

International stock market correlations: A sectoral approach

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Abstract

A lot of studies dealing with international correlations look only at correlations at the market level and often use its time-varying nature as motivation for their work. However, why and how market correlations change is a point that is still not very well understood. As the market is composed of different sectors, we propose to look into this question by studying the behaviour of equity correlations at the sectoral level. We show how sectoral correlation coefficients determine the market correlation coefficient and decompose the latter into two parts; one that represents country factors and one that represents industry factors. This decomposition allows us to get a clear idea on how and why market correlations change over time. We also get some interesting insights such as market level correlations are higher on average than sectoral correlations as well as that sectoral correlations between countries tend to be more stable over time than market level correlations and sectoral correlations within countries. Finally, we present evidence that a few sector correlations related to the Financial, Industrial and Consumer Services segments drive the evolution of the market level correlation.

1 Introduction

The observation that correlations between stock markets tend to vary over time is often used as a motivation for a wide range of papers looking for example at the question of international versus sectoral portfolio diversification as well as for research dealing with economic and financial integration.

The question about the most efficient way to diversify portfolios has been raised by Heston and Rouwenhorst (1994) who present evidence that country factors are the dominant driving force for equity returns and that therefore international diversification is more interesting than sectoral diversification. The same conclusion has been made by Griffin and Karolyi (1998) and Brooks and Del Negro (2004). Other studies such as Baca et al. (2000) and Cavaglia et al. (2000) claim that industry factors start to dominate country factors and that portfolio diversification by sectors is more efficient than by countries.

The benefits of international portfolio diversification depend on the level of international stock market correlations. It is obvious that in a dynamic view with changing importance of country, industry and world factors, these changes will have a direct impact on the correlation coefficients between two stock markets that will therefore fluctuate over time.

Time-varying international correlation coefficients are also often predicted by studies looking at economic and financial integration between countries. Increased economic and financial integration between countries is believed to lead to higher international stock market correlations.

As pointed out by Bekaert and Hodrick (2005) the two issues, the effect of increased integration and changing importance of country, industry and world factors on international stock market correlations, must be linked. It is indeed reasonable to think that a higher degree of integration lowers the importance of country factors and that it increases the importance of industry factors and/or world factors. The conclusion that increasing industry factors are always due to increased economic or financial integration is however dangerous and not always true as shown by Brooks and Del Negro (2004). In their paper, they present evidence that the relative increase of industry factors compared to country factors in the late 1990's was a temporary effect driven by the technology bubble and not a result of an increased market integration.

Based on this background, one can affirm that if increased economic integration has an effect on international stock market correlations, it will do it by modifying the relative importance of country, industry and world factors. Integration is certainly not the only variable that can alter the relative weight of country, industry and world factors and all events changing it will have an effect on international stock market correlation. Therefore, one way to understand how and why the correlations between markets change through time, is first to understand how world, industry and country factors

are related to it and how they change through time. It is straightforward to think that not all industries/sectors are contributing the same way to the market correlation and that there are sectors that play a bigger role than others, i.e. the specific factors associated with certain sectors are more important than others. Once the most important sectors or industry factors identified, the second step is then to identify what the industry factors are and put it together with country and world factors to get a complete model on how and why international correlation change over time.

The present paper is dealing with the first step by identifying the sectors that are driving the international stock market correlations. The second step will be left for further research. We apply an empirical approach to this question by focusing exclusively on sectoral correlations. We show how correlations at the market level and correlations at the sectoral level are linked and that one can compute the market correlation using only sector correlations and index weights. Empirical sectoral correlations are interesting to analyze as they contain a wealth of information on stock market linkages that have not yet been studied.

Another approach would be the one put forward by Bekaert and Hodrick (2005) who look first for the best factor model to fit the stock return comovements between country-industry portfolios and then use this model to answer questions such as the presence of a time trend in correlations and the evolution of country versus industry correlations. They find that the best model is an APT model including world as well as regional (local) factors and allowing for time-varying risk loadings. Based on their model, Bekaert and Hodrick (2005) find that time-varying risk loadings contribute relatively less to changes in correlations than the factor dynamics. However, as the APT factors are extracted using the principal component analysis and although they show some limited evidence that at least the global factors are related to the global Fama-French factors, the identity of the factors remain unclear and the question which factors, industry, country/region or world factors are driving the international stock market is left unanswered.

By opting for a pure empirical study, we do not impose any structure on sector returns that could result in a loss of information. For example, if one takes the dummy method by Heston and Rouwenhorst (1994) as base model this will result in having the same correlation between any two different sectors of two markets as the only relevant factor would be the world factor¹. Only in the case when considering the same sector in both markets you can obtain differences in sector correlations as the industry factor as defined by HR enters. It is clear that in the case of the correlation between the Utility segment in one market and the Technology segment in the other, the degree of comovement will be low as intuitively it is hard to imagine any common factor driving the returns in these two sectors. According to Heston and

¹A detailed discussion of this is given in Appendix C.

Rouwenhorst (1994) the only source of comovement would be the world factor. The same correlation is observed between the industrial segment in one market and the Basic Material in another market. It is however obvious that there exist common factors such as the commodity prices that are relevant for stock returns in both sectors. The dummy method by HR fails to capture such common factors between returns of different sectors. More complicated factor models, such that allow for example different factor loadings contrary to the unit beta implicitly assumed by the HR and with more flexibility in the factor structure can surely be imagined as being more appropriate but these models become quickly heavy to deal with.

We will use two types of sectoral correlations for our analysis; correlations between sectors between countries and correlations between sectors within each country. The first type of correlations clearly depends on factors acting on returns in both country, such as industry factors but also regional factors or world factors. The second type of correlation depends on country factors and world factors, but also to some extent on certain industry factors that are common to certain industries, such as already mentioned above commodity prices that are important for Basic Materials and Industries. Comparing these two types of correlations will allow us to make some interesting conclusions. If for example sector correlations between countries increase more than correlations within countries, then this can be seen as evidence that industry factors and/or regional factors have gained on importance compared to country factors.

We will show in section 5 how the market correlation coefficient can easily be decomposed into two parts; one part that will depend only on sector correlations between countries and index weights and the other part that will depend only on sector correlations within the countries and index weights. This decomposition is especially useful to get a first idea on where changes on the market level come from.

The rest of the paper is organized as follows. In the next section, we present our data set used for this study. Section three will give some general results on market and sector correlations for the countries in our database. Section four addresses the question of the stability over time of the different correlations and show that sector correlations are on average more stable than market correlations with some sectors being more unstable than others. This result indicates that not all sectors are contributing the same way to the evolution of the market correlations. Section five will give details on the decomposition of the market correlation mentioned above, and present evidence on the low importance of index weights in determining the market correlation. Finally in section six, we look at the evolution over time of the different sector correlations and we conclude our work with section seven.

2 Data

Our data set covers market and sector returns from 01.01.1973 to 29.03.2006 for seven major stock markets: the US, the UK, France, Germany, Switzerland, Canada and Japan². All data are from Datastream. For each country we use the Datastream Global index on the market level and the 10 sector indices: Oil and Gas, Basic Materials, Industries, Consumer Goods, Healthcare, Consumer Services, Telecom, Utilities, Financials and Technology³.

Using Datastream indices guarantees that our results are not influenced by different index construction methods and sector classifications used in different countries. As it is common use for these types of study, the return frequency is weekly to avoid the problem of non-synchronous trading, but still having a relatively large number of observations.

Seven stock markets lead to 21 different country pairs to analyze. It is obvious that not every sector is present in each country. For example, the Oil and Gas sector is not present in Germany, Switzerland and Japan. Moreover, some sectors are only of minor importance in some countries. The technology sector in the UK, for example, accounts on average only for 0.86% of the total market capitalization of the British stock market. Finally, some sectors are not present over the whole time period, for example the telecom sector in the UK, which starts only in November 1981 but accounts from then on for 7.7% on average of total market capitalization.

Based on these observations, we impose the following two conditions on a sector in a country to enter our database. First, a sector must have at least 250 observations, which corresponds to about 5 years of data and second, at least 80% of all observed weights of a sector in a country must be higher than 1% of the total market capitalization. The second condition allows us to eliminate sectors which are of minor importance in a country's economy. Based on this two conditions, we finally obtain three countries with ten sectors (US, Canada and France), two countries with 9 (Germany

²The markets for our study are the same as Longin and Solnik (1995) used in their study.

³Here some more details regarding the composition of the different sectors:

Oil / Gas: includes Oil&Gas Producers; Oil Equipment&Services; **Basic Materials:** includes Chemicals; Forestry&Paper; Industrial Metals; Mining; **Industrials:** includes Construction&Materials; Aerospace&Defense; General Industries; Electronic&Electrical Equipment; Industrial Engineering; Industrial Transportation; Support Services; **Consumer Goods:** includes Automobiles&Parts; Beverages; Food Producers; Household Goods; Leisure Goods; Personal Goods; Tobacco; **Healthcare:** includes Healthcare Equipment&Services; Pharmaceuticals&Biotechnology; **Consumer Services:** includes Food&Drug Retailers; General Retailers; Media; Travel&Leisure; **Telecom:** includes Fixed Line Telecommunications; Mobile Telecommunications; **Utilities:** includes Electricity; Gas,Water&Multiutilities; **Financials:** includes Banks; Nonlife Insurance; Life Insurance; Real Estate; General Financial; Equity Investment Instruments; Nonequity Investments Instruments; **Technology:** includes Software&Computer Services; Technology Hardware&Equipment

and Japan) and two countries with 8 sectors (UK and Switzerland). Overall, we have 1753 sector combinations between our seven countries for which the computation of the correlation is possible as well as 263 sector correlations within the countries.

3 Correlation analysis

3.1 Total Market Correlations TMC

Table 1 presents the correlation coefficients at the total market level over the whole period. Without much surprise we observe the highest correlations between countries which are geographically and culturally close to each other like the US and Canada as well as Switzerland and Germany. The lowest correlations are observed when Japanese stock returns are involved. As we have the same countries in our database as Longin and Solnik (1995), it is interesting to compare our total market correlation coefficients to those obtained in their study⁴. The main difference is in the time period studied, our data span the years 1973 to 2006 whereas the period covered in the study by Longin and Solnik (1995) goes from 1960 to 1990. 19 of the 21 correlation coefficients are higher in our study compared to the ones by Longin and Solnik, with important differences when one of the country is Germany. Including the years 1990-2006 to the sample seem to dramatically change the results for Germany, the other differences are less pronounced, however correlation coefficients computed between two European countries are systematically higher in our study compared to Longin and Solnik (1995), giving us a first hint at the possible consequence of the European economic integration.

⁴The results reported by Longin and Solnik (1995) in their paper:

	US	UK	Can	Frae	Ger	Switz
UK	0.50	—				
Can	0.71	0.48	—			
Frae	0.43	0.42	0.42	—		
Ger	0.38	0.34	0.30	0.45	—	
Switz	0.55	0.45	0.46	0.51	0.60	—
Jap	0.30	0.25	0.27	0.26	0.24	0.29

Table 1: Total Market Correlations

The table shows the correlations of returns for seven major stock markets. The correlation were computed over our whole data period, i.e. from 01.01.1973 to 29.03.2006 using Total Market Indices provided by Datastream.

	US	UK	Canada	France	Germany	Switzerland
UK	0.51	–				
Canada	0.72	0.47	–			
France	0.48	0.50	0.44	–		
Germany	0.49	0.44	0.43	0.60	–	
Switzerland	0.54	0.49	0.47	0.59	0.69	–
Japan	0.33	0.29	0.32	0.33	0.36	0.34

3.2 Sector Correlations

We now turn our attention to the correlation at the sectoral level between the seven countries as well as within countries. First we look at the sector comovements between markets. As mentioned in section 2 this analysis results in 1753 correlation coefficients. The highest correlation coefficient is observed for the sector pair US Basic Materials and Canada Basic Materials with 0.70, followed by the pair France Telecom and UK Telecom with 0.68. Interestingly the highest correlation coefficient observed on the sectoral level (out of 1753) is lower than the highest correlation coefficient observed at the total market level (US-Canada with 0.72). We observe 8 negative correlation coefficients out of 1753 (= 0.46%) with the lowest correlation between the pair France Telecom - Japan Utilities with -0.12. Four of these negative correlation coefficients were obtained on sector pairs involving the Japanese Utilities sector combined with either a Telecom or a Technology sector. The other four negative correlations involve Canada Utilities (3 times) and Canada Oil&Gas (once) combined again either with a Technology or a Telecom sector.

We observe no negative correlations when looking at the coefficients between the sectors within the countries. The highest correlation is observed between Industries and Consumer Services in the US with 0.87 followed by the coefficient between Technology and Industries in Japan. The lowest value is obtained between the Telecom and Utilities sectors in Switzerland. When we compute the average sector correlation for each country, we get a rather high average sector correlations for the US (0.60), the UK (0.56), France (0.55), Japan (0.55) and Germany (0.51). The lowest two average sector correlations within the markets are obtained for Switzerland (0.45) and Canada (0.39). As these two countries are relatively small in terms

of economic size compared to its neighbors, the US for Canada, Germany and France for Switzerland, it is reasonable to think that country specific factors play a less important role for these stock markets as they depend to a larger extent on what is going on around them compared for example to the situation of the US or Germany.

In Table 2 we present the average correlation between the different sectors as well as the average correlation at the total market level. For each sector combination, we report two numbers, the first number is the average correlation observed between countries and the number below is the average correlation observed when looking at the different correlations within the markets. By looking at the table we can make two interesting observations, first all sector correlations within the markets are on average higher than the sector correlations between the markets. This shows that country specific factors play a major role in determining the degree of comovement. By taking out this components, i.e. by focusing on the comovement of sectors of different countries, the correlation decreases. The second interesting result is by focusing only on the sector correlations between markets, one sees that the highest average correlation is observed at the total market level. A result that is not as surprising as it seems as we will show in the section where we perform a decomposition of the correlation coefficient of a stock market index. The same result has also been found by Berben and Jansen (2005) which compare however only same sector correlation coefficients to index correlation for 6 country pairs.

Table 2: Average correlations on the sectorial and the total market level
The table shows the average correlation between the ten different sectors as well as the average correlation at the total market level. TM stands for Total Market, OG for Oil&Gas, BM for Basic Materials, Ind for Industries, CG for Consumer Goods, Hlth for Healthcare, CS for Consumer Services, Tel for Telecom, Utl for Utilities, Fin for Financials and Tec for Technology

	TM	OG	BM	Ind	CG	Hlth	CS	Tel	Utl	Fin	Tec
TM	0.47										
OG		0.43									
		-									
BM		0.44	0.38								
		0.56	-								
Ind		0.25	0.37	0.40							
		0.51	0.74	-							
CG		0.25	0.36	0.38	0.39						
		0.39	0.63	0.68	-						
Hlth		0.23	0.31	0.31	0.31	0.34					
		0.47	0.65	0.63	0.56	-					
CS		0.23	0.35	0.38	0.36	0.31	0.37				
		0.47	0.71	0.72	0.63	0.65	-				
Tel		0.13	0.23	0.30	0.25	0.20	0.30	0.36			
		0.22	0.39	0.48	0.37	0.37	0.50	-			
Utl		0.17	0.22	0.21	0.20	0.22	0.20	0.10	0.19		
		0.40	0.42	0.40	0.33	0.41	0.42	0.24	-		
Fin		0.23	0.35	0.37	0.36	0.33	0.35	0.28	0.22	0.40	
		0.47	0.66	0.67	0.56	0.63	0.69	0.47	0.45	-	
Tec		0.17	0.30	0.38	0.32	0.24	0.35	0.33	0.12	0.32	0.43
		0.29	0.49	0.65	0.54	0.43	0.56	0.43	0.23	0.49	-

The highest average correlation at the sectorial level between markets is observed between Basic Materials and Oil&Gas. In general one can see that higher correlations at the sectorial level between markets are usually obtained between the same sectors (Tech-Tech, Oil&Gas-Oil&Gas, Industries-Industries, Financials-Financials, Consumer Goods-Consumer Goods). It is intuitive to attribute this result to the presence of more common factors when looking at same sector combinations rather than different sector combinations. The fact that the highest average correlation is observed between Basic Materials and Oil&Gas can probably be attributed to the fact that the sector Basic Materials is composed by firms with a rather petrol intensive production scheme.

In table 3 we present for each country pair the five highest correlation coefficients observed empirically.

Table 3: The five highest correlation coefficients for each country pair
This table shows the five highest correlation coefficients observed between the ten different sectors as well as the total market level for our 21 country pairs. TM stands for Total Market, O&G for Oil&Gas, BM for Basic Materials, Ind for Industries, CG for Consumer Goods, Hlth for Healthcare, CS for Consumer Services, Tel for Telecom, Utl for Utilities, Fin for Financials and Tech for Technology

	1	2	3	4	5
US-UK	O&G O&G	TM	Fin Fin	Ind Fin	BM BM
US-Can	TM	BM BM	CG CG	Ind BM	CG BM
US-Fra	Tech Tech	TM	Ind Tech	Tech Tel	Tel Tel
US-Ger	Tech Tech	TM	Ind Tech	Ind Ind	CS Tech
US-Swi	TM	Fin Fin	Ind Fin	Ind Ind	CS Fin
US-Jp	Tech Tech	TM	Ind Ind	CS Ind	CS CG
UK-Can	TM	Fin CS	Ind CG	O&G O&G	BM BM
UK-Fra	Tel Tel	CS Tech	Fin Utl	HC Utl	TM
UK-Ger	CS Tech	Tel Tel	TM	Fin BM	Fin Fin
UK-Swi	Fin Fin	TM	Tel Tel	Ind CG	CS CG
UK-Jp	Fin Ind	CS Ind	TM	BM Ind	Fin CG
Can-Fra	Tech Tech	TM	Ind Tech	CG CG	CG Fin
Can-Ger	CG CG	TM	CG Fin	Ind Tech	CG BM
Can-Swi	TM	CG Fin	Ind Fin	CS CG	Ind CG
Can-Jp	Tel Tech	TM	Tel Ind	Tech Tech	BM Ind
Fra-Ger	Tel Tel	TM	Tech Tech	Tech Ind	Fin Fin
Fra-Swi	Fin Fin	TM	Utl Fin	Utl BM	Tech Ind
Fra-Jp	Tech Tech	Tel Tech	Tech Ind	TM	Ind Ind
Ger-Swi	TM	Fin Fin	Ind Fin	Ind Ind	BM Fin
Ger-Jp	TM	Ind Ind	Fin Ind	Ind Tech	BM Ind
Swi-Jp	Ind Ind	Ind Tech	CG Ind	TM	Fin Ind

Table 3 allows us to make three observations. First, we see that the Total Market correlation figures for each country pair among the five highest correlation values observed. For six country pairs it is even the highest value and for another ten country pairs it is the second highest value. Second, same sector combinations are over represented in this table compared to their importance in the overall sample. Leaving out the Total Market correlations, we observe that there are 33 same sector correlation coefficients among the 84 coefficients represented in table 3, which corresponds to around 39%. This is much more than the share of the same sector corre-

lations in the overall sample⁵, which is around 10%, showing that the same sector correlation coefficients tend to be higher than different sector correlations. Third, the sectors which are most present in this table are Industries (41 times), Financials (31 times) and Technology (30 times) indicating that these sectors can be associated with high correlations.

4 Constant or not?

In this section, we turn our interest to the question whether the total market correlation coefficients and the sectoral correlation coefficients were constant over the last 33 years or not. Although it is nowadays considered as a stylized fact that correlations change over time, it is interesting to revisit this issue and to look at the constancy of correlations at the sectoral level.

4.1 Methodology

Two tests have recently been proposed to test for the stability of correlations in a multivariate GARCH setup. The one proposed by Tse (2000) tests the null hypothesis of constant correlation against the alternative of presence of an ARCH effect in correlation. Bera and Kim (2002) also test the null hypothesis of constant correlation but against an unspecified alternative.

We choose to apply the test by Bera and Kim (2002) because at this stage we prefer to not make any assumptions about dynamics in correlations⁶.

The idea of the test is to apply the information matrix test to the constant correlation bivariate GARCH (BGARCH). The test only requires estimates of the constant correlation GARCH model⁷ to construct its statistics, which is quite easy to obtain. Bera and Kim (2002) present several different test statistics. The basic test statistic is given by

$$IM_e = \frac{\left[\sum_{t=1}^T (\widehat{v}_{1t}^{*2} \widehat{v}_{2t}^{*2} - 1 - 2\widehat{\rho}^2) \right]^2}{4T(1 + 4\widehat{\rho}^2 + \widehat{\rho}^4)}$$

where $\widehat{\rho}$ is the estimate of the correlation obtained with the the constant correlation bivariate GARCH and

$$\widehat{v}_t^* = (\widehat{v}_{1t}^*, \widehat{v}_{2t}^*)' = \left(\frac{\varepsilon_{1t}^* - \widehat{\rho}\varepsilon_{2t}^*}{\sqrt{1 - \widehat{\rho}^2}}, \frac{\varepsilon_{2t}^* - \widehat{\rho}\varepsilon_{1t}^*}{\sqrt{1 - \widehat{\rho}^2}} \right)$$

with ε_{it}^* being the standardized residuals. According to Bera and Kim (2002) the variable \widehat{v}_{1t}^* can be thought of as a pure variation of ε_{1t} that

⁵see Appendix A for the number of observations.

⁶Moreover, as mentioned by Bera and Kim (2002), the test by Tse (2000) has the shortcoming of not being able to guarantee that $|\rho_t| < 1$.

⁷This model was first presented by Bollerslev (1990).

cannot be explained by ε_{2t} . This statistic IM_e , under the null hypothesis of constant correlation, asymptotically follows a chi-square distribution with one degree of freedom.

As it is derived using a bivariate normal distribution, it is not very efficient if the data are non-normal, as criticized by Tse (2000). To correct this, Bera and Kim (2002) propose a studentized version of this statistic, given by

$$IM_s = \frac{\left[\sum_{t=1}^T (\eta_t) \right]^2}{\sum_{t=1}^T (\eta_t - \bar{\eta})^2}$$

where $\eta_t = \widehat{v}_{1t}^* \widehat{v}_{2t}^* - 1 - 2\widehat{\rho}^2$ and $\bar{\eta} = \sum \eta_t / T$, which, as they show in their article, has good finite-sample behavior, even if the data are generated from a non-normal distribution.

We are going to use this latter statistic IM_s to test for constancy in correlations in our sample. For each country pair we estimate for each possible sector combination a constant correlation bivariate GARCH(1,1)⁸ and compute IM_s .

4.2 Total Market Correlations

We apply the above mentioned test first to the correlations between our seven countries at the total market level. Table 4 gives the result of the test.

Table 4: BeraKim Test Results for the Total Market Correlations
This table shows the IM_s statistics for the different total market correlations over the whole period. ** and * indicates the significance at 5% level, resp. 10% level.

	US	UK	Canada	France	Germany	Switzerland
UK	3.10*	-				
Canada	2.20	4.31**	-			
France	2.29	7.55**	2.43	-		
Germany	6.28**	10.48**	6.13**	5.35**	-	
Switzerland	2.33	3.83*	3.78*	7.11**	3.69*	-
Japan	2.83*	1.12	2.17	3.94**	4.88**	2.82*

We observe that 9 out of the 21 correlation coefficients are not constant at the 5% level and that another 6 coefficients are not constant at the 10%

⁸We use a BGARCH(1,1) for all of our data, as otherwise it would become computationally extremely heavy.

level. Overall, 71% of the correlation coefficient on the total market level exhibit some non-constancy. One of the most striking feature in table 4 is that all correlation coefficients between two European markets show non-constancy. This could be due to the economic integration of these markets. Leaving out Switzerland, which is not a member of the European Union, we observe that all correlation coefficients between EU countries (UK, France and Germany) are highly non-constant (at the 5% level). We also observe that all correlations with Germany are non-constant. We suspect that the reunification of West and East Germany could be the reason behind this, as this event changed the German economic structure.

4.3 Sector correlations

We apply the same test to all different sector correlations between the markets and all sector correlations within the markets that can be computed using our data base. Of the 1753 correlation coefficients between the markets, 273 sector correlations show non-constancy at the 5% level and another 303 correlations at the 10% level. Overall, only 33% of all sector correlations between countries exhibit some non-constancy compared to the 71% of the total market correlations coefficients. However, out of the 263 sector correlations within the countries there are 127 that show non-constancy at the 5% level and 46 others at the 10% level. 66% of all sector correlations within countries are non-constant at least at the 10% level, which is the double compared to the sector correlations between countries. Sector correlations between countries have a tendency to be more constant than sector correlations within countries and at the total market level.

Table 5 gives us the percentage of non-constant sector correlations for each country pair. The country pair with the highest fraction of non-constant sector correlation is UK-Germany with 76% of all sector correlations presenting non-constancy at the 10% level. Not surprisingly it is the same country pair with the highest test statistic on the total market level (see Table 4). Other country pairs with rather high fractions of non-constant sector correlations are UK-France and France-Germany, with 59% and 56% at the 10% level. This results hints again at some possible influence of the integration process in the European Union.

Table 5: Percentage of non-constant sector correlations per country pair
This table gives the percentage of non-constant sector correlation coefficients for each country pair.

Panel A: 5% level	US	UK	Canada	France	Germany	Switzerland
UK	1%	-				
Canada	19%	6%	-			
France	7%	34%	7%	-		
Germany	27%	51%	11%	39%	-	
Switzerland	1%	17%	4%	43%	32%	-
Japan	2%	1%	1%	7%	16%	8%
Panel B: 10% level	US	UK	Canada	France	Germany	Switzerland
UK	24%	-				
Canada	47%	21%	-			
France	24%	59%	23%	-		
Germany	39%	76%	30%	56%	-	
Switzerland	23%	38%	20%	46%	47%	-
Japan	23%	13%	8%	28%	33%	17%

When looking at the percentage of non-constant coefficients within the markets, we observe a high fraction of non-constancy for the US, UK and Japan with respectively 91%, 86% and 86% at the ten percent level and rather low percentages for Switzerland and Canada with respectively 32% and 33% at the ten percent level. Finally, we have Germany with a percentage of 78% and France with 58% of non-constant sector correlations at the ten percent level.

After having looked at the percentage of non-constant sector correlations per country pair and within each market, we now turn our attention to the question which sector combination shows the least constant correlation coefficients. Table 6 summarizes the percentage of non-constant correlation coefficients for each possible sector correlations across all countries.

As mentioned above, 16% of all sector correlations show non-constancy at the 5% level and 33% at the 10% level. By looking at table 6, we see that there are some big differences depending on the sector combination.

Table 6: Percentage of non-constant correlation coefficients per sector combination

This table gives the percentage of non-constant correlation coefficients for each sector combination. TM stands for Total Market, O&G for Oil&Gas, BM for Basic Materials, Ind for Industries, CG for Consumer Goods, Hlth for Healthcare, CS for Consumer Services, Tel for Telecom, Utl for Utilities, Fin for Financials and Tech for Technology

Panel A: 5% level	O&G	BM	Ind	CG	Hlth	CS	Tel	Utl	Fin	Tech
O&G	17%									
BM	17%	38%								
Ind	8%	33%	29%							
CG	5%	25%	31%	20%						
Hlthc	21%	21%	19%	17%	29%					
CS.	13%	31%	29%	31%	24%	38%				
Tel	0%	2%	2%	0%	10%	0%	5%			
Utl	0%	10%	5%	3%	7%	2%	0%	0%		
Fin	8%	43%	36%	36%	21%	36%	7%	2%	48%	
Tech	12%	3%	7%	12%	10%	7%	3%	3%	13%	0%
Panel B: 10% level	O&G	BM	Ind	CG	Hlth	CS	Tel	Utl	Fin	Tech
O&G	50%									
BM	29%	62%								
Ind	46%	64%	71%							
CG	19%	42%	56%	40%						
Hlthc	38%	40%	48%	36%	52%					
CS.	25%	52%	64%	50%	45%	76%				
Tel	13%	17%	7%	8%	19%	17%	5%			
Utl	17%	14%	17%	8%	21%	14%	2%	5%		
Fin	42%	60%	50%	42%	50%	67%	21%	24%	67%	
Tech	24%	23%	23%	28%	20%	30%	10%	7%	27%	20%

Correlations between Telecom or Utilities sectors seem to be constant over time, only 5% of all observations on Telecom-Telecom correlations and Utilities-Utilities are not constant. Other sector combinations are more subject to changing correlations. Consumer Services-Consumer Services, Industries-Industries and Financials-Financials show the highest proportions of not constant correlation coefficients with 76%, 71% and 67% at the 10% level.

Based on this analysis we can make some first conclusions. Total market correlations show a rather strong tendency to vary through time. This non-constancy seems mostly be due to the non-constancy of sector correlations within the markets. A few sector combinations between countries have a higher proportion of non-constant coefficients than most others, especially same sector combinations such as Consumer Services-Consumer Services, Industries-Industries and Financials-Financials, contributing probably a great deal to the non-constancy at the market level. How these non-constant sectoral correlations contribute exactly to the evolution of the correlation at the market level can however not be explained using only this test. It could be for example that although sector correlations within countries are unstable, they do not influence in a major way the market correlations as some sector correlations may have increased whereas others have decreased over time, the two effects offsetting each other when taken together. To really understand the results we obtained in this section, we must study the interaction between sector correlations and market correlations and then look at the evolution over time. This is what we are doing in the following two sections.

5 A correlation decomposition

In this part, we will now show in an easy way how sector correlations and the correlation at the market level are linked. Formally, we start from the definition of the correlation coefficient between country A with m sectors and country B with n sectors:

$$\rho_{A,B} = \frac{Cov(A,B)}{\sigma_A \sigma_B}$$

which can be rewritten as

$$\rho_{A,B} = \frac{\sum_{i=1}^m \sum_{j=1}^n w_i z_j \sigma_{A_i} \sigma_{B_j} \rho_{A_i, B_j}}{\sigma_A \sigma_B}$$

where w_i is the weight of sector i in country A, z_j the weight of sector j in country B and ρ_{A_i, B_j} the correlation coefficient between sector i in country A and sector j in country B.

Using the fact that the correlation coefficient can be obtained using standardized returns, we standardize all sector returns and we can then write

$$\rho_{A,B} = \frac{1}{\tilde{\sigma}_A \tilde{\sigma}_B} \sum_{i=1}^m \sum_{j=1}^n w_i z_j \rho_{A_i, B_j} \quad (1)$$

where

$$\tilde{\sigma}_A = \sqrt{\sum_{i=1}^m \sum_{j=1}^m w_i w_j \rho_{A_i, A_j}} \quad (2)$$

with $\rho_{A_i, A_j} = 1$ if $i = j$.

So the correlation coefficient between two stock market indices can be decomposed into two parts: a weighted average of the sector correlations and the inverse of the product of the markets standard deviation computed by using standardized sector returns. We will call the first part *WSC* for the weighted sector correlations and the second part *IPM* for the inverse of the product of the market standard deviations. The total market correlation will be called *TMC*.

Based on this decomposition we see that sector correlations influence the correlation between two markets in two ways: first we have the correlations between the sectors of the two countries, and second we have the correlations between sectors of the same country in the inverse of the product of the market standard deviations. We can claim therefore that not only the sector correlations between countries influence the correlation between two markets, but also the sector correlations within the countries are important.

From equation (1) and (2) we also see that the index weights influence the index correlation in two ways. Changes in index weights modify the *WSC* but also the *IPM*. The overall effect is difficult to forecast.

By adopting the same approach as Heston and Rouwenhorst, meaning that stock returns are determined by an industry effect, a country effect and some stock-specific effect, one possible interpretation of the *IPM* is that it reflects the country factor in stock returns. The lower the country factor, i.e. the lower the correlation between sectors in the same country, the higher the *IPM* and therefore we have a higher stock market correlation *TMC* everything else equal. The *WSC* part on the other hand reflects the importance of the industry factors, captured by the correlation between the sectors⁹. If the *WSC* part increases everything else equal, we observe an increase of the correlation between two stock markets. As mentioned in the introduction however, we believe that HR approach is a good way to explain sector correlations in an easy way, but that it does not capture sufficiently the complexity of the links between sectors.

⁹A small proof of this interpretation can be found in appendix.

5.1 The importance of sector correlations within a country

We can show the importance of the sector correlations within a country by looking at the following example. Imagine a country A which has the same sector correlations with country B as with country C . Moreover assume that country B and C have the same industrial structure such that $w_i^B = w_i^C$. We therefore have

$$\sum_{i=1}^m \sum_{j=1}^n w_i z_j \rho_{A_i, B_j} = \sum_{i=1}^m \sum_{j=1}^n w_i z_j \rho_{A_i, C_j} = \bar{\rho}$$

and

$$\rho_{A,j} = \frac{1}{\tilde{\sigma}_A \tilde{\sigma}_j} \bar{\rho} \quad \text{with } j = B, C$$

The possible difference in the correlation coefficient between country A and B and the correlation coefficient between country A and C would then come from the term $\tilde{\sigma}_j$ which depends strongly on the magnitude of the sector correlations within country B and C . Assume that $\bar{\rho} = 0.3$, $\tilde{\sigma}_A = 0.8$ and that country B has on average higher sector correlations than country C such that we have $\tilde{\sigma}_B = 0.9 > \tilde{\sigma}_C = 0.7$. We obtain then a correlation of 0.42 between country A and country B and a correlation of 0.54 between A and C . The difference of 0.12 only being due to the difference in inner country sector correlations.

5.2 Empirical application

Table 7 shows the total market correlations obtained by applying equation (1) to our data. We use the average weights over the whole time period for the different sector weights. This approach as well as our restrictions on sector inclusion in our data as explained in section 2 lead to a small bias in our computations of the total market correlation. As shown in table 8, this bias is however quite small, as the correlation coefficient using the decomposition approach has an average absolute difference of only 3.1% compared to the correlation coefficient computed using the index returns provided by Datastream.

Table 7: Total Market Correlations computed using the simple decomposition

This table shows the correlations between the returns of seven major stock markets computed using the simple decomposition approach. The correlations were computed over our whole data period, i.e. from 01.01.1973 to 29.03.2006 using the Total Market Indices provided by Datastream. The weights used for the different sectors were the average weights over the whole period.

	US	UK	Canada	France	Germany	Switzerland
UK	0.51	–				
Canada	0.72	0.49	–			
France	0.49	0.53	0.46	–		
Germany	0.48	0.46	0.45	0.63	–	
Switzerland	0.54	0.52	0.49	0.61	0.70	–
Japan	0.32	0.31	0.31	0.34	0.36	0.35

Tables 9 and 10 present the results when we decompose the total market correlation coefficient into two parts, the weighted average of sector correlations WSC (table 9) and the inverse of the product of the market standard deviations using standardized sector returns IPM (table 10).

By looking at the WSC , we see that also on the sector level, the highest correlations are observed between countries that are geographically and culturally close (Germany-Switzerland with 0.48 and US-Canada with 0.41). The weighted average sector correlations are lowest when one of the two countries is Japan.

Table 8: Difference between correlations computed using sector return data and using index return data

This table shows the absolute difference in % between total market correlations computed using sector return data and equation (1) and using index return data.

	US	UK	Canada	France	Germany	Switzerland
UK	1.4%	–				
Canada	0.3%	4.5%	–			
France	1.8%	5.7%	6.2%	–		
Germany	1.8%	4.4%	4.4%	5.7%	–	
Switzerland	0.0%	4.7%	4.4%	5.0%	1.8%	–
Japan	3.4%	4.4%	1.2%	2.6%	0.7%	1.1%

Table 9: Weighted average sector correlations: WSC

	US	UK	Canada	France	Germany	Switzerland
UK	0.34	–				
Canada	0.41	0.28	–			
France	0.31	0.34	0.26	–		
Germany	0.32	0.31	0.26	0.41	–	
Switzerland	0.36	0.35	0.29	0.40	0.48	–
Japan	0.21	0.20	0.18	0.21	0.24	0.23

Table 10: The inverse of the product of market standard deviations: IPM

	US	UK	Canada	France	Germany	Switzerland
UK	1.50	–				
Canada	1.76	1.74	–			
France	1.56	1.54	1.80	–		
Germany	1.50	1.48	1.73	1.54	–	
Switzerland	1.48	1.46	1.71	1.52	1.46	–
Japan	1.53	1.51	1.77	1.57	1.51	1.49

The *IPM* is on average 1.58, meaning that the total market correlation coefficient is on average 58% higher than the weighted average sector correlation coefficient. This gives us the explanation why in section 3 we observe that the total market correlation is on average higher than any sector correlation. It is of course possible to observe a higher sector correlation than the total market correlation, however this sector correlation must be much higher than the average sector correlation to make it happen.

Based on table 9 and 10 we can explain why total market correlation between Canada and the other markets is relatively high, we observe for example a higher correlation between the UK and Canada than between the UK and Germany - both being EU members. The *IPM* part is particularly high when Canada is involved; 1.75 on average when one of the country is Canada compared to 1.51 if we exclude Canada. This is due to the low average sector correlations within the Canadian markets compared to the sector correlations within the other markets. Although Switzerland has a low average sector correlation, the values of the *IPM* part involving the Swiss market, are below average - between 1.46 and 1.52, when excluding the one with Canada. This result is due to the industrial structure of the

Swiss market¹⁰ with its high concentration in the Healthcare sector and the Financial sector. These two sectors account over the period from 01.01.1973 to 29.03.2006 on average for three quarters of the Swiss market capitalization. Therefore the difference between a weighted average and a simple average is particularly important for the Swiss market.

5.3 Index weights

As mentioned above, it is clear from equation (1) and (2) that not only the sector correlations - between markets and within markets - play a role in determining the correlation between two stock indices, but that we also have to consider the index weights. Again as in the case of the sector correlations, index weights influence the *WSC* part as well as the *IPM* part.

In order to have an idea of the importance of different index weights on the total market correlation, we compute the coefficients using equal sector weights for each country and compare the results to the one we obtain using the average sector weights over the whole time period.

By looking at the average sector weights given in appendix B, we already see that the biggest difference is to be expected for correlation coefficients where one of the two countries is Switzerland, as this market has the most unequal structure.

We first compute the total market correlation coefficients using equal sector weights and compare them to the ones reported in table 7. The results are given in table 11 for the coefficients and in table 12 for differences with respect to table 7. The average difference is 4.8%, meaning that if we would have used equal sector weights to compute the index correlations, our average error would have been quite low. By looking at the errors by country pair, we see that there are some cases where the change from the real average weights to equal weights has some importance for the coefficients. Country pairs where we either have Canada, Switzerland or both show some rather large differences compared to the others - up to 11.3% for the country pair Canada - Germany.

We can actually explain this rather low average difference, by looking at the two parts *WSC* and *IPM* in the case of real average weights and equal weights. The differences in percentage between *WSC* and *IPM* computed using equal sector returns and *WSC* and *IPM* computed using average sector weights over the whole period are given in table 13.¹¹

¹⁰ An overview of the sector weights for each country is given in Appendix B.

¹¹ The tables with the values of *WSC* and *IPM* for each country pair using equal sector weights are given in appendix C.

Table 11: Total Market Correlations computed using the simple decomposition and equal sector weights

Correlations between the returns of seven major stock markets computed using the simple decomposition approach and equal sector weights.

	US	UK	Canada	France	Germany	Switzerland
UK	0.52	–				
Canada	0.75	0.52	–			
France	0.51	0.56	0.51	–		
Germany	0.48	0.48	0.50	0.65	–	
Switzerland	0.55	0.55	0.54	0.67	0.71	–
Japan	0.33	0.32	0.33	0.34	0.36	0.37

Table 12: Absolute differences between correlations computed using average index weights and equal index weights

Difference in % between total market correlations computed using sector return data and average sector weights (table 5) and total market correlations computed using sector return data and equal sector weights (table 8)

	US	UK	Canada	France	Germany	Switzerland
UK	1.3%	–				
Canada	4.4%	6.7%	–			
France	3.2%	5.6%	11.0%	–		
Germany	1.2%	2.7%	11.3%	3.9%	–	
Switzerland	1.5%	6.6%	10.9%	9.6%	2.2%	–
Japan	1.1%	3.3%	5.7%	1.1%	–1.0%	6.9%

Table 13: Differences in WSC and IPM

<i>WSC</i>	US	UK	Canada	France	Germany	Switzerland
UK	-5.2%					
Canada	-1.3%	-3.4%				
France	0.8%	-1.3%	4.9%			
Germany	-7.4%	-10.1%	-1.5%	-5.1%		
Switzerland	-10.9%	-10.4%	-5.8%	-3.9%	-16.1%	
Japan	-4.0%	-6.1%	-2.9%	-4.0%	-12.0%	-8.8%
<i>IPM</i>	US	UK	Canada	France	Germany	Switzerland
UK	6.8%					
Canada	5.7%	10.5%				
France	2.4%	7.0%	5.9%			
Germany	9.3%	14.2%	13.0%	9.4%		
Switzerland	13.9%	19.0%	17.8%	14.0%	21.7%	
Japan	5.2%	10.0%	8.8%	5.4%	12.5%	17.2%

Based on these results we see that changing index weights from the average weights to equal weights has in general opposite effects on *WSC* and *IPM*. We observe a decrease in the weighted average of sectoral correlations of -5.5% on average and an increase in the *IPM* part of 10.9% on average. The decrease in *WSC* is not observed for two country pairs: US-France and France-Canada.

The change in weights seem to affect *WSC* and *IPM* in opposite ways resulting in a rather low change in the total market correlation coefficient¹². It is intuitive that we observe opposite effects on *WSC* and *IPM*, as for example giving less weight to sectors with rather high correlations lowers the *WSC* part and increases the *IPM* part, but it is not necessary the case as shown by the fact that the decrease in *WSC* is not observed for two country pairs: US-France and France-Canada.

From table 13, we can also deduce that sectors with rather high correlations coefficients have larger weights in the indices as changing the weights from the actual average ones to equal ones lowers the *WSC* part.

6 Evolution over time

In section 4 we looked at the constancy of the correlation coefficients using a test applied over the whole time period. Now we are going to change our

¹²We obtain the total effect by computing $1.109 * 0.945 = 1.048 = 4.8\%$.

methodology and we will now look at the evolution over time of the index correlation coefficients and the sector correlations.

6.1 Methodology

There exist different ways to compute time-varying correlation¹³. The easiest and most often used is the rolling correlation estimator. As pointed out by Engle (2002) it is a very convenient estimator for a conditional correlation coefficient as it always lies in the interval $[-1, 1]$. However, the problem with this estimator is that it gives equal weights to all observations which lie inside the rolling window and zero to all observations outside the window and that there is no clear condition on how to determine the length of the window.

Multivariate GARCH models are the obvious choice to estimate time-varying conditional correlation coefficients. Several different methodologies have been proposed, such as the principle component GARCH method or different forms of the so called BEKK GARCH. Among one of the latest approaches to estimate in a convenient way time varying correlations in a multivariate setting is the Dynamic Conditional Correlation GARCH model presented by Engle (2002). This model allows to estimate large correlations matrices in a relatively fast way. It is based on a two stage procedure where first univariate GARCH models for each asset's variance are estimated, and then in a second step using transformed residuals from the first step a time-varying conditional correlation coefficient. Engle proposes the following dynamic correlation structure:

$$Q_t = (1 - \sum_{m=1}^M \alpha_m - \sum_{n=1}^N \beta_n) \bar{Q} + \sum_{m=1}^M \alpha_m (\epsilon_{t-m} \epsilon'_{t-m}) + \sum_{n=1}^N \beta_n Q_{t-n}$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}$$

where R_t is the time-varying correlation matrix, \bar{Q} the unconditional covariance of the standardized residuals resulting from the first stage estimation and Q_t^* a diagonal matrix composed of the square root of the diagonal elements of Q_t . The elements of R_t will thus be of the form $\rho_{ijt} = \frac{q_{ijt}}{\sqrt{q_{ii}q_{jj}}}$. In their working paper, Engle and Sheppard (2001) show that the requirements for positive definiteness of the conditional covariance are the same for the DCC model as for a univariate GARCH process.

The DCC model was proposed with the idea in mind of estimating large correlation matrices in the case of several assets as it reduces the number of parameters considerably compared to other multivariate GARCH specifications.

¹³A good overview is provided in the article by Engle (2002).

In our case this model would allow us to estimate all sector correlations between two countries as well as the correlations between sectors in the same country at once. However, by doing this we would impose the same dynamic structure on all correlation coefficients. Possible existing differences in the evolution through time in sector correlations would become impossible to detect.

Therefore, we use the DCC GARCH methodology individually on all our 1753 sector correlations as well as on all sector correlations within the countries in order to capture possible difference in the behavior of the sector correlations coefficients.

It is clear that the main argument of the DCC GARCH model, its easiness in estimating large correlation matrices in a multivariate setting, does not apply anymore in these bivariate settings such as ours. There are numerous ways on how to estimate time-varying correlations in this situation. However, we stick to the DCC GARCH model because Engle compares in his article (2002) different correlation estimators in a bivariate setting and concludes that the DCC model performs overall better than the others.

6.2 Average evolution of TMC, WSC and IPM

As mentioned above we estimate the DCC model for all 1753 sector combinations possible between our 7 countries as well as all sector combinations within each country. Based on these results we construct then the Total Market Correlation using the formulas described in section 5. We first average then all our computations of the *TMC* and its two parts *WSC* and *IPM* across all 21 country pairs to get a first impression of the average evolution through time. A graphical representation is given in figure 1.

From 1973 through October 1987 the correlation between stock markets was quite low and rather constant, it fluctuates between 0.23 and 0.44. The October 1987 crash is highly visible in this figure and it is interesting to see that it had a permanent effect on the correlation between stock markets. Although there is a decrease in correlation after the crash, we never find the pre-crash levels. Another spike in the correlation coefficient is observed around August 1990, the month in which the Iraqi invasion of Kuwait took place. Again this spike wears slowly off and we observe then a steady increase in correlation starting with the Asian crisis in October 1997. Interestingly, the effect of the September 11, 2001 is not as visible as the effects of other major events. The correlation coefficients increased following the terrorist attacks, but not as dramatically as for example after the 1987 crash. During 2003 till 2005 we observe a downward tendency in correlation, but it seems as in 2006 this tendency has stopped and the correlation coefficient actually start increasing again.

From 1973 to 1998 the *WSC* part has a very similar pattern as the Total Market correlation and looks to be the factor that drives the changes

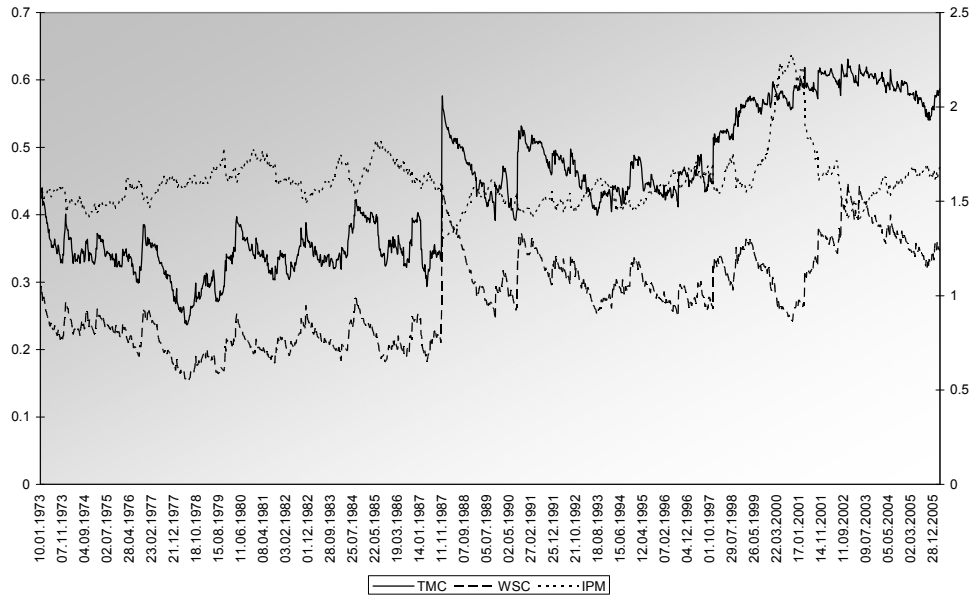


Figure 1: Average of TMC, WSC and IPM across all country pairs

in TMC. This is confirmed when looking at the rolling correlations between TMC and WSC and IPM that are shown in figure 2. The correlations are computed using a 200 observations window (about 4 years). We observe a very high correlation between TMC and WSC until the late nineties where it decreases heavily before it comes to very high values again in 2005. The IPM presents most of the time a negative correlation with the TMC , which means that an increase in the IPM part is usually accompanied by a drop in the TMC . An increase in the IPM part comes from a decrease in sector correlations within the countries; when sector correlations between markets decrease at the same time, then we observe a decrease in the WSC part at the same time. A negative correlation between TMC and IPM and a positive correlation between TMC and WSC shows that if sector correlations within countries and between countries evolve in the same direction, the WSC part dominates the IPM part and determines the movement of the TMC . This picture is observed for the period from 1976 to 1979. In 1979, we observe an increase in correlation between the TMC and the IPM part leading to positive values for the years 1980 to 1982. As we compute rolling correlations over 4 years, we capture the second oil crisis from 1979 during this period of positive correlation between TMC and IPM and it seems the most likely explanation for this episode. In figure 1, we see that starting in spring 1979 an increase in TMC , WSC and IPM over the next few months. The increase of the oil price was most likely perceived as a factor

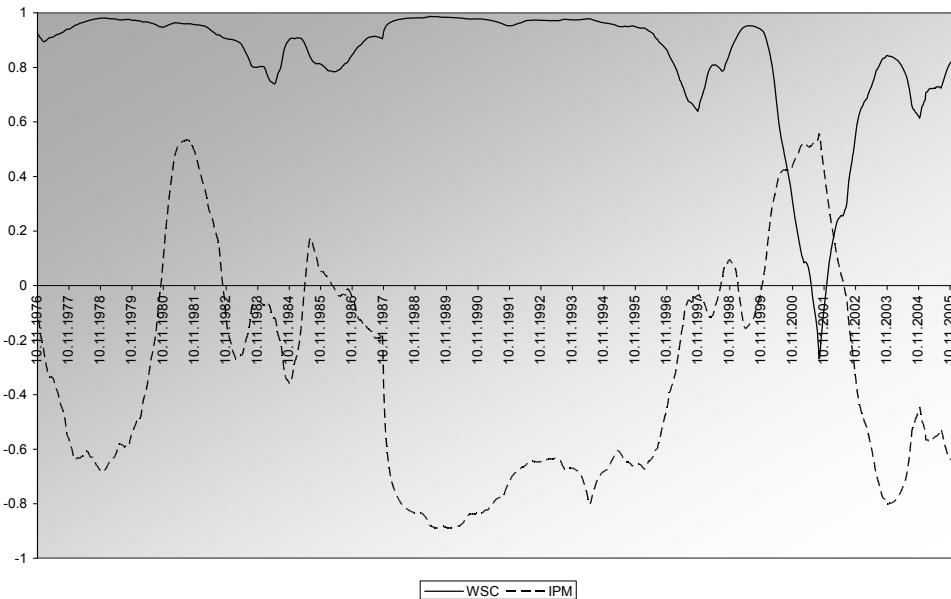


Figure 2: Rolling correlations between TMC and WSC/IPM (200 weeks window)

affecting the whole economy, therefore it can be considered as a world factor that increased in importance, pushing down the importance of country factors. The decrease of importance of country factors must have been more important relatively to the increase in the world factor, driving down sector correlations within countries and therefore leading to an increase in the IPM . At the same time we see that the WSC increased, indicating that the increase in the world factor was more important than any possible decrease in industry factors. The overall effect on TMC was an increase. Another small episode of positive correlations between TMC and IPM is observed during the mid eighties. After the stock market crash 1987 we observe highly negative correlations between the TMC and IPM .

In late 1998 when the dot-com-bubble started to build up, things change considerably. In figure 1, we observe a dramatic decrease in WSC and a steep increase in IPM , the overall effect on the TMC being quite low. For the first time in our data period, we observe that the TMC pattern is detached from the WSC as also shown by the dramatic decrease in the correlation between these two in figure 2. One could think that the dot-com-bubble led to important changes in index weights, giving more weight to the Technology and Telecom sectors and that the decline in WSC is due to this assuming that these two sectors have rather low correlations with other sectors. However, this argument does not hold, because if we compare

the equally weighted average of sector correlations, where changing index weights have no importance, to the weighted average of sector correlations using the index weights of the sector, we find the same pattern. Sector correlations decreased during the building up of the dot-com-bubble leading to the conclusion that industry factors became less important during this period. A phenomenon that was not observed during the second oil crisis.

The *IPM* increased sharply indicating that the sector correlations within the countries must have gone down or said in an other way country factors became less important as well during the dot-com-bubble. We also find again the pattern in correlation between *TMC* and *IPM* that was observed during the second oil crisis.

The simultaneous change in *WSC* and *IPM* in opposite directions is not an observation that is limited to this period; it is something that is visible all along our time period. We think that these changes are due to some world factors affecting all sector correlations the same way.

What makes the period around the dot-com-bubble interesting is that these changes in *WSC* and *IPM* compensate each other and that there is almost no effect on the Total Market correlation coefficient. In all other cases where we have important changes in *WSC* and *IPM*, we assist at a change in *TMC*, reflecting the change in the *WSC* part. The changes in *WSC* determine the changes in *TMC*, changes in *IPM* being without much effect on *TMC*.

During the building up of the dot-com-bubble, some other factor(s) lowered all sector correlations between countries as well as within countries. These factors had a slightly different effect on *WSC* and *IPM* than before, as they affect *IPM* more than *WSC* resulting in a very small overall effect on the index correlation coefficient.

We suspect that one of the most likely factor to have provoked this is the exaggerated enthusiasm for the new economy during this period, pushing back all other factors and affecting all sectors in more or less the same way. A lot was said at this time about how the world will become a village, how the frontiers will become less and less important. It is obvious to think that such a view of the future would result in an important decrease of the importance of country factors in stock returns. It also affected the importance of the industry factors, but less than the country factors.

After the burst of the dot-com-bubble, we observe an increase in *WSC* and a decrease in *IPM*, which can be interpreted as the disappearing of this world factor that lowered all correlation coefficients at the sectoral level.

Linear trend regressions on *TMC*, *WSC* and *IPM* over the whole period show that all three have a significant upward trend. The *TMC* has an increasing trend of about 0.9% per year, resulting in an increase of around 34% over the whole period. The *WSC* part is characterized by an upward trend of around 0.5% per year and an overall increase of roughly 17%. Finally, the *IPM* part has a trend of plus 0.3% per year, leading to an increase

of 11% over the observed period. It is clear that the positive trend of the *IPM* is driven by the spike around the dot-com-bubble. Taking this episode out, we don't find any significant trend for *IPM*. Although there is no robust trend for *IPM*, the average value over the last two years is about 7% higher than it was at the beginning in 1973.

Figure 1 together with these trend regressions show that stock market correlations have increased over the last 33 years and that this increase was largely driven by an increase in the sector correlations between countries. The industry factors have gained in importance, reflected by the increase in *WSC*, whereas the country factors remained more or less constant when not taking into account the dot-com-bubble period. If one includes this episode, the conclusion on the *IPM* would change from constant country factors to decreasing country factors as the *IPM* shows an upward trend and therefore the sector correlations within a country must have decreased. This result, increasing importance of sector factors and decreasing importance of country factors, has been found by other authors using other methodologies.

The time trend regressions are particularly interesting when they are performed using only European data. The trend for the *TMC* is in this case equal to plus 1.6% per year, which comes down to an increase of almost 67% of the total market correlation coefficients over the last 33 years. The *WSC* part increased by 0.9% per year or 34% over the whole period. Finally, the *IPM* part went up by 0.56% per year which equals to plus 19% since 1973. Again, the *IPM* part results must be interpreted carefully, but changes in correlation were more pronounced in Europe, a result that is most likely due to the European integration.

6.3 Sector Correlations

After having looked at the general evolution of stock market correlations over the last 33 years, we now look at the evolution of sector correlations over time. For each of the 55 possible sector combinations, we take the average over our 21 country pairs and compute the correlation between the average sector correlation and the total market correlation. This allows us to identify the sector correlations that have had a similar pattern through time like the average index correlation.

Based on figure 1 and figure 2, it is clear that the pattern of the sector correlations between markets determine the pattern of the index correlation and not the other way round. By looking at the individual sector correlation coefficient, we can identify the sector combinations that are most determining for the *TMC*.

The results are presented in table 14, where we show the 15 highest values and the 5 lowest values. The correlation coefficients between the different sector correlations and the total market correlations lie between 0.77 for the sector combination that is most correlated with *TMC* and 0.05 for the one

that is least correlated. It is very interesting to compare these results to the ones we obtained by applying the Bera-Kim test for constancy in correlation to the different sector correlation coefficients. We can do this by looking at column 4 in table 18, where we report the rank of the different sector correlations in table 14, panel B. This table gave us the percentage of observed non-constancy in each sector combination possible across all our 21 country pairs. Rank 1 is given to the sector correlation with the highest percentage of non-constant observations and rank 55 to the sector correlations with the lowest percentage. By confronting these ranks with the results from the DCC correlation modelling, we observe that the sector correlations presenting a high number of non-constant observations show up on top in table 18. The highest correlation coefficient on average between the *TMC* and the sectors, is found for the sector correlation Industries-Industries, which is the sector combination with the second highest percentage of non-constant observations. On second place we find Financials-Financials, again a sector combination that is characterized by a high level of non-constancy.

The sector pair with the highest number of non-constant coefficient, which is Consumer Services-Consumer Services, is found at position 5 in table 14. Except for combinations where one of both sectors are the Technology sector, we find in the top 15 of the highest correlation coefficient with the *TMC* only sector pairs that show a rather high number of non-constant coefficients when applying the Bera-Kim test. The technology sector seems to be an exception, with a quite low number of non-constant coefficients, but a rather high correlation with the *TMC*.

The same pattern is visible when focusing on the bottom of table 18, where we have the lowest 5 correlation coefficients between sectors and the *TMC*. A low correlation is associated with a low number of non-constant observations. The lowest correlation is observed for the sector pair Utilities-Telecom, the same sector pair for which we observed the lowest percentage of non-constant observations in table 6.

Table 14: DCC sector correlation analysis and comparison with the Bera-Kim Test Results

Rank DCC	Correlation with TMC	Sector Combination	Rank Bera-Kim
1	0.7685	IND IND	2
2	0.7584	FIN FIN	3
3	0.7394	IND CS	5
4	0.7354	TECH TECH	37
5	0.7339	CS CS	1
6	0.7263	CS TECH	26
7	0.71	BM BM	7
8	0.6984	IND TECH	33
9	0.6928	IND FIN	12
10	0.6902	BM IND	8
11	0.688	CS FIN	3
12	0.659	BM FIN	8
13	0.6578	CG FIN	19
14	0.657	BM CS	10
15	0.6422	HLTH HLTH	10
⋮	⋮	⋮	⋮
51	0.1675	OG TECH	52
52	0.1414	OG TEL	47
53	0.1181	UTL UTL	53
54	0.0978	UTL TECH	52
55	0.0462	TEL UTL	55

Overall, the results of the two methodologies fit together well and allow us to draw the conclusion that the pattern of the index correlation coefficient is determined by a few sector correlations. As shown with the Bera-Kim test, there are a some sector pairs that present a high fraction of non-constancy in their correlation and we observe that these are exactly the sector pairs for which we observe the highest correlation with *TMC*. These sector pairs seem to be responsible for the time-varying nature of the index correlation coefficient. By looking at these different sector combinations, we see that same sector pairs, like Industries-Industries, Consumer Services-Consumer Services etc., play an important role in it. Moreover, we can say that the three sectors Industries, Consumer Services and Financials play a major role in determining the index correlation as they figure in twelve of the top 15 coefficient in table 14 and present the highest fraction of non-constant observations as shown in table 16. Basic Materials and Technology are two sectors which are also of importance.

7 Conclusion

The goal of this study is to shed some light on the time-varying nature of international stock market correlation coefficients by using a bottom up approach with the computation of correlations using only coefficients at the sectoral level. This method allows us to show that market correlations are on average higher than correlations at the sectoral level; a result that can be explained by the presence of the sector correlations within each country. Using the test proposed by Bera and Kim (2002) we find that sectoral correlations between markets are more stable over time than correlations at the market level as well as sector correlations within countries. Sectors such as Industry, Financials and Consumer Services present however a rather high proportion of inconstant correlation coefficients. The same sectors show up on top of the list of the most correlated sectors correlations with the market correlations as presented in the section 6. This leads us to the conclusion that these sectors are important in determining the correlation and its evolution at the market level.

Our results show that focusing at the sectoral level in studying international stock correlations gives some interesting insights. We think that there is still more to find on the behavior of international stock market correlations using this approach and that this is certainly a suitable way to understand what drives correlations.

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Appendix A: Number of observations

A.1 Number of sectors per country pair

	US	UK	Canada	France	Germany	Switzerland
UK	80	-				
Canada	100	80	-			
France	100	80	100	-		
Germany	90	72	90	90	-	
Switzerland	80	64	80	80	72	-
Japan	90	72	90	90	81	72

A.2 Number of observations per sector pair

	O&G	BM	Ind	CG	Hlth	CS	Tel	Utl	Fin	Tech
O&G	6									
BM	24	21								
Ind	24	42	21							
CG	21	36	36	15						
Hlth	24	42	42	36	21					
CS	24	42	42	36	42	21				
Tel	24	42	42	36	42	42	21			
Utl	24	42	42	36	42	42	42	21		
Fin	24	42	42	36	42	42	42	42	21	
Tech	17	30	30	25	30	30	30	30	30	10

Appendix B: Sector weights per country

Weights	O&G	BM	Ind	CG	Hlth	CS	Tel	Utl	Fin	Tech	
US	Mean	10.5%	6.4%	9.1%	4.6%	18.2%	12.4%	7.0%	6.7%	11.8%	12.3%
	Min	3.9%	1.4%	7.3%	1.2%	12.1%	7.8%	2.6%	2.5%	4.9%	6.3%
	Max	26.3%	11.9%	12.0%	8.6%	27.6%	16.2%	11.2%	10.6%	23.3%	34.8%
UK	Mean	15.4%	9.4%	7.0%	17.4%	16.7%	7.7%	4.6%	21.6%		
	Min	7.8%	2.1%	1.7%	7.8%	12.1%	0.9%	2.3%	12.9%		
	Max	27.6%	14.6%	16.5%	22.7%	23.6%	23.0%	7.2%	29.8%		
Canada	Mean	21.3%	15.7%	6.7%	1.3%	6.4%	8.6%	2.9%	5.4%	20.0%	12.5%
	Min	3.3%	4.7%	1.5%	0.5%	0.1%	0.0%	0.8%	2.2%	4.5%	1.9%
	Max	35.7%	40.7%	17.1%	2.2%	12.3%	14.4%	14.3%	16.9%	34.1%	39.2%
France	Mean	15.0%	11.0%	15.7%	11.1%	14.8%	8.3%	7.6%	2.6%	11.8%	5.2%
	Min	6.3%	2.7%	3.9%	3.6%	7.4%	3.4%	3.3%	0.8%	1.7%	0.4%
	Max	39.2%	16.9%	26.2%	26.6%	24.1%	16.0%	15.1%	11.2%	22.7%	15.0%
Germany	Mean	16.1%	16.1%	22.6%	11.1%	5.0%	4.0%	5.9%	4.9%	27.9%	5.1%
	Min	5.8%	5.8%	8.8%	6.3%	2.8%	2.2%	2.0%	1.7%	15.7%	0.5%
	Max	22.4%	22.4%	30.3%	18.9%	10.1%	7.9%	32.2%	13.2%	40.9%	14.9%
Switzerland	Mean	4.7%	4.7%	9.3%	2.5%	51.4%	4.2%	3.3%	3.3%	24.5%	
	Min	2.9%	2.9%	3.0%	0.6%	32.0%	0.9%	1.9%	0.4%	9.3%	
	Max	7.2%	7.2%	15.9%	16.9%	73.6%	8.1%	4.8%	8.7%	39.7%	
Japan	Mean	13.3%	13.3%	12.8%	11.3%	7.5%	13.3%	5.5%	5.6%	22.9%	6.8%
	Min	5.2%	5.2%	5.8%	4.3%	4.1%	9.5%	0.1%	2.1%	13.3%	3.0%
	Max	21.7%	21.7%	18.2%	21.3%	10.9%	16.1%	16.7%	11.9%	37.1%	19.0%

Appendix C: Link between Heston and Rouwenhorst 1994 and WSC/IPM

Return decomposition like Heston and Rouwenhorst 1994

$$R_{ict} = \alpha_t + \beta_{it} + \gamma_{ct} + \varepsilon_{ict}$$

where R_{ict} is the return of an equity (local industry) that belongs to the global industry i and to country c . α_t is the return common to all assets (sectors), β_{it} is the industry effect specific to the global industry i and γ_{ct} the country effect specific to country c . ε_{ict} is a firm (local industry) specific disturbance. By construction, we have

$$\begin{aligned} Cov(\beta_{it}, \beta_{jt}) &= 0 \\ Cov(\beta_{it}, \gamma_{ct}) &= 0 \\ Cov(\gamma_{ct}, \gamma_{dt}) &= 0 \\ Cov(\beta_{it}, \varepsilon_{ict}) &= 0 \\ Cov(\gamma_{ct}, \varepsilon_{ict}) &= 0 \\ Cov(\varepsilon_{ict}, \varepsilon_{jct}) &= 0 \end{aligned}$$

not so sure about, but should be verified as well

$$\begin{aligned} Cov(\alpha_t, \beta_{it}) &= 0 \\ Cov(\alpha_t, \gamma_{ct}) &= 0 \end{aligned}$$

Individual security/local sector covariances

In this context, how can we describe the return covariances:

Case 1 Different industry sector and different country

$$\begin{aligned} R_{ict} &= \alpha_t + \beta_{it} + \gamma_{ct} + \varepsilon_{ict} \\ R_{jbt} &= \alpha_t + \beta_{jt} + \gamma_{bt} + \varepsilon_{jbt} \end{aligned}$$

$$\begin{aligned} Cov(R_{ict}, R_{jbt}) &= Cov(\alpha_t + \beta_{it} + \gamma_{ct} + \varepsilon_{ict}, \alpha_t + \beta_{jt} + \gamma_{bt} + \varepsilon_{jbt}) \\ &= \underbrace{Cov(\alpha_t, \alpha_t)}_{=Var(\alpha_t)} + \underbrace{Cov(\alpha_t, \beta_{jt})}_{=0} + \underbrace{Cov(\alpha_t, \gamma_{bt})}_{=0} + \underbrace{Cov(\alpha_t, \varepsilon_{jbt})}_{=0} \\ &\quad + \underbrace{Cov(\beta_{it}, \alpha_t)}_{=0} + \underbrace{Cov(\beta_{it}, \beta_{jt})}_{=0} + \underbrace{Cov(\beta_{it}, \gamma_{bt})}_{=0} + \underbrace{Cov(\beta_{it}, \varepsilon_{jbt})}_{=0} \\ &\quad + \underbrace{Cov(\gamma_{ct}, \alpha_t)}_{=0} + \underbrace{Cov(\gamma_{ct}, \beta_{jt})}_{=0} + \underbrace{Cov(\gamma_{ct}, \gamma_{bt})}_{=0} + \underbrace{Cov(\gamma_{ct}, \varepsilon_{jbt})}_{=0} \\ &\quad + \underbrace{Cov(\varepsilon_{ict}, \alpha_t)}_{=0} + \underbrace{Cov(\varepsilon_{ict}, \beta_{jt})}_{=0} + \underbrace{Cov(\varepsilon_{ict}, \gamma_{bt})}_{=0} + \underbrace{Cov(\varepsilon_{ict}, \varepsilon_{jbt})}_{=0} \\ &= Var(\alpha_t) \end{aligned}$$

Case 2 *Same industry sector and different countries*

$$\begin{aligned} R_{ict} &= \alpha_t + \beta_{it} + \gamma_{ct} + \varepsilon_{ict} \\ R_{jbt} &= \alpha_t + \beta_{it} + \gamma_{bt} + \varepsilon_{jbt} \end{aligned}$$

$$\begin{aligned} Cov(R_{ict}, R_{jbt}) &= Cov(\alpha_t + \beta_{it} + \gamma_{ct} + \varepsilon_{ict}, \alpha_t + \beta_{it} + \gamma_{bt} + \varepsilon_{jbt}) \\ &= \underbrace{Cov(\alpha_t, \alpha_t)}_{=Var(\alpha_t)} + \underbrace{Cov(\alpha_t, \beta_{it})}_{=0} + \underbrace{Cov(\alpha_t, \gamma_{bt})}_{=0} + \underbrace{Cov(\alpha_t, \varepsilon_{jbt})}_{=0} \\ &\quad + \underbrace{Cov(\beta_{it}, \alpha_t)}_{=0} + \underbrace{Cov(\beta_{it}, \beta_{it})}_{=Var(\beta_{it})} + \underbrace{Cov(\beta_{it}, \gamma_{bt})}_{=0} + \underbrace{Cov(\beta_{it}, \varepsilon_{jbt})}_{=0} \\ &\quad + \underbrace{Cov(\gamma_{ct}, \alpha_t)}_{=0} + \underbrace{Cov(\gamma_{ct}, \beta_{it})}_{=0} + \underbrace{Cov(\gamma_{ct}, \gamma_{bt})}_{=0} + \underbrace{Cov(\gamma_{ct}, \varepsilon_{jbt})}_{=0} \\ &\quad + \underbrace{Cov(\varepsilon_{ict}, \alpha_t)}_{=0} + \underbrace{Cov(\varepsilon_{ict}, \beta_{it})}_{=0} + \underbrace{Cov(\varepsilon_{ict}, \gamma_{bt})}_{=0} + \underbrace{Cov(\varepsilon_{ict}, \varepsilon_{jbt})}_{=0} \\ &= Var(\alpha_t) + Var(\beta_{it}) \end{aligned}$$

Case 3 *Different industry sector, but same country*

$$\begin{aligned} R_{ict} &= \alpha_t + \beta_{it} + \gamma_{ct} + \varepsilon_{ict} \\ R_{jct} &= \alpha_t + \beta_{jt} + \gamma_{ct} + \varepsilon_{jct} \end{aligned}$$

$$\begin{aligned} Cov(R_{ict}, R_{jct}) &= Cov(\alpha_t + \beta_{it} + \gamma_{ct} + \varepsilon_{ict}, \alpha_t + \beta_{jt} + \gamma_{ct} + \varepsilon_{jct}) \\ &= Var(\alpha_t) + Var(\gamma_{ct}) \end{aligned}$$

WSC part

The WSC part is defined as :

$$\sum_{i=1}^m \sum_{j=1}^n w_i z_j \rho_{A_i, B_j}$$

The correlation between two sectors A and B is given by:

$$\rho_{A_i, B_j} = \frac{\sum_{i=1}^k \sum_{j=1}^l a_i b_j Cov(R_i, R_j)}{\sigma_A \sigma_B}$$

with a_i being the weight of stock i in the sector index and R_i its return. Using standardized sector returns, we get:

$$\rho_{A_i, B_j} = \sum_{i=1}^k \sum_{j=1}^l a_i b_j Cov(R_i, R_j)$$

Using the Heston and Rouwenhorst return decomposition we obtain for the correlation between two different sectors between two different countries the following expression

$$\rho_{A_i, B_j} = \sum_{i=1}^k \sum_{j=1}^l a_i b_j \text{Var}(\alpha_t) = \text{Var}(\alpha_t)$$

and for the correlation between two same sectors between two different countries the expression below

$$\rho_{A_i, B_j} = \sum_{i=1}^k \sum_{j=1}^l a_i b_j (\text{Var}(\alpha_t) + \text{Var}(\beta_{it})) = \text{Var}(\alpha_t) + \text{Var}(\beta_{it})$$

The WSC part in our correlation formula can thus be expressed by

$$WSC = \sum_{i=1}^m \sum_{\substack{j=1 \\ j \neq i}}^n w_i z_j \text{Var}(\alpha_t) + \sum_{i=1}^m \sum_{\substack{j=1 \\ j=i}}^n w_i z_j (\text{Var}(\alpha_t) + \text{Var}(\beta_{it}))$$

which can be rewritten as:

$$WSC = \text{Var}(\alpha_t) + \sum_{i=1}^{\min(m,n)} w_i z_i \text{Var}(\beta_{it})$$

This formula shows clearly that the WSC part is related to industry factors ($\text{Var}(\beta_{it})$) and the so-called world factor $\text{Var}(\alpha_t)$.

IPM part

The IPM part is defined as:

$$IPM = \frac{1}{\tilde{\sigma}_A \tilde{\sigma}_B}$$

with

$$\tilde{\sigma}_A = \sqrt{\sum_{i=1}^m \sum_{j=1}^m w_i w_j \rho_{A_i, A_j}} = \sqrt{\sum_{\substack{i=1 \\ i \neq j}}^m \sum_{j=1}^m w_i w_j \rho_{A_i, A_j} + \sum_{i=1}^m w_i w_j}$$

where ρ_{A_i, A_j} is the correlation between two sectors in the same country. Such a correlation is defined by:

$$\rho_{A_i, A_j} = \frac{\sum_{i=1}^k \sum_{j=1}^l a_i b_j \text{Cov}(R_i, R_j)}{\sigma_A \sigma_B}$$

with a_i being the weight of stock i in the sector index and R_i its return. Again using standardized sector returns, we get:

$$\rho_{A_i, A_j} = \sum_{i=1}^k \sum_{j=1}^l a_i b_j \text{Cov}(R_i, R_j)$$

Using the Heston and Rouwenhorst approach, the covariance between two stocks, belonging to different sectors is given by:

$$\text{Cov}(R_{iAt}, R_{jAt}) = \text{Var}(\alpha_t) + \text{Var}(\gamma_{At})$$

therefore the correlation between two sectors in the same country is equal to:

$$\rho_{A_i, A_j} = \sum_{i=1}^k \sum_{j=1}^l a_i b_j (\text{Var}(\alpha_t) + \text{Var}(\gamma_{At})) = \text{Var}(\alpha_t) + \text{Var}(\gamma_{At})$$

and we get

$$\tilde{\sigma}_A = \sqrt{\sum_{i=1}^m \sum_{\substack{j=1 \\ i \neq j}}^m w_i w_j (\text{Var}(\alpha_t) + \text{Var}(\gamma_{At})) + \sum_{i=1}^m w_i w_j}$$

The IPM part is thus equal to:

$$IPM = \frac{1}{\tilde{\sigma}_A \tilde{\sigma}_B} = \frac{1}{\sqrt{\sum_{i=1}^m \sum_{\substack{j=1 \\ i \neq j}}^m w_i w_j (\text{Var}(\alpha_t) + \text{Var}(\gamma_{At})) + \sum_{i=1}^m w_i w_j} * \sqrt{\sum_{i=1}^m \sum_{\substack{j=1 \\ i \neq j}}^m w_i w_j (\text{Var}(\alpha_t) + \text{Var}(\gamma_{Bt})) + \sum_{i=1}^m w_i w_j}}$$

We see that the IPM part depends on the world factor $\text{Var}(\alpha_t)$ as well as the country factors $\text{Var}(\gamma_{At})$ and $\text{Var}(\gamma_{Bt})$.

Appendix D: *WSC* and *IPM* values for each country pair using equal sector weights

WSC computed using the simple decomposition approach and equal sector weights.

	US	UK	Canada	France	Germany	Switzerland
UK	0.32	–				
Canada	0.40	0.27	–			
France	0.32	0.34	0.27	–		
Germany	0.29	0.28	0.26	0.39	–	
Switzerland	0.32	0.32	0.27	0.39	0.40	–
Japan	0.20	0.19	0.17	0.21	0.21	0.21

IPM computed using the simple decomposition approach and equal sector weights.

	US	UK	Canada	France	Germany	Switzerland
UK	1.61	–				
Canada	1.86	1.92	–			
France	1.60	1.65	1.91	–		
Germany	1.64	1.70	1.96	1.69	–	
Switzerland	1.69	1.74	2.01	1.73	1.78	–
Japan	1.61	1.66	1.92	1.65	1.70	1.74