large slump in GDP. Instead, the interaction of a frictional housing market and volatility shocks accounts for a large fraction of the probability of experiencing a crisis, and its corresponding fall in GDP growth.

Finally, I study the time variation in the borrowing margin $\frac{q_{w,t}}{q_{f,t}}$. As long as $q_{w,t}$ equals $q_{f,t}$ when the housing market is perfectly liquid, the borrowing margin
is constant and equals 1 in the economies without search frictions. Instead, Table 4 shows that once I allow for a frictional housing market, the borrowing margin changes over time. In the “Search Frictions” economy, the standard deviation of the loan-to-value ratios is 2.2\%. When I add volatility shocks, the standard deviation becomes 3.6\%, implying that volatility shocks amplify the variation in the borrowing margin by around 64\%. Thereby, volatility shocks can be accounted for as a possible foundation of the financial shocks a la Jermann and Quadrini (2012), in which loan-to-value ratios exogenously move over time. While financial shocks are not always known to the econometrician, this model provides a theory of time varying borrowing margins which can be tested using the observable series on housing market liquidity.

5 Concluding Remarks

In this paper I document a new stylized fact on the dynamics of aggregate volatility around financial crises. I find that financial crises coincide with a sudden peak in volatility which follows a long period (of approximately 4 years) of low aggregate fluctuations. This new stylized fact is consistent with the financial instability hypothesis of Minsky (1992) and the volatility paradox of Brunnermeier and Sannikov (2014). I show that volatility shocks do affect the housing market, and especially the level of liquidity, suggesting that the relationship between financial crises and volatility could stem from the housing market.

I argue that shocks to the volatility of total factor productivity are a source of financial instability, and account for both the building up of risk and the burst of financial crises. I develop a DSGE model which there are two main features beside the volatility shocks: a collateral constraint and a frictional housing market. On one hand, households’ borrowing capacity is limited by the collateral value of
their housing stock. The constraint is occasionally binding, generating a highly non-linear dynamics. Indeed, the economy experiences a financial crisis only in the rate states in which the constraint binds and households fire sale. On the other hand, the search frictions determine the equilibrium liquidity of the collateral. In this environment, volatility shocks affect the frequency of financial crises by altering the collateral liquidity. Periods of low volatility trigger a boom in housing investment, heating up the housing market. A higher liquidity of the housing market boosts a credit expansion, which is reinforced by the investment boom into a positive spiral. If leverage rises enough, then a sudden peak in volatility dries up the liquidity of the housing market and reverses the investment and credit boom into a bust. In a quantitative exercise, I feed into the model the stochastic volatility of the Solow residual of the U.S. economy estimated using Bayesian methods. I find that the interaction of time varying volatility and search frictions in the housing market increases both the frequency and the magnitude of financial crises by 62% and 71%, respectively. When I shut down either the stochastic volatility or the search frictions, the frequency of crises becomes much more negligible.

Furthermore, I introduce a novel mechanism into the class of models with a collateral constraint. In these models, agents can borrow up to an exogenous fraction - an exogenous borrowing margin - of their collateral value. Instead, in this paper the borrowing margin is endogenous, and depends on the liquidity of the housing market. In hot markets, households can borrow up to a larger fraction of their collateral value. This approach incorporates the interaction of market and funding liquidity of Brunnermeier and Pedersen (2009) into a DSGE model. Interestingly, I find that volatility shocks generate sizable variations in the borrowing margin and can provide a rationale for the financial shocks a lá Jermann and Quadrini (2012).
Finally, in this paper I study the quantitative relevance of volatility shocks in generating financial crises from a normative side. In a companion paper, Rachedi (2014), I show that the interaction of volatility shocks and a frictional housing is also relevant for macro-prudential regulation. I show that only a Pigouvian taxation contingent on housing market liquidity can avoid the occurrence of financial crises, while policies contingent on house prices are not effective. These findings point out to the importance of housing market liquidity in the propagation of systemic risk, and suggest that housing market liquidity rather than house prices is the relevant variable for understanding the condition of the real estate market.
References


A Data

A.1 Aggregate Volatility and Financial Crises

I build a panel of 20 developed countries from 1980 until 2013. Extending the panel back to the 60’s or 70’s does not alter the results because in those years the 20 developed countries under investigation experienced almost no financial crisis. The countries covered are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States.

Financial Crises: I take the dates of financial crises from multiple sources, that is, Bordo et al. (2001), Caprio and Klingebiel (2003), Reinhart and Rogoff (2009), Laeven and Valencia (2012), Schularick and Taylor (2012), Jorda et al. (2013b). Financial crises are defined as credit crunches in which the financial sector experiences large losses and bank runs, that eventually lead to a spike in bankruptcies, forced merged and government intervention. I follow most of the dating procedure used in Schularick and Taylor (2012) and Jorda et al. (2013b). I refer to these two papers for further detailed discussions on the dating of financial crises.

Normal Recessions: The dates of normal recessions are instead given by the OECD recession indicators. For the United States, I follow the dates provided by NBER. Overall, the panel covers 30 events of financial crises and 118 events of normal recessions. The dates of financial crises and normal recessions by country are reported in Table I.

Stock Market Volatility: The measure of aggregate volatility is based on the volatility of stock market returns. For each of the 20 countries of the panel, I consider the representative stock market index, I take daily returns and compute a measure of dispersion (either the variance or the interquantile range) within a period (either a year or a quarter). The stock market indexes are the following: MSCI for Australia, MSCI for Austria, MSCI for Belgium, TSX for Canada, MSCI for Denmark, MSCI for Finland, MSCI for France, DAX for Germany, ATHEX for Greece, MSCI for Ireland, MSCI for
Italy, NIKKEI for Japan, MSCI for Netherlands, MSCI for Norway, MSCI for Portugal, MSCI for Spain, MSCI for Sweden, MSCI for Switzerland, FTSE for the UK, DJIA for the US. The source of the data is Datastream.

**Credit to the Private Nonfinancial Sector:** I take the series on private credit from the “Long Series on Total Credit and Domestic Bank Credit to the Private Nonfinancial Sector” of the Bank for International Settlements. For each country, I take the adjusted for breaks nominal quarterly series. I take the series in which the lending sector is any sector and the borrowing sector is the private nonfinancial sector. Real values are derived by dividing the credit series by the CPI. Annual observations are computed by averaging the quarterly values within a year.

**Gross Domestic Product:** I take the series of real GDP for the United States from the Bureau of Economic Analysis, series ID GDPC1. For all the other countries, I take the series of nominal GDP from the “Main Economic Indicators” database of the OECD. I compute the real series by dividing the nominal GDP series by the CPI.

**Interest Rates:** The interest rates series is the ex-post real interest rate, where I consider the long term rates provided by the “International Financial Statistics” of the IMF, divided by next period inflation rate.

**Inflation Rates:** The inflation rate is computed as the log difference of the CPI series for each country. Namely, I consider the annual series of the Consumer Price Index All Items from the “Main Economic Indicators” database of the OECD.

**House Prices:** Real house prices are mostly taken from the International House Price database of FED Dallas, which is borrowed from Mack and Martinez-Garcia (2011). For Austria, Greece and Portugal, I have taken the quarterly series of house prices from the Property Price Statistics of the Bank for International Settlements (BIS). For Austria, I consider the series of “Residential Property Prices, All Flats (Vienna), per square meter”, for Greece I consider the series of “Residential Property Prices, All Flats (Other Cities), per dwelling”, and for Portugal I consider the series. The real annual prices are taken by deflating with the according CPI series the nominal series, which has been aggregated at the annual level by taking the average over the
four quarterly observations per year. For Portugal, I take the monthly series from the Property Price Statistics of the BIS, considering the series of “Residential Property Prices, All Dwellings, per square meter”. The annual series is computed by taking the average over the twelve observations per year.

**Ratio of the population aged 65 and above over total population:** I take this ratio from the World Development Indicators of the World Bank. The Indicator code is `SP.POP.65UP.TO.ZS`.

**Openness:** The openness variable is built by summing the ratio of import and export over total GDP. The shares of imports and exports over GDP are taken from the World Development Indicators. They indicator codes are `NE.IMP.GNFS.ZS` and `NE.EXP.GNFS.ZS`, respectively.

**Financial Development:** Financial development is proxied by the ratio of private credit by deposit money bank to GDP. The series is taken from the Financial Development and Structure Dataset of the World Bank. The indicator code is `pcrdbgdp`.

### A.2 SVAR: Volatility Shocks and the Housing Market

The VAR is estimated using with monthly data from January 1963 until December 2013 on the level of S&P 500 returns, an indicator of volatility, the Federal Funds Rate, the consumer price index, industrial production and three variables on the housing markets related to price, quantity and liquidity. Each series but the volatility indicator is taken in logarithm and detrended with a band-pass filter that removed frequencies below 18 months and above 96 months. The VAR includes a set of 12 lags.

**S&P 500 returns:** I take the logarithmic returns of the series of S&P 500 Stock Price Index provided by S&P Dow Jones Indices LLC.

**Indicator of Volatility:** The indicator of volatility is borrowed by Bloom (2009). The measure of volatility is an indicator function which equals one in the events in which the VIX index (or the volatility of daily returns within a month in case the VIX data is not available) is at least 1.65 standard deviations above its long run trend, as proxied by the HP-filtered trend. In total, I have 15 observations of extreme volatility.
in my sample. For further details on the construction of the series, I refer to Bloom (2009).

Federal Funds Rate: The series is the Effective Federal Funds Rate provided by the Board of Governors of the Federal Reserve System. The FED-FRED indicator code is \textit{FEDFUNDS}.

Consumer Price Index: The series is the Consumer Price Index for All Urban Consumers: All Items provided by the Bureau of Labor Statistics. The FED-FRED indicator code is \textit{CPIAUCSL}.

Industrial Production: The series is the Industrial Production Index provided by the Board of Governors of the Federal Reserve System. The FED-FRED indicator code is \textit{INDPRO}.

House Price: The series is the Median and Average Sales Prices of New Homes Sold provided by the Census Bureau. The series refers to new, single-family houses only. The FED-FRED indicator code is \textit{MSPNHSUS}. In the robustness checks, I also use the series of the Conventional Mortgage Home Price Index provided by Freddie Mac, which starts in January 1975.

Quantity of Houses Sold: The series is the Number of Houses Sold provided by the Census Bureau. The series refers to new, single-family houses only. The FED-FRED indicator code is \textit{HSN1F}.

Liquidity of the Housing Market: The series is the Monthly Supply of Home provided by the Census Bureau. The series refers to new, single-family houses only. The series indicates the expected time of the market of houses put up on sale. The months’ supply indicates how long the current for sale inventory would last given the current sales rate if no additional new houses were built. The FED-FRED indicator code is \textit{MSACSR}.
B  Empirical Evidence - Robustness Checks

B.1  Dynamics around Financial Crises

Figure B.1 reports the dynamics of aggregate volatility around recessions and financial crises, by using different measures of volatility.

Figure B.1: Different Measures of Aggregate Volatility.

(a) Volatility - Interquantile Range

(b) Volatility - HP Filtered Data

(c) Volatility - Quarterly Data

(d) Volatility - GDP Growth Rates

Note: The figure plots the dynamics of different measures of aggregate volatility around recessions and financial crises (9 year window). In Panel (a) the volatility is measured as the interquantile range of daily stock market returns within a year. In Panel (b) the volatility refers to the HP-filtered ($\lambda = 6.25$) variance of daily returns within a year. In Panel (c) the volatility is the variance of daily returns with a quarter. In Panel (d) the volatility refers to the variance of quarterly GDP growth rates computed over a moving window of 20 quarters. The continuous line indicates the dynamics around financial crises, while the dynamics around normal recession is presented in a dashed line. The dates of financial crises are taken from Reinhart and Rogoff (2009). Normal recessions are derived from the OECD recession indicators.

First, I check that the time-varying pattern on volatility around crises is robust to the presence of outliers in daily returns. I compute volatility as the interquantile range of daily returns within a year and report it in Panel (a). Second, instead of taking the logarithmic difference of the variance from its long run mean, in Panel (b) I use...
HP-filtered data ($\lambda = 6.25$). Third, I show that the pattern of volatility is robust to changing the frequency upon which to compute the measure of dispersion. In Panel (c) I show the dynamics of the variance of daily returns within a quarter. Finally, in Panel (d) I display the dynamics of a measure of variance which applies to GDP growth rates, by computing the variance over rolling windows of 20 quarters.

### B.2 SVAR and the House Price

Figure B.2 shows that the impulse response functions of the housing market variables do not change even when considering a different measure of the house price, that is, the CMHPI series from Freddie Mac.

Figure B.2: Volatility Shocks and the Housing Market.

(a) Volatility 
(b) House Price 
(c) House Sales 
(d) Time on the Market

Note: VAR estimated from January 1975 to December 2013. The dashed lines are 1 standard-error bands around the response to a volatility shock. The coordinates indicate percent deviations from the baseline.
C Proofs

C.1 Utility Function of the Household

Consider the following maximization problem

$$\max_{c_{i,o,t}, l_{i,o,t}, c_{i,n,t}, l_{i,n,t}} \sum_{t=0}^{\infty} \beta^t \left[ \left( c_{i,o,t}^{1-\xi} (1 + (1 + \xi)) - \mu \frac{(1 - l_{i,o,t})^{1+\omega}}{1 + \omega} \right) h_{i,t} + \left( c_{i,n,t}^{1-\xi} - \mu \frac{(1 - l_{i,n,t})^{1+\mu}}{1 + \mu} \right) (1 - h_{i,t}) \right]$$

s.t. \( h_{i,t} c_{i,o,t} + (1 - h_{i,n,t}) + \kappa e_i^2 + qf_i P_b e_i + d_{i,t} = [z_t F(n_i h_{i,t}) - n_i w_t] + (1 - h_{i,t}) l_{i,o,t} - (1 - h_{i,t}) l_{i,n,t}) w_t + qw_t (s_{i,t} + \psi) h_{i,t} + \frac{d_{i,t+1}}{R} + \pi_{i,t} \)

that is the problem of a household that wants to maximize the lifetime utility of its members, out of which a fraction \( h_{i,t} \) is a homeowner, consumes \( c_{i,o,t} \) and enjoys leisure \( l_{i,o,t} \), while a fraction \( 1 - h_{i,t} \) does not own a house, consumes \( c_{i,n,t} \) and enjoys leisure \( l_{i,n,t} \). Taking the first order condition with respect \( c_{i,o,t} \) and \( c_{i,n,t} \) yields

\[(1 + (1 + \xi))^\xi c_{i,o,t}^{1-\xi} (1 - \xi) = \lambda_t\]

and

\[c_{i,n,t}^{1-\xi} (1 - \xi) = \lambda_t\]

where \( \lambda_t \) is the Lagrange multiplier associated to the budget constraint. Combining the two conditions, and using the definition of total consumption within the household \( c_{i,t} = h_{i,t} c_{i,o,t} + (1 - h_{i,t}) c_{i,n,t} \) yields

\[c_{i,n,t} = \frac{c_{i,t}}{1 + (1 + \xi) h_{i,t}}\]

and

\[c_{i,o,t} = \frac{[1 + (1 + \xi)] c_{i,t}}{1 + (1 + \xi) h_{i,t}}\]
Substituting the last two equations in the utility function of the household leads to the result of Equation (1).

C.2 Equilibrium Borrowing Constraint

The derivation of the equilibrium borrowing constraint closely follows Appendix A.3 of Bianchi and Mendoza (2013). The borrowing constraint arises in equilibrium as an incentive compatibility constraint which grounds on a limited enforceability of debt, that is, households lack of commitment to repay their debt. I consider the following environment:

1. Loans are signed with lenders in a competitive environment;
2. Financial contracts are not exclusive;
3. There is no informational friction between lenders and households;
4. Households borrowing during the first stage of each period of the model, that is, just after the realization of the shocks, and before production takes place;
5. Households lack of commitment in repaying the debt only during the first stage of the problem;
6. If households renege on their debt, lenders seize their collateral asset $h_{i,t}$ and sell it to real estate agencies at the price $q_{w,t}$;
7. After reneging on debt, households can immediately access again financial market at no penalty, and can purchase again its housing stock at competitive prices.
8. All the household homeowners are hit by the mismatch shock period by period.
In this environment, the households that borrow an amount \( b_{i,t+1} \) and renege on paying it back, it gains \( b_{i,t+1} \) and loses \( q_{w,t} h_{i,t} \). The value of repaying the due debt is then

\[
V^r (h, d; H, D, \epsilon_z, \epsilon_\sigma) = \max_{c,n,e,s,d'} \left[ \xi^{-\xi} \left( 1 + (1 + \xi) h \right)^\xi - \mu \frac{n^{1+\omega}}{1+\omega} \right] + \beta \mathbb{E}_{\epsilon_z, \epsilon_\sigma} \left[ V (h', d'; H', D', \epsilon_z', \epsilon_\sigma) \right]
\]

s.t. \( c + d + C_h = zF (n, h) + \frac{d'}{R} + \pi + G_h \)

\( C_h = \kappa e^2 + q_f (H, D, \epsilon_z, \epsilon_\sigma) P_b (H, D, \epsilon_z, \epsilon_\sigma) e \)

\( G_h = q_w (H, D, \epsilon_z, \epsilon_\sigma) h \)

\( \frac{d'}{R} + \nu w (H, D, \epsilon_z, \epsilon_\sigma) n \leq q_w (H, D, \epsilon_z, \epsilon_\sigma) h \)

\( D' = \Gamma_D (H, D, \epsilon_z, \epsilon_\sigma) \)

\( H' = \Gamma_H (H, D, \epsilon_z, \epsilon_\sigma) \)

Instead, if households borrow \( b_{i,t+1} \) at the beginning of the period and defaults, then their value function equals

\[
V^d (h, d, b; H, D, \epsilon_z, \epsilon_\sigma) = \max_{c,n,e,s,d'} \left[ \xi^{-\xi} \left( 1 + (1 + \xi) h \right)^\xi - \mu \frac{n^{1+\omega}}{1+\omega} \right] + \beta \mathbb{E}_{\epsilon_z, \epsilon_\sigma} \left[ V (h', d'; H', D', \epsilon_z', \epsilon_\sigma) \right]
\]

s.t. \( c + d + C_h = b + zF (n, h) + \frac{d'}{R} + \pi \)

\( C_h = \kappa e^2 + q_f (H, D, \epsilon_z, \epsilon_\sigma) P_b (H, D, \epsilon_z, \epsilon_\sigma) e \)

\( \frac{d'}{R} + \nu w (H, D, \epsilon_z, \epsilon_\sigma) n \leq q_w (H, D, \epsilon_z, \epsilon_\sigma) h \)

\( D' = \Gamma_D (H, D, \epsilon_z, \epsilon_\sigma) \)

\( H' = \Gamma_H (H, D, \epsilon_z, \epsilon_\sigma) \)

Hence, from the budget constraints of the two problems it is evident that households repay their debt if and only if \( b_{i,t+1} \leq q_v, h_{i,t} \).
C.3 Proof of Proposition 1.

The borrowing margin \( \frac{q_{w,t}}{q_{h,t}} \) equals

\[
\frac{q_{w,t}}{q_{f,t}} = P_{s,t} + (1 - P_{s,t}) \mathbb{E}_t [\Lambda_{t,t+1} q_{w,t+1}]
\]

In a steady-state equilibrium, the borrowing margin equals

\[
\frac{q_w}{q_f} = P_s + (1 - P_s) \beta \frac{q_w}{q_f} = \frac{P_s}{1 - (1 - P_s) \beta}
\]

since \( \Lambda_{t,t+1} = \beta \frac{U_{t+1}}{U_c} \), and \( U_{c,t+1} = U_c \) along the steady-state. Thus, the derivative of the margin with respect to a change in the current level of the liquidity of the frictional housing market, measured in terms of probability of selling a house is

\[
\frac{\partial q_w}{\partial P_s} = \frac{1 - \beta}{[1 - (1 - P_s) \beta]^2} > 0 \quad \forall \beta \in (0, 1), \ P_s \in (0, 1)
\]

C.4 Proof of Proposition 2.

I use the equation of house price \( q_{w,t} \) given by the condition (25) to characterize the expected equity premium associated to the investment in housing

\[
\mathbb{E}_t [R_{t+1}^{ep}] = \mathbb{E}_t [R_{t+1}^h - R]
\]
where \( R_{t+1}^h = \frac{z_{t+1} F_{h_{t+1}} + q_{f,t+1}}{q_{f,t}} \) denotes the cum-dividend return on housing investment.

The equity premium reads

\[
\mathbb{E}_t [R_{t+1}^{ep}] = \frac{1}{\mathbb{E}_t [\Lambda_{t,t+1}]} \left\{ \frac{\phi_t}{U_{ct}} + \mathbb{E}_t \left[ \Lambda_{t,t+1} \Delta q_{f,t+1} \left( 1 - \frac{q_{w,t+1}}{q_{f,t+1}} \right) \right] + \mathbb{E}_t \left[ \Lambda_{t,t+1} \Omega_t^B \right] + \right. \\
+ \mathbb{E}_t \left[ \Lambda_{t,t+1} \Omega_t^M \right] - \mathbb{E}_t \left[ \Lambda_{t,t+1} \Delta q_{f,t+1} \frac{U_{h_{t+1}}}{U_{ct+1} q_{f,t+1}} \right] - \mathbb{E}_t \left[ \Lambda_{t,t+1} \Delta q_{f,t+1} \frac{V_{h_{t+1}}}{q_{f,t+1}} \right] - \\
\left. \mathbb{E}_t \left[ \Lambda_{t,t+1} \Delta q_{f,t+1} \frac{q_{w,t+1}}{U_{ct+1} q_{f,t+1}} \right] - C_t \left[ R_{t+1}^{ep} \Lambda_{t,t+1} \right] \right\} 
\]

(32)

where

\[
\Omega_t^B = (1 - \zeta) \mathbb{E}_t \left[ \Lambda_{t,t+1} \Delta q_{f,t+1} \left( \frac{z_{t+1} F_{h_{t+1}} + V_{h_{t+1}}}{q_{f,t+1}} + \frac{U_{h_{t+1}}}{U_{ct+1} q_{f,t+1}} + \frac{\phi_{t+1} q_{w,t+1}}{U_{ct+1} q_{f,t+1}} \right) \right] 
\]

and

\[
\Omega_t^M = \zeta \psi \mathbb{E}_t \left[ \Lambda_{t,t+1} \Delta q_{f,t+1} \frac{V_{h_{t+1}}}{q_{f,t+1}} \right] - (1 - \zeta) \mathbb{E}_t \left[ \Lambda_{t,t+1} \Delta q_{f,t+1} \frac{q_{w,t+1}}{q_{f,t+1}} \right] 
\]

Formula (32) highlights that the premium, and therefore the house price, depends on collateral values and search frictions. Indeed, in standard asset pricing conditions, the equity return depends only the level of risk, that is, the covariance between households’ stochastic discount factor and the equity premium. Here, the equity premium is also increasing in the current Lagrange multiplier associated to the borrowing constraint \( \phi_t \) and the search frictions as measured by the margin of the borrowing constraint. On one hand, when the borrowing constraint binds, the equity premium rises, and the house price \( q_{f,t} \) declines. Thus, borrowing constrained households that are forced to fire sales depress the current house price. On the other hand, when the future probability of selling houses in the frictional market decreases, tightening the borrowing margin, the equity premium rises and therefore the house price declines. So, a liquidity freeze lowers the house price. In either case, there is also an indirect effect. The high equity return
in the states in which the borrowing constraint binds and the liquidity of the housing market is low tends to be associated by disproportionately higher levels of households’ marginal utility of consumption. This co-movement amplifies the component of the equity premium due to risk and further depresses the house price. House price \( q_{h,t} \) also depends on the marginal utility derived from occupying such an asset. Henceforth, a negative co-movement between such flow of utility and the stochastic discount factor represents an additional mechanism that accounts for a low house price. The level of house prices is even affected by a component due to the Nash bargaining, the mismatch shock, the continuation value of the match on the frictional market and an interaction term between the collateral and search frictions.

C.5 Proof of Proposition 3.

TBA

C.6 Proof of Proposition 4.

Since household members always exert a positive effort \( e_t > 0, \forall t \), in equilibrium household homeowners voluntarily sell houses on the Walrasian market if and only if

\[
q_{f,t} + \frac{2\kappa e_t}{P_{b,t}} \leq \mathbb{E}_t \left[ A_{t,t+1} \left( (s_{t+1} + \psi)q_{w,t+1} + (1 - s_{t+1} - \psi) V_{t+1}^H + z_{t+1} F_{t+1} + U_{ht+1} + \frac{U_{ht+1}}{U_{ct+1}} \frac{q_{w,t+1}}{U_{ct+1}} \right) \right] \leq q_{w,t}
\]

which follows combining the optimal conditions (29) and (30). Basically, in equilibrium household members both exert effort on the frictional market and sell houses on the Walrasian market if and only if the gain of selling the houses on the Walrasian market is larger than the continuation value of owning a house which is larger than the cost of searching for and buying houses on the frictional market. Hence, in equilibrium \( s_t > 0 \) if and only if

\[
q_{w,t} \geq q_{f,t} + \frac{2\kappa e_t}{P_{b,t}}
\]
This condition is never fulfilled, because $q_{w,t} \leq q_{f,t}$, as shown in Proposition 1, and 
\[ \frac{2\kappa_{e_t}}{P_{b,t}} > 0 \] as shown in Proposition 3. Therefore, in equilibrium $s_t = 0$.

D Supplementary Appendix

D.1 The Dynamics around Crises and Recessions

This Appendix shows the dynamics of macroeconomic variables around financial crises and recessions, when the typical shape of the variable is measured by the mean across events, and not by the median. Figure D.3 shows that using either the mean or the median, as in Figure 1, does not change the dynamics of either volatility or the other three macroeconomic variables around the events of interest.

D.2 The Role of Time-Varying Volatility

Although the dynamics of volatility is related with the realizations of financial crises, the relationship could be statistically insignificant once controlling for other macroeconomic variables. Therefore, I provide panel data estimates in which I check the role of volatility once considered together with a wide set of covariates. Namely, I run an OLS linear probability model where the dependent variable is a dummy that equals 1 in the year of a financial crisis. The benchmark regression then considers as independent variables four lags of volatility, house price growth, credit growth and GDP growth. I also include countries’ characteristics such as the ratio of population with age above 65 years and the degrees of openness and financial development. Finally, I always control for country and year fixed effects. As in Fogli and Perri (2013), I include the share of the population aged 65 or above because Jaimovich and Siu (2009) show that demographic factors do affect the level of volatility in G7 countries. The presence of countries’ openness is due to Di Giovanni and Levchenko (2009), who show instead that volatility is positively affected by the degree of trade openness of a country. Instead, Bekaert et al. (2006) show that a high degree of financial development is positively related to
lower consumption growth volatility. Finally, the presence of country and fixed effects controls for unobserved country characteristics and international aggregate shocks that could affect the frequency of financial crisis. The results reported in Column (1) of Table D.3 show the coefficient of the third lag of volatility is negative and significant at the 5% confidence level. Hence, a low level of volatility is related with the realization of a financial crisis three years afterwards. In addition, a one percent decrease of volatility below its long-run mean increases the probability of experiencing a crisis three years afterwards by 6.34%. Therefore, the effect of low volatility on the frequency of financial crisis is both statistically and economically significant. Also the two years lag of house price growth is statistically significant, though just at the 10% level. In this case, a one percent increases of real estate price above its long-run mean increases the probability
of experiencing a crisis three years afterwards by 45.10\%. Thus, Column (1) supports the hypothesis that both low volatility and house price boom are related with financial crises, even once controlling for other covariates. In Column (2) I define the product of volatility and house price growth as a new variable to test whether the interaction of low fluctuations and a real estate boom is also related with the burst of a crisis. I add the new interaction term to the set of covariates of Column (1). Column (2) shows that the third lag of volatility keep being significant at the 5\% level, and its point estimate has just slightly decreased. Surprisingly, the addition of the interaction makes the role of house price growth per se to be statistically insignificant. Instead, the third lag of the interaction is significant at the 5\% level. A one percent decrease of the level of the interaction below its long-run mean increases the probability of experiencing a crisis three years afterwards by 51.74\%. Note that these results are robust even when controlling for further covariates, such as the level of interest rates in Column (3) and the inflation rate in Column (4).

Table D.2 replicates the same analysis using as dependent variable a dummy that equals 1 in the dates of normal recessions. The results show that in this case there is no significant relationship between low level of volatility, house price booms and the probability of experiencing a recession.

D.3 Estimation of the Stochastic Volatility

D.3.1 Solow Residuals

TBA

D.3.2 The Sequential Particle Filter

TBA

D.3.3 The Particle Smoother

TBA
D.3.4 Metropolis Hastings Algorithm

TBA

D.3.5 Convergence Diagnostics

TBA

D.3.6 Testing for ARCH Effects

D.4 Computational Algorithm

Additional References


Table D.1: Volatility and Financial Crises - OLS Linear Probability Model

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<th>(3) Crises</th>
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| R\(^2\)                           | 0.3598     | 0.3694     | 0.3708     | 0.3702     |
| Observations                       | 552        | 552        | 551        | 552        |

Note: The dependent variable is a dummy variable which equals one at the starting dates of financial crises. Volatility is the volatility of countries’ stock market index, and equals the variance of daily returns within a year. All regressions include country and fixed effects, and control for four lags of real credit growth and real GDP growth, a measure of openness (the sum of import and export ratio to GDP), a measure of financial development (share of private credit by deposit money bank to GDP), and the share of population with more than or equal to 65 years. Column (3) includes four lags of the real interest rate. Column (4) includes four lags of the inflation rate. Standard errors are reported in brackets. ***, ** and * denote significance at the 1%, 5% and 10%, respectively.
Table D.2: Volatility and Recessions - OLS Linear Probability Model

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>(1) Recessions</th>
<th>(2) Recessions</th>
<th>(3) Recessions</th>
<th>(4) Recessions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility(_{t-1})</td>
<td>0.0131</td>
<td>0.0047</td>
<td>0.0125</td>
<td>0.0082</td>
</tr>
<tr>
<td></td>
<td>(0.0423)</td>
<td>(0.0429)</td>
<td>(0.0434)</td>
<td>(0.0428)</td>
</tr>
<tr>
<td>Volatility(_{t-2})</td>
<td>-0.0352</td>
<td>-0.0369</td>
<td>-0.0346</td>
<td>-0.0292</td>
</tr>
<tr>
<td></td>
<td>(0.0460)</td>
<td>(0.0467)</td>
<td>(0.0469)</td>
<td>(0.0467)</td>
</tr>
<tr>
<td>Volatility(_{t-3})</td>
<td>0.0090</td>
<td>0.0067</td>
<td>0.0051</td>
<td>0.0017</td>
</tr>
<tr>
<td></td>
<td>(0.0448)</td>
<td>(0.0455)</td>
<td>(0.0456)</td>
<td>(0.0456)</td>
</tr>
<tr>
<td>Volatility(_{t-4})</td>
<td>0.0018</td>
<td>0.0070</td>
<td>-0.0026</td>
<td>0.0068</td>
</tr>
<tr>
<td></td>
<td>(0.0391)</td>
<td>(0.0400)</td>
<td>(0.0402)</td>
<td>(0.0399)</td>
</tr>
<tr>
<td>House Price Growth(_{t-1})</td>
<td>-0.4280</td>
<td>-0.5305</td>
<td>-0.5569</td>
<td>-0.5314</td>
</tr>
<tr>
<td></td>
<td>(0.3951)</td>
<td>(0.4034)</td>
<td>(0.4077)</td>
<td>(0.4033)</td>
</tr>
<tr>
<td>House Price Growth(_{t-2})</td>
<td>0.3980</td>
<td>0.4308</td>
<td>0.3872</td>
<td>0.3456</td>
</tr>
<tr>
<td></td>
<td>(0.4746)</td>
<td>(0.4826)</td>
<td>(0.4921)</td>
<td>(0.4841)</td>
</tr>
<tr>
<td>House Price Growth(_{t-3})</td>
<td>-0.6039</td>
<td>-0.6126</td>
<td>-0.6241</td>
<td>-0.6879</td>
</tr>
<tr>
<td></td>
<td>(0.4438)</td>
<td>(0.4568)</td>
<td>(0.4590)</td>
<td>(0.4574)</td>
</tr>
<tr>
<td>House Price Growth(_{t-4})</td>
<td>0.3594</td>
<td>0.3508</td>
<td>0.4511</td>
<td>0.4313</td>
</tr>
<tr>
<td></td>
<td>(0.3742)</td>
<td>(0.3815)</td>
<td>(0.3851)</td>
<td>(0.3833)</td>
</tr>
<tr>
<td>House Price Growth × Volatility(_{t-1})</td>
<td>0.5197</td>
<td>0.4642</td>
<td>0.5506</td>
<td>0.5506</td>
</tr>
<tr>
<td></td>
<td>(0.4517)</td>
<td>(0.4569)</td>
<td>(0.4504)</td>
<td>(0.4504)</td>
</tr>
<tr>
<td>House Price Growth × Volatility(_{t-2})</td>
<td>0.2146</td>
<td>0.1297</td>
<td>0.2457</td>
<td>0.2457</td>
</tr>
<tr>
<td></td>
<td>(0.4588)</td>
<td>(0.4680)</td>
<td>(0.4580)</td>
<td>(0.4580)</td>
</tr>
<tr>
<td>House Price Growth × Volatility(_{t-3})</td>
<td>0.2176</td>
<td>0.2091</td>
<td>0.2244</td>
<td>0.2244</td>
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<tr>
<td></td>
<td>(0.4441)</td>
<td>(0.4486)</td>
<td>(0.4437)</td>
<td>(0.4437)</td>
</tr>
<tr>
<td>House Price Growth × Volatility(_{t-4})</td>
<td>0.0584</td>
<td>0.0032</td>
<td>0.0096</td>
<td>0.0096</td>
</tr>
<tr>
<td></td>
<td>(0.4117)</td>
<td>(0.4116)</td>
<td>(0.4118)</td>
<td>(0.4118)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R²</th>
<th>0.1815</th>
<th>0.2938</th>
<th>0.2934</th>
<th>0.2941</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>552</td>
<td>551</td>
<td>552</td>
<td>551</td>
</tr>
</tbody>
</table>

Note: The dependent variable is a dummy variable which equals one at the starting dates of recessions. Volatility is the volatility of countries' stock market index, and equals the variance of daily returns within a year. All regressions include country and fixed effects, and control for four lags of real credit growth and real GDP growth, a measure of openness (the sum of import and export ratio to GDP), a measure of financial development (share of private credit by deposit money bank to GDP), and the share of population with more than or equal to 65 years. Column (3) includes four lags of the real interest rate. Column (4) includes four lags of the inflation rate. Standard errors are reported in brackets in brackets. ***, ** and * denote significance at the 1%, 5% and 10%, respectively.