Security Design with Status Concerns*

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Abstract

This paper studies security design in a dynamic economy in the presence of status concerns. Our setting involves an entrepreneur with status concerns who has an idea for a project requiring an initial investment, which she raises by issuing a security to a financier. We characterize analytically the optimal security and find that its payoff profile is considerably similar to that of a convertible security. In contrast to existing explanations for convertible securities, ours does not rely on agency problems or asymmetric information. The more volatile the project is, the more similar the optimal security is to a convertible security. This is consistent with the observation that convertible securities are primarily used to finance relatively volatile projects. Our analysis uncovers the entrepreneur's and financier's risk attitudes as factors that can explain why convertible securities have different conversion ratios. While the paper focuses on convertible securities for expositional purposes, our analysis is potentially relevant for understanding a broader class of hybrid securities as well as executive compensation contracts.

1. Introduction

Financial instruments play a fundamental role in the economy by facilitating interaction between entrepreneurs, those with project ideas requiring investments, and financiers, those who wish to invest their funds.[1] An appropriately designed financial instrument, or a *security*, allows an entrepreneur to access a financier's funds in exchange for providing the financier with a claim on future profits. There is a voluminous security design and financial contracting literature examining how the choice of security depends on various salient features underlying security issuance. However, while there is substantial evidence that individuals in general, and entrepreneurs in particular, care about status, little is known about the effects of status concerns on optimal security design. This is somewhat surprising given how extensively status concerns have been studied in other areas of economics and finance. This paper contributes towards filling this gap.

We show that a status-driven entrepreneur issues a security that closely resembles a convertible security, which is a prevalent form of financing for many firms, notably riskier ones. A key property of convertible securities is that they combine features of both equity and debt. In contrast to existing explanations for convertible securities, which focus on agency problems, ours is the first, to our knowledge, that highlights a role for convertible securities unrelated to mitigating agency problems. Though this paper focuses on security design, we believe that our analytical framework can be straightforwardly adopted to also provide an explanation for widely used compensation packages that include both a fixed wage and a performance-based bonus. The payoff profile of such a package resembles that of a convertible security, whereby the fixed and performance-related components of the package correspond to the debt and equity components of the security, respectively (see Remark 1 of Section 5).

It has long been recognized that people tend to care about their status in society. There is now abundant evidence, obtained in various settings, that concern for status is an important factor behind people's behavior (see Frank (1985) and Heffetz and Frank (2011) for summaries of the evidence). There are also studies that directly support the main premise

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[1]We use the terms "entrepreneur" and "financier" in a broad sense. "Entrepreneur" may refer to an individual working at a start-up firm or to a CEO of an established company. "Financier" may refer to an individual investor such as a venture capitalist, to a financial organization such as a bank, or to the financial market as a whole.

of this paper—that entrepreneurs care about status. According to the 2011 High Impact Entrepreneurship Global Report, a comprehensive cross-country study of entrepreneurship, the idea that successful entrepreneurs have high status has wide support among both entrepreneurs and non-entrepreneurs.[2] Becker, Murphy, and Werning (2005) argue that entrepreneurship as an activity is especially appealing in countries in which entrepreneurial success leads to high status. Begley and Tan (2001) provide empirical support for this argument. An entrepreneur, in this paper, can also refer to a CEO of a company (see footnote 1), and the prevalence of status concerns among CEOs is also well-documented. Maug, Niessen-Ruenzi, and Zhivotova (2013) find that CEOs working at prestigious firms accept lower compensation, and argue that this is because they value the high social status attached to working at such firms. Shemesh (2014) presents evidence that the risk-taking behavior of CEOs is affected by status concerns. Other works exploring the behavior of CEOs through the prism of status concerns include Wade, Porac, Pollock, and Graffin (2006), Malmendier and Tate (2009), and Goel and Thakor (2010).

We develop a dynamic continuous-time framework in which an entrepreneur has a project idea that requires a certain initial investment. A key novelty of our work is that the entrepreneur has status concerns. Following the seminal idea of Friedman and Savage (1948)—and in line with the formal analysis of Patel and Subrahmanyam (1978), Gregory (1980), and Robson (1992)—we model such concerns by considering a utility function featuring a local convexity region. Briefly (more details are given in Section 2.2), the idea is that the marginal utility decreases in wealth up to a certain level of wealth due to the standard satiation mechanism. However, once wealth reaches this level, the marginal utility *increases* as the individual switches from consuming "low status" to "high status" or "luxury" goods (a related concept is "conspicuous consumption").

The project value is modeled as a dynamic random process whose characteristics are controlled by the entrepreneur. In particular, she dynamically chooses a parameter that we refer to as the novelty of the product developed in this project, where a higher product novelty corresponds to a higher mean growth rate and volatility of the project value. To make the initial investment required to start the project, the entrepreneur can either use her own resources—the *internal financing case*—or can issue a security to a financier who has enough money to purchase it—the *external financing case*. The security specifies the

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[2]This report is available online at http://www.gemconsortium.org/docs/download/295.

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payoff to the financier at a future date as a function of the project value at that date. The financier has standard constant relative risk aversion (CRRA) preferences, and so does not exhibit status concerns. She chooses to buy the security if the resulting expected utility exceeds her reservation utility. To isolate the effects of status concerns, we do not consider various other features that may be at work in the interaction between entrepreneurs and financiers. Notably, we assume that there are no agency conflicts between the entrepreneur and the financier, which makes our model distinctly different from the voluminous research on financial contracting focusing on asymmetric information or moral hazard(as discussed below). We also note that, though we tackle a non-standard security design problem, in that preferences are not (globally) concave, we are able to solve analytically for all quantities of interest.

We first examine the simpler internal financing case, and find that the entrepreneur chooses the product novelty so as to avoid middle status. In particular, she substantially increases the novelty when the project value is in the middle status region. The reason is that in the middle status region the entrepreneur is mainly affected by the convex part of her utility function, inducing her to raise the project volatility by developing a more novel product. This status-driven behavior is crucially different from that when status concerns are absent, in which case the entrepreneur's choice of product novelty does not change over time. That is, the entrepreneur who cares about status is more actively involved in managing the project than the one without status concerns.

In the external financing case, our main focus, we show that the optimal security issued by the entrepreneur to finance the project is considerably similar to a convertible security, which is a hybrid security that has features of both equity and debt. Absent status concerns, on the other hand, the optimal security is equity-like. Status concerns induce the entrepreneur to introduce a debt-like component into the security because doing so makes middle status less likely, which is what she aims to achieve in the presence of status concerns. While in the internal financing case the entrepreneur could avoid middle status only by adjusting the product novelty, with external financing she has an additional tool at her disposal—the ability to structure the security. The debt-like component in the optimal security, paying an (almost) fixed amount to the financier, occurs for middle-status project values. This implies that an increase in the project value accrues fully to the entrepreneur, and is not shared with the financier as would be the case if the optimal security had an equity-like component in

the middle-status region. As a result, the implication is that the debt-like component helps the entrepreneur to reach a high-status region sooner than would the equity-like component, thus reducing the likelihood of middle status.

We find that the optimal security is affected by the project volatility: the higher is the project volatility, the more the optimal security resembles a convertible security. Hence, our model predicts that convertible securities are likely to be more prevalent among more volatile firms. This prediction is consistent with the evidence (discussed in Section 5) that convertible securities are mainly used by start-up firms and small firms, which are known to be relatively volatile. The intuition is that, when the project volatility is high, the entrepreneur has relatively little control of the project dynamics, and so she focuses on the choice of security as a mechanism to avoid middle status. This is why she optimally chooses a convertible-like security. When the project volatility is low, on the other hand, the entrepreneur can effectively avoid middle status by dynamically managing the project, implying that the incentives to issue a convertible security are weak.

Our model also provides a possible explanation, based on risk aversion, of why convertible securities have different conversion ratios. A conversion ratio determines what share of the project the financier will have if she chooses to convert the security into equity. We find that the more risk averse the financier is, the lower is the conversion ratio implied by the optimal security. On the contrary, the more risk averse the entrepreneur is, the higher is the conversion ratio. A higher conversion ratio implies that the financier will hold a larger share of the project if she converts, implying a higher exposure to the project risk. A more risk averse entrepreneur prefers a lower project risk, and the optimal security reflects this preference through a lower conversion ratio. Analogous reasoning explains a positive relation between the entrepreneur's risk aversion and the conversion ratio.

Our paper contributes to the literature aiming to explain the use of convertible securities. A common theme of existing work is that convertible securities help to mitigate various agency problems, typically arising in settings with asymmetric information (adverse selection or moral hazard). In particular, convertible securities are shown to mitigate the asset substitution problem (Green (1984)), window-dressing behavior (Cornelli and Yosha (2003)), inefficient investment (Schmidt (2003)), the underinvestment problem (Lyandres and Zhdanov (2014)), and other asymmetric information problems (Constantinides and Grundy

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(1989), Stein (1992), Repullo and Suarez (2004), Hellmann (2006), Chakraborty and Yilmaz (2011)). Given this literature, it may be tempting to conclude that convertible securities are only valuable for overcoming agency conflicts. Our analysis shows that convertible securities also have an economic role when such conflicts are absent, as is the case in our model.

Ours and the above studies focusing on convertible securities are part of the broader security design and financial contracting literature that seeks to rationalize the multitude of financial contracts used in reality. Excellent reviews of this literature are provided in Allen and Winton (1995), Hart (2001), Biais, Mariotti, and Rochet (2013), and Sannikov (2012). Kaplan and Stromberg (2003) compare financial contracting models' ability to explain contracts used in reality. Several studies in this area, notably Cadenillas, Cvitanic, and Zapatero (2007) and Bolton and Harris (2013), consider, like us, settings without agency problems, but they do not explain the use of convertible securities.

Our work also contributes to the growing literature investigating the role of status concerns in various areas of economics and finance. Several recent examples are Ball, Eckel, Grossman, and Zame (2001), Becker, Murphy, and Werning (2005), Moldovanu, Sela, and Shi (2007), Auriol and Renault (2008), Besley and Ghatak (2008), Wendner and Goulder (2008), Roussanov (2010), Dijk, Holmen, and Kirchler (2014), Georgarakos, Haliassos, and Pasini (2014), and Hong, Jiang, Wang, and Zhao (2014).

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the internal financing case, where the entrepreneur finances the project herself. Section 4 characterizes the optimal security in the external financing case. Section 5 examines the properties of the optimal security. Section 6 concludes. Appendix A presents all proofs, and Appendix B examines settings with alternative preference specifications.

2. Model

We consider a dynamic continuous-time economy with full information and no agency problems. There are two agents in the economy: an *entrepreneur* with status concerns who has a project idea but no funds to develop the project, and a *financier* who has the required funds. The main goal of this paper is to study the implications of status concerns for security design. Towards this end, we examine the *optimal security* issued by the status-driven

entrepreneur to the financier. The future project value is uncertain and follows a random process whose parameters are controlled by the entrepreneur. For the model to be non-vacuous, we assume the financier cannot manage the project herself without the entrepreneur, or technically speaking, the financier is not able to trade the project risk. Hence, the financier faces an incomplete market for this risk; for the entrepreneur, on the other hand, the market is complete. Some of the above key features make our framework similar to that of Cadenillas, Cvitanic, and Zapatero (2007), as elaborated at the end of Section 2.3.

We view our model as being applicable to both private and publicly traded companies seeking financing for their projects. Given this general specification, here entrepreneur may refer to a start-up firm, and the financier to a venture capitalist or angel investor; alternatively, the entrepreneur may be a CEO of a publicly traded company and the financier a bank or the financial market. Accordingly, we use the term "security" in a broad sense to mean either a non-tradable (private) financial agreement or a publicly traded financial instrument. We now provide a detailed description of the model.

2.1. Project Value Dynamics

We consider an entrepreneur with a unique project idea that requires initial investment V_0. After this investment is made, the entrepreneur starts developing the project. We model her daily activities by positing that she dynamically chooses the process ϕ in the dynamics of the project value V governed by

$$dV_t/V_t = \phi_t \mu dt + \phi_t \sigma d\omega_t, \tag{1}$$

where $\mu, \sigma > 0$ are the *project mean growth rate* and *volatility*, respectively, and ω is a standard Brownian motion representing the uncertainty in the economy. Specification (1) formalizes the idea of "nothing ventured, nothing gained." Specifically, if the entrepreneur wants to raise the project's expected growth rate (first term), she needs to increase the parameter ϕ, implying that she is also raising the project's riskiness (second term). To elaborate further, when working on a project, the entrepreneur develops a certain product that she plans to sell at a profit in the market. In the process of development, she can dynamically choose the direction of the project—that is by choosing how novel the product

is going to be relative to existing products in the market. The more novel the product is, the higher are the expected future profits due to less intense competition, also raising the expected growth rate of the project value. At the same time, the future demand for novel products is less predictable, and so product novelty is associated with a higher project riskiness. Given this, we refer to the parameter ϕ as the *product novelty*.[3]

2.2. Status Concerns

A key feature of our model relative to existing security design models is that the entrepreneur is driven by status concerns. To model status concerns, we follow the classical insight of Friedman and Savage (1948) that the craving for higher status can be captured using a concave-convex-concave utility function. The idea is as follows. The traditional argument for a concave utility function, or equivalently for marginal utility decreasing in wealth, relies on the divisibility of consumption goods. In this case, one can consume the same set of goods regardless of one's wealth, and higher wealth simply increases the quantity of each good in the consumption bundle.

Clearly, in reality, consumption bundles of relatively poor and wealthy people are quite different—wealthy people consume various (non-divisible) "status" goods (e.g., private jets, yachts, memberships of elite golf clubs) that less wealthy people do not consume as they cannot afford them. A key point of Friedman and Savage is that, when an individual's wealth becomes sufficiently high that she can afford status goods, her marginal utility increases as, instead of becoming satiated due to buying more of the same basic goods, she switches to status goods. As wealth increases further, the normal satiation mechanism kicks in, and the marginal utility starts decreasing. While Friedman and Savage's argument is mainly descriptive in nature, Robson (1992) shows formally that concave-convex-concave preferences arise endogenously when individuals care about wealth both directly and indirectly through their status determined by their relative wealth.[4] Moreover, Becker, Murphy, and Werning

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[3]Though the entrepreneur in our model is allowed to choose a negative value of the novelty parameter $\phi_t < 0$, in all model calibrations presented later we find that she never actually does this. Choosing $\phi_t < 0$ leads to a declining expected project value, which essentially means destroying the project. Moreover, we note that a process similar to (1) appears in dynamic asset pricing models when one considers the financial wealth dynamics of an investor. We make use of this analogy when developing an approach to solving the model, as explained in the Appendix.

[4]Other papers deriving endogenously concave-convex-concave preferences include Patel and Subrahmanyam (1978) and Gregory (1980).

(2005) justify using preferences with convexities when modeling entrepreneurs' behavior: "[S]tart-ups and other entrepreneurial efforts...are much more common and less well rewarded than would be expected from the usual assumptions of risk aversion and diminishing marginal utility of income."

Given the above, we posit that the entrepreneur's utility function $u_E(\cdot)$ over her wealth $W_{E\tau}$ at some future date τ is

$$
u_E(W_{E\tau}) = \begin{cases} \dfrac{(W_{E\tau})^{1-\gamma_E}}{1-\gamma_E} & W_{E\tau} < L, \\[4mm] \dfrac{(W_{E\tau} - \alpha)^{1-\gamma_E}}{1-\gamma_E} + B, & W_{E\tau} \geq L, \end{cases} \tag{2}
$$

where $\gamma_E, L > 0$, $\alpha \in [0, L)$, and $B = (L^{1-\gamma_E} - (L-\alpha)^{1-\gamma_E})/(1-\gamma_E) \geq 0$ ensures continuity of preferences. The parameter α represents the *status concerns*—the higher α is, the stronger is the entrepreneur's desire to achieve high status, and so the more pronounced is the convexity region in the utility. The special case of $\alpha = 0$ corresponds to a standard CRRA utility function with no status concerns. Figure 1 presents typical shapes of preferences with status concerns. In line with Friedman and Savage, in what follows we refer—going from left to right in Figure 1—to the region of low wealth with concave utility as the *low-status* region, to the region of moderate wealth with convex utility as the *middle-status* region, and to the region of high wealth with concave utility as the *high-status* region. The position of the middle-status region is determined by the parameter L; henceforth, we refer to L as the *status level* (of wealth). The parameter γ_E represents the entrepreneur's *risk aversion* when her wealth is in the low or high-status region. To demonstrate that our subsequent results are not driven by the specific functional form of (2), in Appendix B we consider settings with alternative concave-convex-concave preference specifications.[5]

The other agent is a financier who can potentially provide funds to the entrepreneur to make the initial investments required to launch the project. To focus on the effects of the entrepreneur's status concerns, we assume that the financier has a standard CRRA utility function $u_F(\cdot)$ over her wealth W_{FT} at some future date T, and does not exhibit status

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[5]Our results are not driven either by the kink in the utility function (2) (seen in Figure 1), which is not present in Friedman and Savage, who propose a smooth concave-convex-concave utility function. We explain this point in the proof of Proposition 1 (Appendix A).
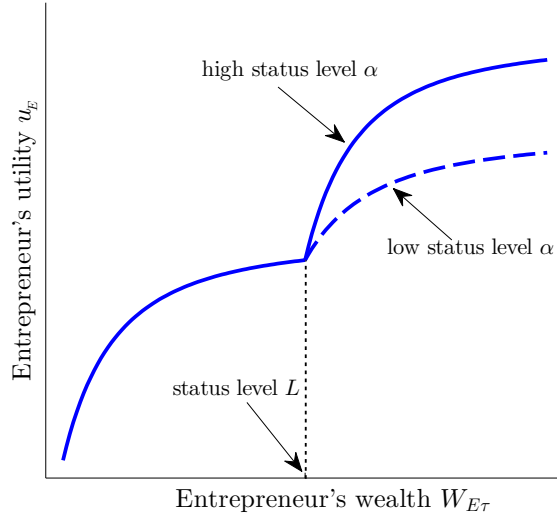
Figure 1: Entrepreneur's Utility Function $u_E(W_{E\tau})$

concerns:[6]

$$u_F(W_{FT}) = \frac{(W_{FT})^{1-\gamma_F}}{1 - \gamma_F}, \qquad (3)$$

where $\gamma_F > 0$ is the financier's relative risk aversion.

2.3. External and Internal Financing

The main goal of this paper is to analyze the *external financing case* in which the entrepreneur has no money of her own to make the required initial investment V_0, and so she turns to the financier for the money. In return for receiving V_0, the entrepreneur offers the financier a state-contingent claim, or a *security*, represented by a function $W_{FT}(V_T)$. We do not impose any restrictions on the shape of the function $W_{FT}(V_T)$, and so the set of admissible securities consists of virtually all possible securities.[7] The security specifies the amount W_{FT} that the financier will receive at date T for each possible realization of the project value $V_T > 0$. As we abstract away from agency problems, the entrepreneur cannot distort or misreport the true project value V_T and cannot refuse to pay the full amount $W_{FT}(V_T)$. Moreover, the

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[6]As noted earlier, the financier in our model may represent a group of investors (venture capital fund investors, financial market investors, etc) whose money is pooled together and given to the entrepreneur. Even if individual investors in the group exhibit status concerns, it is not clear how these features will be aggregated when we combine these investors into a single representative investor, the financier in our model.

[7]In some studies, the function $W_{FT}(V_T)$ is alternatively referred to as a sharing rule, as it specifies how the project value will be shared between the involved parties. We also note that our results would not change if the financing offer were made by the financier to the entrepreneur.

entrepreneur cannot run with the money before paying back the financier, which we model by assuming that the entrepreneur's horizon is longer than that of the security, $\tau > T$.

The financier accepts the offer if the resulting expected utility is not lower than her (commonly known) reservation utility $\bar{u}_F$, which reflects factors such as her outside investment opportunities and her bargaining power. We assume that this reservation utility is not prohibitively high from the entrepreneur's perspective, and so the financing transaction between the entrepreneur and the financier takes place.[8] At the payoff date T, the entrepreneur pays the financier the required amount (out of the project value) and continues managing the project until her horizon τ, at which time she consumes the project value V_τ.

The optimal security $W^*_{FT}(V_T)$ and the optimal product novelty process ϕ^*_t, $t \in [0, \tau)$ are such that the financier accepts the security, and the corresponding time-τ project value V^*_τ maximizes the entrepreneur's expected utility (2), as formalized in Definition 1.

Definition 1. *In the external financing case, the optimal security, $W^*_{FT}(V_T)$, and the product novelty process, $\phi^*_t > 0$, $t \in [0, \tau]$, are determined as the solution to the problem*

$$\max_{\phi_t, W_{FT}} E[u_E(V_\tau)] \tag{4}$$

$$subject\ to \quad dV_t = V_t \phi_t \mu dt + V_t \phi_t \sigma d\omega_t - W_{FT} dI_t,$$

$$E[u_F(W_{FT})] \geq \bar{u}_F, \tag{5}$$

where I_t is a step function $I_t \equiv 1_{\{t=T\}}$.

While our main focus is on the external financing case, we also examine the *internal financing* case in which the entrepreneur finances the project out of her own resources, without interacting with the financier. Besides being of independent interest, this analysis allows us to describe in a clear way the economic mechanism that turns out to be important for understanding the optimal security in the external financing case, and serves as a benchmark for comparison. Definition 2 characterizes the optimal behavior of the entrepreneur under internal financing.

Definition 2. *In the internal financing case, the entrepreneur's dynamic choice of the prod-*

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[8]If this were not the case, the entrepreneur would likely search for another, less demanding, financier. Our model can be interpreted as corresponding to the situation where the search process has already taken place and the entrepreneur has identified a suitable financier.

uct novelty, $\phi_t^* > 0$, $t \in [0, \tau]$, is given by the solution to the problem

$$\max_{\phi_t} E[u_E(V_\tau)] \tag{6}$$

$$subject \ to \quad dV_t = V_t \phi_t \mu dt + V_t \phi_t \sigma d\omega_t.$$

Having presented our model, we now relate it to the framework of Cadenillas, Cvitanic, and Zapatero (2007). Like these authors, we examine the optimal security in a setting with full information and no agency problems. While agency problems are a key feature in most studies of security design and financial contracting, we abstract from this feature so as to focus on the effects of status concerns. The main difference between Cadenillas et al. and our work is that they do not consider status concerns, our main focus. Moreover, there is no costly effort in our model, i.e., the entrepreneur does not derive disutility from running the project, whereas this feature is present in Cadenillas et al. (and also in a large body of studies in this area). While the assumption that work reduces one's utility is appropriate in various situations, we believe that not including this feature is less problematic in our particular setting. Indeed, real entrepreneurs commonly enjoy working on their projects, and typically devote their full effort to them without perceiving it as a utility-reducing activity. This argument is also made in Schmidt (2003): "Typically, the problem is not to get the entrepreneur to work hard enough, but rather to induce her to allocate her effort efficiently." Finally, in our model, the entrepreneur's choice of project volatility determines the project's mean growth rate, whereas in Cadenillas et al. the two are chosen separately. Because, unlike these authors, we do not have costly effort, allowing the entrepreneur to choose the two parameters separately would lead to her choosing an infinite mean growth rate in our setting.

3. Entrepreneur's Behavior with Internal Financing

In this Section, we investigate how the entrepreneur manages the project in the internal financing case, when she has sufficient resources to launch the project herself and does not, therefore, issue financial securities. The mechanisms underlying the entrepreneur's optimal behavior in this case are key to understanding the structure of the optimal security (Section

4).

Proposition 1 presents the optimal product novelty and the corresponding project value in closed form. It turns out that the economic state variable in the solution is the state-price process ξ given by $\xi_t = \exp\left(-\mu(\omega_t - \mu t/(2\sigma))/\sigma\right)$, as is typically the case in asset pricing models. A low ξ_t is associated with good states (high realization of uncertainty ω_t), and a high ξ_t is associated with bad states (low ω_t).

Proposition 1. *Under internal financing, the optimal product novelty is given by*

$$\phi_t^* = \frac{\mu}{\sigma^2 V_t^*}\left[\frac{K_{1t}}{\gamma_E}(y\xi_t)^{-1/\gamma_E} + \frac{\alpha}{K_{3t}}n\left(\frac{\ln\frac{B}{\alpha y \xi_t} - K_{2t}}{K_{3t}}\right)\right], \tag{7}$$

and the optimal project value, V_t^*, is given by

$$V_t^* = K_{1t}(y\xi_t)^{-1/\gamma_E} + \alpha N\left(\frac{\ln\frac{B}{\alpha y \xi_t} - K_{2t}}{K_{3t}}\right), \tag{8}$$

where $N(\cdot)$ and $n(\cdot)$ are the standard normal cumulative distribution function and probability density function, respectively, y is given in the Appendix, the constant B is as defined in equation (2), and the deterministic quantities K_{1t}, K_{2t}, and K_{3t} are given by

$$K_{1t} \equiv e^{(1-\gamma_E)(\tau-t)\mu^2/(2\gamma_E^2\sigma^2)}, \; K_{2t} \equiv (\tau - t)\mu^2/(2\sigma^2), \; K_{3t} \equiv \sqrt{\tau - t}\mu/\sigma. \tag{9}$$

Figure 2 plots the optimal product novelty (7) and project value (8) when the entrepreneur has status concerns $\alpha > 0$ (solid lines) and in the benchmark case of no status concerns $\alpha = 0$ (dashed lines). From panel (a), we see a sharp distinction between the cases of status concerns and no status concerns in terms of how actively the entrepreneur manages the project. In particular, the status-driven entrepreneur adjusts the product novelty depending on the project value, whereas without status concerns she simply chooses a constant product novelty. With status concerns, the entrepreneur substantially increases the novelty for intermediate project values, while for relatively high and low project values the optimal product novelty is close to that with no status concerns. To understand why, we recall the earlier discussion of the three status regions (Section 2.2), and place $\underline{L}$ and $\overline{L}$ onto the x-axis in panel (a) and y-axis in panel (b) to mark the boundaries of these regions. The low, middle, and high-status

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regions correspond to, respectively, $V_t < \underline{L}$, $\underline{L} \le V_t \le \overline{L}$, and $V_t > \overline{L}$.[9] The corresponding threshold values for the state-price process are marked by $\underline{\xi}$ and $\overline{\xi}$ for the case with status concerns and by $\underline{\xi}^b$ and $\overline{\xi}^b$ for the case without status concerns. For middle-status project values, the entrepreneur is mainly driven by the convex part of her utility function. This creates incentives to increase the project volatility, leading to the profile in panel (a).

(a) Product novelty
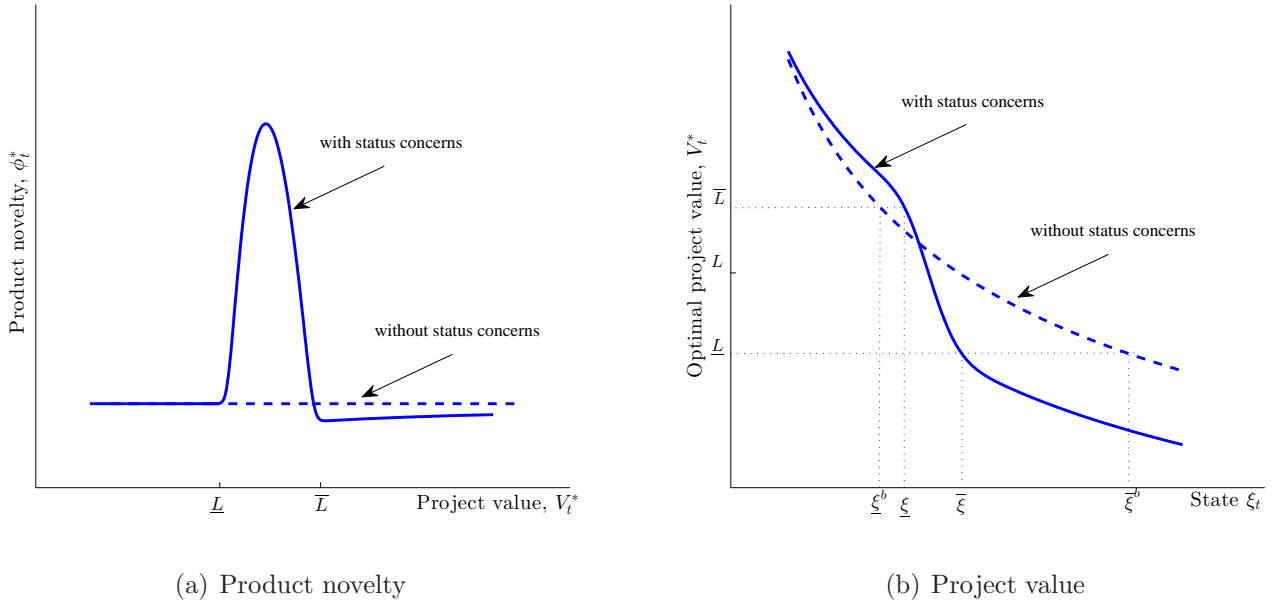
(b) Project value

Figure 2: Optimal Product Novelty and Project Value with Internal Financing.
Panel (a) depicts the time-t optimal product novelty, ϕ_t^*, and panel (b) depicts the optimal project value, V_t^*, in the internal financing case. In both panels, the solid line corresponds to the case of status concerns and the dashed line to the case of no status concerns.[10]

Panel (b) of Figure 2 depicts the corresponding optimal project value. It reveals that the key outcome of increasing the product novelty and, consequently, the project volatility in the middle-status region is that, at any date t, the middle status becomes less likely than if the volatility were kept constant. That is, although the middle-status region lies in between the low and high-status regions (in terms of wealth), the status-driven entrepreneur behaves as if the middle-status region is the least desirable, in trying to avoid middle status. Indeed, from panel (b) we see that the set of states of nature for which this region occurs with status concerns, $[\underline{\xi}, \overline{\xi}]$, is narrower than the corresponding set without status concerns, $[\underline{\xi}^b, \overline{\xi}^b]$. The

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[9]While the entrepreneur's status is realized at her horizon τ (when her consumption takes place), the entrepreneur can compute her *expected* status at any prior date $t < \tau$. Accordingly, we refer to the region $V_t < \underline{L}$ as the low-status region because, when $V_t < \underline{L}$, the entrepreneur expects to have low status at date τ, and analogously for the two other regions.

[10]The parameter values are $\gamma_E = 3$, $\alpha = 0.5$ for the solid line and $\alpha = 0$ for the dashed line, $L = 2$, $V_0 = 3$, $\mu = 0.1$, $\sigma = 0.8$, $t = 3.5$, $\tau = 4$, and so $B = 0.0972$.

likelihoods of high and low status, on the other hand, increase due to status concerns. The entrepreneur's desire to avoid middle-status wealth is a mechanism that will play a key role in helping us to understand the structure of the optimal security in the external financing case. We now present a detailed analysis of that case.

4.   Optimal Security with External Financing

In this Section, we consider the external financing case and characterize analytically the optimal security issued by the entrepreneur whose preferences feature convexity, reflecting status concerns. Studying models with such preferences and developing the (non-standard) techniques to solve them have proved valuable in various areas of finance. However, to our knowledge there are no models with such preferences in the security design and financial contracting literature. Our paper is a first step in that direction.

Proposition 2. *The optimal security $W_{FT}^*(V_T)$ is given parametrically through a pair of functions $(W_{FT}(x), V_T(x))$ where the parameter x varies from 0 to $+\infty$. The functions $W_{FT}(x)$ and $V_T(x)$ are*

$$W_{FT}(x) = (\bar{u}_F(1-\gamma_F))^{-1/(\gamma_F-1)}\, e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F}, \tag{10}$$

$$V_T(x) = K_{1T}g(x)^{-1/\gamma_E} + \alpha N\left(\frac{\ln(B/\alpha)-\ln g(x)-K_{2T}}{K_{3T}}\right) + (\bar{u}_F(1-\gamma_F))^{-1/(\gamma_F-1)}\, e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F}, \tag{11}$$

where the function $g(x)$ is implicitly given by

$$\frac{K_{1T}K_{3T}\, g(x)^{(\gamma_E-1)/\gamma_E} + \gamma_E\, B\, n\left(\frac{\ln(B/\alpha)-\ln g(x)+K_{2T}}{K_{3T}}\right)}{K_{1T}K_{3T}\, g(x)^{-1/\gamma_E} + \gamma_E\, \alpha\, n\left(\frac{\ln(B/\alpha)-\ln g(x)-K_{2T}}{K_{3T}}\right)} = zx. \tag{12}$$

In the above, $N(\cdot)$ and $n(\cdot)$ are the standard normal cumulative distribution function and probability density function, respectively, the constant B is as given in equation (2), the quantities K_{1T}, K_{2T}, and K_{3T} are as given in Proposition 1, and z is provided in the Appendix.

Panel (a) of Figure 3 plots the optimal security when status concerns are present (solid line) and absent (dotted line). We see that the status-driven entrepreneur finances the project
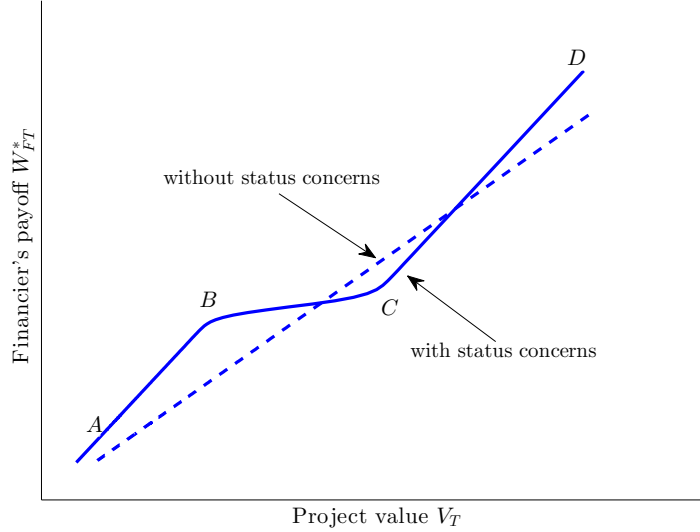
(a) Optimal security
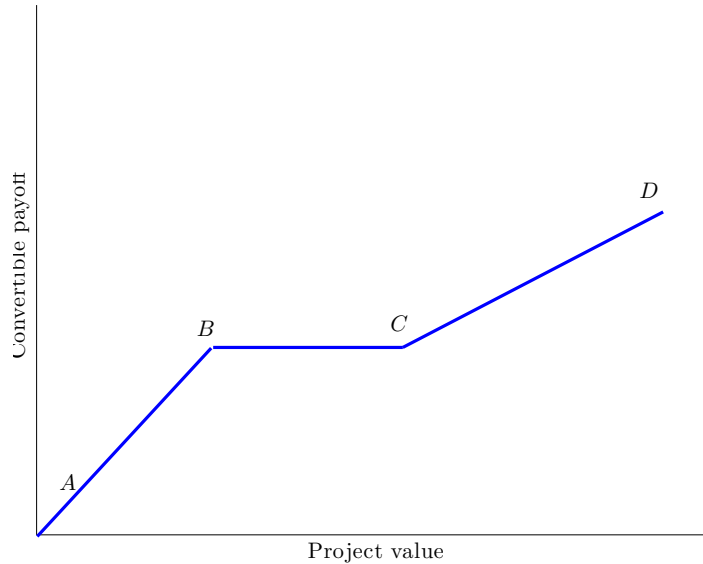
(b) Payoff of an actual convertible security

Figure 3: Optimal Security and Convertible Security. Panel (a) depicts the optimal security W_{FT}^* in the external financing case of our model, while panel (b) depicts the payoff profile of an actual convertible security. In panel (a), the solid line corresponds to the case of status concerns and the dashed line to the case of no status concerns.[12]

by offering to the financier a security that is considerably similar to a convertible security, which has the payoff profile depicted in panel (b). A key feature of such a security is its hybrid nature, in that it exhibits attributes of both equity and debt (but cannot be replicated by a (static) mix of equity and debt): the *A-B-C* segment corresponds to debt and the *C-D* segment corresponds to equity. The slope of *C-D* is determined by the conversion ratio,

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[12]The parameter values are $\alpha = 1.5$ for the solid line and $\alpha = 0$ for the dashed line, $\gamma_F = 3$, $u_F = -0.5$, $T = 3$, and the other parameters are as in Figure 2.

namely the number of equity shares into which a convertible security can be converted.[11] Absent status concerns ($\alpha = 0$, dotted line in panel (a)), the optimal security is similar to equity. Hence, concern for status is indeed a key ingredient for generating a convertible-like optimal security. The existing explanations of convertible securities center around agency problems under asymmetric information (as discussed in the Introduction). Given this, one might be tempted to conclude that mitigating such problems is the only reason convertible securities are used. Our paper is the first (to our knowledge) to show that convertible securities can also have an economic role without agency conflicts involving moral hazard or asymmetric information.

The intuition is as follows. For relatively low project values, the entrepreneur chooses an equity-like payoff profile, given by the segment A-B in panel (a) of Figure 3. The reason is that the entrepreneur is most driven by status concerns when her status remains unclear, i.e., when she has a middle status. When the project value is low, the entrepreneur has a relatively high chance of having a low status, making her incentives similar to those of an entrepreneur without status concerns. Hence, she opts for an equity-like payoff profile. Analogously, for relatively high project values, the entrepreneur is not greatly affected by status concerns, thus choosing the equity-like payoff given by the segment C-D in panel (a).

For intermediate project values, the entrepreneur chooses the debt-like payoff profile given by the segment B-C in panel (a). This is because the entrepreneur has a middle status in this region of project values, and choosing an almost flat payoff profile allows her to better avoid middle status. As explained in Section 3, avoiding middle status is a key mechanism driving the entrepreneur with status concerns. An almost flat payoff segment means that the entrepreneur keeps almost all of the incremental project value to herself, which ensures, as explained via a numerical example below, that her wealth reaches the high-status region for a lower value of this increment—and so leaves the middle-status region sooner—than if the segment had a steeper slope. While in the internal financing case of Section 3 the entrepreneur could affect her status through a single channel only, by altering the product novelty ϕ_t, with external financing there is an additional channel—the choice of security

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[11]For a more detailed description of a convertible security and its payoff profile, see for example Section 24-6 in Brealey, Myers, and Allen (2010). It is worth noting that hereafter we use a generic term "convertible security," rather than specifying a particular security, because there are several real-world instruments that, while different in certain aspects, have features of both equity and debt. Two notable examples are convertible bonds and convertible preferred stocks. We view our analysis as potentially relevant to all such instruments, including hybrid securities and executive compensation contracts.

offered to the financier. By introducing a debt-like component into the optimal security for middle-status project values, the entrepreneur uses this channel to avoid middle status.

To see in more detail how this channel works, consider the following numerical example. First, we set the boundary values between the low, middle, and high-status regions to be $\underline{L} = \$100K$ and $\overline{L} = \$200K$ (these boundaries are as in Section 3). That is, the entrepreneur has a low status when her wealth is below $\$100K$, a middle status when her wealth is between $\$100K$ and $\$200K$, and a high status when her wealth exceeds $\$200K$. Consider two securities a and b. Security a is an equity claim on the project value V_T and pays $1/2$ of V_T. Security b additionally has a debt-like component and pays $1/2$ of V_T for V_T below $\$200K$, or a fixed amount $\$100K$ for V_T higher than $\$200K$. If at time 0 the entrepreneur issues security a to finance the project, she is going to have a middle status at time T for V_T between $\$200K$ and $\$400K$, whereas if she instead issues security b she is going to have a middle status for V_T between $\$200K$ and $\$300K$.[13] Hence, sitting at time 0, the entrepreneur prefers security b (with a debt-like segment) over security a (without such a segment) because the former implies a narrower set of time-T project values under which the entrepreneur will have a middle status.

We note several differences between our optimal security (panel (a) of Figure 3) and an actual convertible security (panel (b) of Figure 3). First, the payoff profile of a convertible security is piecewise linear, whereas that of our security is a smooth non-linear function. Second, segment A-B of a convertible security is a 45-degree line starting from the origin, meaning that the security issuer pays the total project value V_T to the security holder (which corresponds to default by the security issuer). As for our optimal security, if we approximated segment A-B by a straight line, its slope would be less than 45 degrees (for any parameter values), implying that the security issuer (entrepreneur) retains a share in the project after paying the security holder (financier). In our model, the entrepreneur's marginal utility tends to infinity as wealth approaches zero (as seen from specification (2)), and moreover the entrepreneur has no other sources of wealth besides the project. Hence, the optimal security cannot prescribe that the entrepreneur pays the full project value to the financier as this would

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[13]In particular, if security a is issued, the entrepreneur's own wealth at time T is $1/2$ of the project value V_T (what is left after she pays $1/2$ of V_T to the financier). Hence, the entrepreneur's own wealth is in the middle-status region $[\$100K, \$200K]$ for $V_T \in [\$200K, \$400K]$. If, instead, security b is issued, for $V_T = \$200K$, the entrepreneur's own wealth is $1/2$ of V_T and so equals $\$100K$. For $V_T = \$300K$, the entrepreneur pays $\$100K$ to the financier, retaining $\$200K$. Hence, the entrepreneur has a middle status for $V_T \in [\$200K, \$300K]$.

leave the entrepreneur with no wealth.[14] Despite the above differences, we view our model as providing an explanation for convertible securities. In reality, securities used to finance projects are primarily standard securities (or a combination thereof), presumably because the parties involved in such financing transactions prefer to deal with familiar securities whose properties are well understood. Hence, for a given theoretically optimal security, the security used in the *real* transaction is likely to be one of the standard securities (or a portfolio thereof) with payoff structure as close as possible to that of the theoretically optimal security. Applying this reasoning in our context, our model predicts that real status-driven entrepreneurs will issue convertible securities to finance their projects.

# 5. Properties of the Optimal Security

In this Section, we examine how the shape of the optimal security is affected by the model parameters. One among them—the project volatility parameter σ—stands out as being particularly relevant for relating our model to empirical evidence on the use of convertible securities.[15] Issuing convertible securities is a predominant form of financing for—relatively *more* volatile—start-up firms (Sahlman (1990), Gompers (1999), Kaplan and Stromberg (2003)), while for—relatively *less* volatile—established publicly traded companies, convertible securities do not constitute the main source of external financing (which are equity and debt). A similar relation between the use of convertibles and volatility is observed when one considers only publicly traded companies. For example, summarizing the relevant evidence, Brennan and Schwartz (1988) point out that "companies issuing convertible bonds tend to be characterized by higher market and earnings variability, higher business and or financial risk." A similar observation is made in Brealey, Myers, and Allen (2010), who note that "convertibles tend to be issued by the smaller and more speculative firms," as well as in

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[14]There are at least two ways that our model could be extended to address this issue. The first would be to consider an extension where the entrepreneur has other sources of income (e.g., housing wealth, savings), in which case the entrepreneur would be able to pay the full project value without leaving herself with zero wealth. The second would be to consider an alternative preference specification for which the utility function would not be undefined at zero and negative wealth levels, such as an exponential utility function. We leave these extensions for future work.

[15]Other key parameters of our model are related to agents' preferences. However, existing theoretical works on convertible securities typically disregard the role of preferences by assuming risk neutrality. As empirical research tends to be driven by theory, the possible linkages between the use of convertible securities and the preference parameters of the agents issuing or buying such securities remain largely unexplored empirically.

Noddings, Christoph, and Noddings (2001).[16]
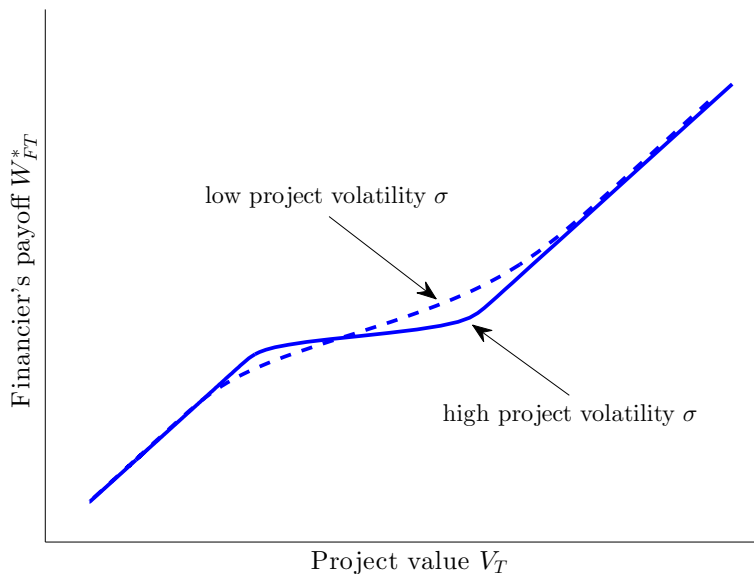
Figure 4: Effect of Project Volatility on Optimal Security. This figure depicts the optimal security for relatively high project volatility (solid line) and relatively low project volatility (dashed line).[17]

To determine whether our model can provide an explanation for these findings, in Figure 4 we depict the optimal security for varying levels of project volatility σ. The figure reveals that, as the project volatility increases (going from a dashed to a solid line), the similarity between the optimal security and a convertible security becomes more pronounced. Hence, our model predicts that incentives to issue convertible securities are stronger for riskier projects, consistent with the evidence. To understand the intuition, recall that the entrepreneur seeks to avoid middle status, and to do so she makes appropriate choices of the product novelty ϕ_t and the security $W_{fT}^*(V_T)$. Controlling her status through the product novelty can be done effectively (i.e., with sufficient precision) for low levels of project volatility, but is less effective for high volatilities.[18] Given this, for low volatilities, avoiding middle status is achieved

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[16]Noddings, Christoph, and Noddings (2001) consider publicly traded companies in the U.S. that have issued either convertible debt or convertible preferred stocks. Out of the companies using convertible debt, 58% are small-cap companies, 27% are middle-cap companies, and 15% are large-cap companies. For companies using convertible preferred stocks, the corresponding numbers are 47%, 39%, and 14%. This evidence, combined with the well-documented regularity that smaller companies tend to be more volatile than larger ones, suggests that more volatile companies are more likely to issue convertible securities.

[17]In Figure 4, $\sigma = 0.2$ for the dashed line and $\sigma = 0.8$ for the solid line. In panel (a) of Figure 5, $\gamma_E = 3$ for the dashed line and $\gamma_E = 5$ for the solid line; in panel (b), $\gamma_F = 3$ for the dashed line and $\gamma_F = 5$ for the solid line. In panel (a) of Figure 6, $L = 2$ for the dashed line and $L = 2.5$ for the solid line; in panel (b), $u_F = -1$ for the dashed line and $u_F = -0.5$ for the solid line. The other parameter values are as in Figure 3.

[18]For example, consider the limiting case when the project volatility is zero, $\sigma = 0$. In this case, the entrepreneur has perfect control over her status as she can achieve any desired project value at a given date.

mainly through the product novelty channel, whereas for high volatilities, when this channel is ineffective, the entrepreneur focuses on the second channel and issues a convertible-like security.

(a) Varying entrepreneur's risk aversion

(b) Varying financier's risk aversion
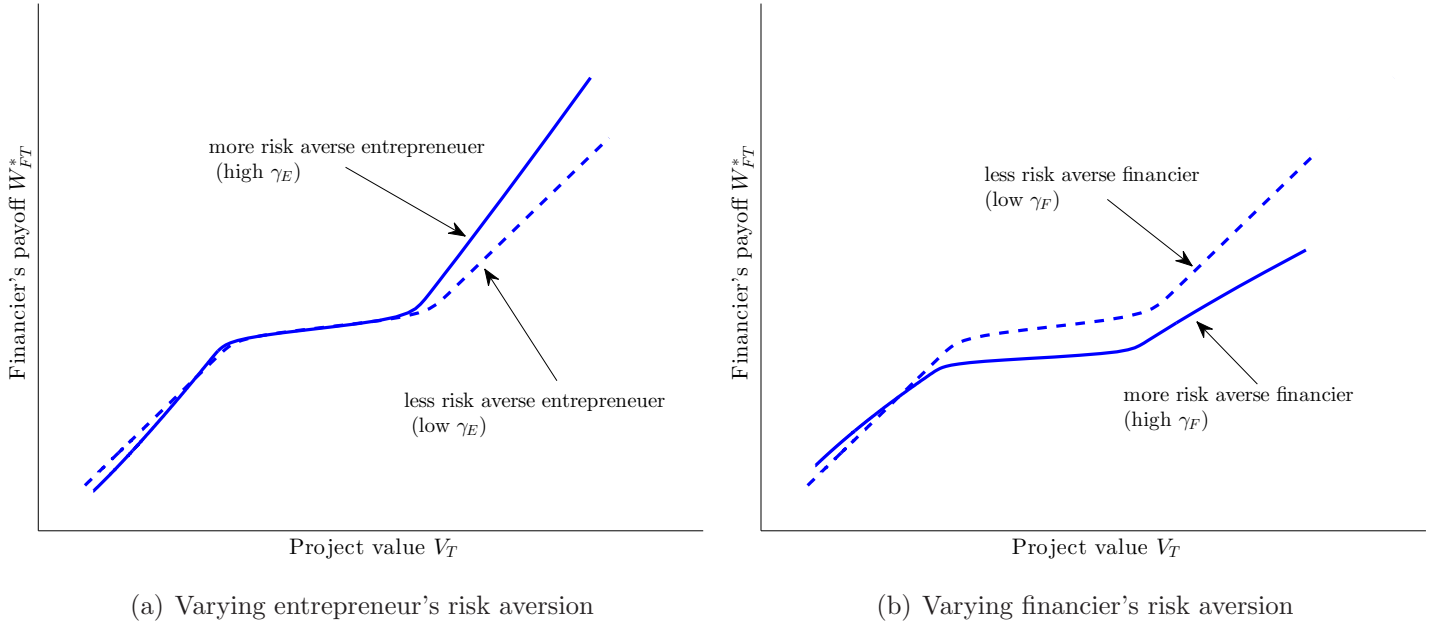
Figure 5: Effect of Risk Aversion on Optimal Security. Panel (a) depicts the optimal security when the entrepreneur is relatively more risk averse (solid line) and relatively less risk averse (dashed line). Panel (b) depicts the optimal security when the financier is relatively more risk averse (solid line) and relatively less risk averse (dashed line). The parameter values are provided in footnote 17.

We next explore how the optimal security depends on the entrepreneur's and the financier's risk aversion, γ_E and γ_F, respectively. The resulting optimal securities are depicted in Figure 5. From panel (a), we see that a higher level of risk aversion for the entrepreneur leads to a steeper payoff profile of the optimal security. When she is more risk averse, the entrepreneur prefers a security that implies a lower variability of her wealth. A steeper payoff profile of the security allows the entrepreneur to shift the wealth variability from herself to the financier. Analogous reasoning explains why a higher level of risk aversion for the financier implies a flatter payoff profile of the optimal security, as depicted in panel (b). In both panels, the most notable effect is that the slope of the right-most segment of the optimal security changes. As highlighted earlier (discussion of segment C-D in panel (b) of Figure 3), this slope is linked to the conversion ratio of a convertible security. Therefore, our

Indeed, if at time t_1 the project value is $V_{t_1} = V_1$ and the desired value at a later time $t_2 > t_1$ is $V_{t_2} = V_2$, then this can be achieved by setting the product novelty to $\phi_t = (\ln(V_2/V_1))/(t_2 - t_1)$, obtained by solving (1) for $\sigma = 0$. As the project volatility increases, however, the entrepreneur may not be able to achieve the desired precision when attempting to guide the project value into a certain region.

analysis reveals that risk aversion heterogeneity, across both entrepreneurs and financiers, is a possible explanation for convertible securities having different conversion ratios in reality.

(a) Effect of status level L (b) Effect of financier's reservation utility $\bar{u}_F$
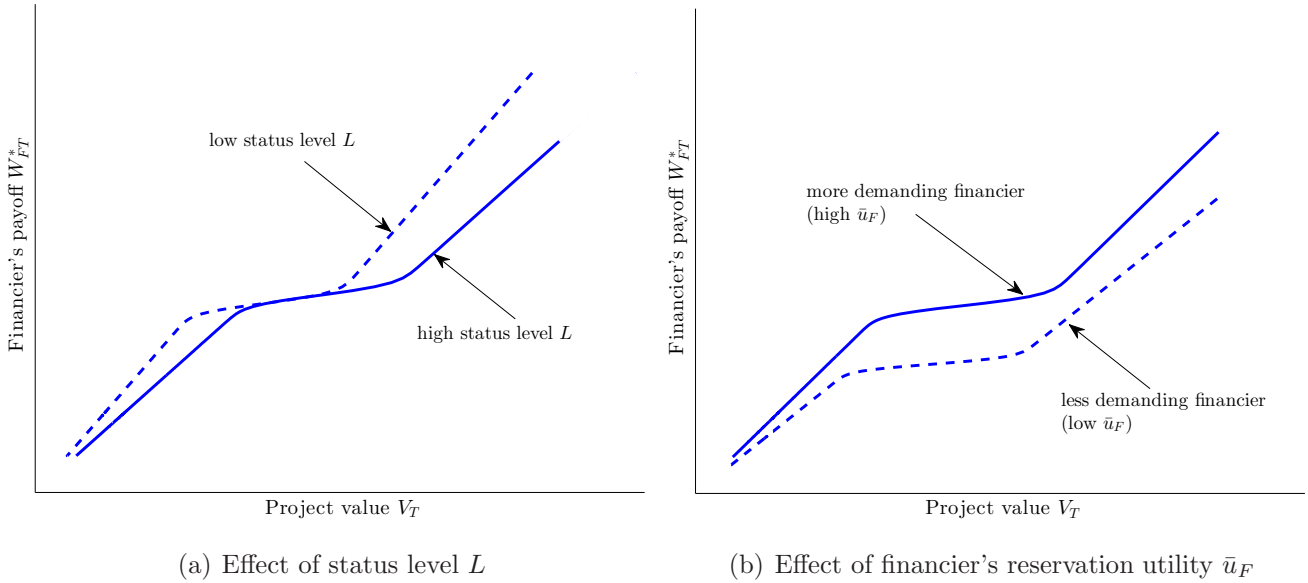
Figure 6: Effect of Status Level and Reservation Utility on Optimal Security.
Panel (a) depicts the optimal security when the status level (of wealth) is relatively high (solid line) and relatively low (dashed line). Panel (b) depicts the optimal security when the financier's reservation utility is relatively high (solid line) and relatively low (dashed line). The parameter values are provided in footnote 17.

Finally, we investigate the behavior of the optimal security with respect to the status level (of wealth) L and the financier's reservation utility $\bar{u}_F$. What constitutes low and high status could well differ across individuals, and by varying the status level we can look at entrepreneurs with different perceptions of status. By varying the financier's reservation utility, we explore how the optimal security depends on the distribution of bargaining power between the entrepreneur and the financier—an increase in the reservation utility of the financier corresponds to an increase in her bargaining power. The results are depicted in panels (a) and (b) of Figure 6, respectively. As discussed in Section 4, the middle segment of the optimal security corresponds to project values for which the entrepreneur has middle status. Accordingly, as the status level L increases, the middle segment occurs for higher project values, as seen in panel (a). When the financier's reservation utility $\bar{u}_F$ increases, she needs to be offered a higher expected payment to finance the project. This is why the higher $\bar{u}_F$ is, the higher is the payoff of the optimal security for each project value V_T^*, as seen in panel (b).

Remark 1. *Status-based explanation for compensation with fixed and performance-related*

21

components. We believe that the model developed in this paper can be adopted to provide a status-based explanation for compensation packages with both fixed and performance-related components, which are widely used in practice. Towards this, one can consider a model that builds on the above analytical framework but adopts it to a different context. Here is a brief outline of the model. There is an employer who needs to hire a worker to manage a certain project. The employer cares about status as in our main analysis, thereby having preferences featuring convexity; the worker has no status concerns, having standard concave preferences. The employer offers the worker a compensation package that maximizes the employer's expected utility while providing the worker with a certain reservation utility. One aspect in which this modified model is different is that the person who will manage the project, the worker, *receives* the offer, whereas in the main model the person who will manage the project, the entrepreneur, *makes* the offer. However, as noted in footnote 7, this does not affect our main implications, and the shape of the optimal compensation package in the modified model will be similar to that of a convertible security, as depicted by the solid line in Figure 3(a). Looking at this figure, if the outcome of the worker's actions is average, she receives an almost fixed wage (segment *B-C* in Figure 3(a)), whereas if the outcome is sufficiently high she additionally receives a performance-dependent bonus (segment *C-D*). If her work is unsuccessful, she may be fired or the company may default, implying compensation below the promised fixed wage (segment *A-B*).

6. Conclusion

This paper develops a continuous-time security design framework in which a status-driven entrepreneur dynamically manages a project. We characterize analytically the entrepreneur's behavior in the internal financing case, where the project is financed by the entrepreneur, and in the external finance case, where the entrepreneur issues a security to finance the project. In the external financing case, our main focus, we show that the optimal security is considerably similar to a convertible security. We also provide a possible explanation of why convertible securities are mainly used by relatively volatile firms (start-ups, small firms), and also why convertible securities have different conversion ratios.

While in our model managing a project involves a single choice, the product novelty, in

reality doing so involves multiple activities, such as research, development, marketing, examining the business models of competitors, etc. The entrepreneur has to allocate available resources across these activities. To account for this, one could generalize our framework by assuming that the project value is driven by multiple random factors, and that the entrepreneur chooses her loadings on these factors. We believe that our main insights would remain valid for this generalization. In our model the entrepreneur is not allowed to redeem the security before its maturity, whereas some real-world convertible securities include call provisions allowing the issuers to do so. Incorporating this feature into our model could be an interesting generalization. Finally, as elaborated in footnote 14, it would be of interest to extend our model by assuming that the entrepreneur has other sources of income besides the project.

Appendix A: Proofs

Proof of Proposition 1. In proving Propositions 1-2 and B1-B2, we employ martingale methods in a continuous-time complete market setting. These methods are particularly popular in portfolio choice and asset pricing models. Accordingly, to make our analysis easier to relate to such familiar models, we consider an investment problem that is methodologically analogous to the problem faced by our entrepreneur. Specifically, consider an investor who dynamically allocates her wealth between two assets, cash and a risky asset following a geometric Brownian motion with a mean return μ and volatility σ. If we denote the investor's time-t wealth by V_t and the wealth share invested in the risky asset by ϕ_t, then, as is well-known in the continuous-time finance literature, the dynamic process for wealth V_t is given by the process (1). Hence, the investor with the same preferences as the entrepreneur optimally chooses the same risky wealth share as the optimal product novelty chosen by the entrepreneur.[19]

As discussed in the portfolio choice literature (see, e.g., Carpenter (2000), Basak, Pavlova, and Shapiro (2007)), to solve our non-concave optimization problem in Definitions 1 and 2, we convert it into an equivalent concave problem by concavifying the entrepreneur's preferences. To do so, we replace the convex part of the utility function (corresponding to middle status) with a linear segment $a + b * W_{E\tau}$ that is tangent to both the low-status segment of the utility function (top line in specification (2)) and the high-status segment (bottom line in (2)). Denoting the tangency points by $\underline{L}$ and $\overline{L}$, respectively, the parameters a and b of the linear segment are obtained by solving the following system of equations:

---

[19]Though we employ the investment setting because of its familiarity, one can think of examples where the entrepreneur's job is indeed similar to that of an investor. In particular, this would be the case if we considered a project consisting of several sub-projects with different expected growth rates and levels of riskiness. The entrepreneur would need to choose dynamically how to allocate her resources between these sub-projects, and would therefore, similarly to an investor, form a (optimal) portfolio of sub-projects. Aghion and Stein (2008) formally explore a similar idea by considering a model in which a firm manager allocates one unit of effort between two strategies, one increasing sales growth and the other improving profit margins. In their model, there are no status concerns and the manager does not issue financial securities, which are the key ingredients of our work.

$$\frac{\underline{L}^{1-\overline{\gamma}_E}}{1-\gamma_E} = a + b\underline{L},$$

$$\frac{(\overline{L}-\alpha)^{1-\gamma_E}}{1-\gamma_E} + B = a + b\overline{L}, \tag{A1}$$

$$\underline{L}^{-\gamma_E} = b,$$

$$(\overline{L}-\alpha)^{-\gamma_E} = b.$$

The first and second equations in this system ensure that the concavified utility function is continuous at the points $\underline{L}$ and $\overline{L}$. The third and fourth equations ensure that the utility function is smooth at $\underline{L}$ and $\overline{L}$. Solving the system, we obtain

$$a = \frac{\gamma_E(B/\alpha)^{1-1/\gamma_E}}{1-\gamma_E}, \; b = B/\alpha, \; \underline{L} = (B/\alpha)^{-1/\gamma_E}, \; \overline{L} = (B/\alpha)^{-1/\gamma_E} + \alpha. \tag{A2}$$

Hence, the concavified utility function of the entrepreneur is

$$u_E(W_{E\tau}) = \begin{cases} \dfrac{(W_{E\tau})^{1-\gamma_E}}{1-\gamma_E} & W_{E\tau} < (B/\alpha)^{-1/\gamma_E}, \\ W_{E\tau}(B/\alpha) + \dfrac{\gamma_E(B/\alpha)^{1-1/\gamma_E}}{1-\gamma_E}, & (B/\alpha)^{-1/\gamma_E} \le W_{E\tau} \le (B/\alpha)^{-1/\gamma_E} + \alpha, \\ \dfrac{(W_{E\tau}-\alpha)^{1-\gamma_E}}{1-\gamma_E} + B, & W_{E\tau} > (B/\alpha)^{-1/\gamma_E} + \alpha. \end{cases} \tag{A3}$$

Before proceeding with the solution, let us clarify the point made in footnote 5 that the presence of the kink in the utility function (2) does not affect our results. Because of the concavification, the shape of the utility function (2) between the wealth levels $\underline{L} < L$ and $\overline{L} > L$ is essentially irrelevant. Hence, the kink at status level L can be smoothed so that the concavified utility function (A3)—and hence all our results—remain unchanged.

Given the utility function (A3), the first-order condition with respect to time-τ project value V_τ, after some manipulation, is

$$V_\tau^* = \begin{cases} (y\xi_\tau)^{-1/\gamma_E} & y\xi_\tau > B/\alpha, \\ (y\xi_\tau)^{-1/\gamma_E} + \alpha & y\xi_\tau \le B/\alpha, \end{cases} \tag{A4}$$

where y is the Lagrange multiplier computed from the condition that the project value V_τ^*

is feasible: $E[\xi_\tau V_\tau^*] = V_0$, and ξ is as given in Section 3. To compute the optimal time-t project value V_t^* for $t < \tau$, we use the fact that the process for $\xi_t V_t^*$ is a martingale, and so $V_t^* = E_t[\xi_\tau V_\tau^*]/\xi_t$. Substituting herein expression (A4), we obtain

$$V_t^* = E_t[(y\xi_\tau)^{-1/\gamma_E}\xi_\tau]/\xi_t + \alpha E_t[\xi_\tau \mathbb{1}_{\{\xi_\tau \leq B/\alpha\}}]/\xi_t. \tag{A5}$$

To compute the two expectations in (A5), we use the fact that the state-price process ξ_t is lognormally distributed, implying that ξ_τ^{1-1/γ_E} is also lognormally distributed with mean growth rate $(1 - \gamma_E)\mu^2/(2\gamma_E^2\sigma^2)$, and so

$$E_t[(y\xi_\tau)^{-1/\gamma_E}\xi_\tau]/\xi_t = (y\xi_t)^{-1/\gamma_E} e^{(1-\gamma_E)(\tau-T)\mu^2/(2\gamma_E^2\sigma^2)}. \tag{A6}$$

As ξ_τ is lognormally distributed, its truncated expected value can also be computed explicitly (e.g., Chapter 19 in Greene (2011)):

$$E_t[\xi_\tau \mathbb{1}_{\{\xi_\tau \leq B/\alpha\}}]/\xi_t = N\left(\frac{\ln\frac{B}{\alpha y\xi_t} - (\tau-t)\mu^2/(2\sigma^2)}{\sqrt{\tau-t}\mu/\sigma}\right). \tag{A7}$$

Substituting (A6) and (A7) into (A5) yields (8).

Applying Itô's Lemma to (8), after some algebra, we obtain the diffusion term in the dynamic process for V_t^* as

$$-(\xi_t\mu/\sigma)\frac{\partial V_t^*}{\partial \xi_t}d\omega_t = (\xi_t\mu/\sigma)\left[\frac{K_1(t)}{\gamma_E}(y\xi_t)^{-1/\gamma_E} + \frac{\alpha}{K_3(t)}n\left(\frac{\ln\frac{B}{\alpha y\xi_t} - K_2(t)}{K_3(t)}\right)\right]d\omega_t. \tag{A8}$$

From (1), the diffusion term is $\phi_t^*\sigma V_t^* d\omega_t$, which after equating with (A8) and rearranging yields (7). *Q.E.D.*

Proof of Proposition 2. During the time period $(T, \tau]$, after the financier is paid, the setting is analogous to that with internal financing, which means that the entrepreneur's behavior is as characterized in Proposition 1. Given this, we compute the entrepreneur's time-T indirect utility function $v_E(W_{ET}) = E_T[u_E(V_\tau^*)]$ by using the optimality condition (A4) and taking into account that the time-τ project value V_τ^* has to be feasible given the time-T project value $V_\tau = W_{ET}$. This implies that the Lagrange multiplier y is now implicitly given by $E_T[\xi_\tau V_\tau^*] = \xi_T W_{ET}$. In particular, the entrepreneur's time-T indirect utility is given

by

$$
\begin{aligned}
v_E(W_{ET}) &= E_T\left[\frac{(y\xi_\tau)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E}\mathbb{1}_{\{y\xi_\tau>B/\alpha\}}\right] + E_T\left[\left(\frac{(y\xi_\tau)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E}+B\right)\mathbb{1}_{\{y\xi_\tau\leq B/\alpha\}}\right] \\
&= \frac{(y\xi_T)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E}K_{1T} + B*N\left(\frac{\ln(B/\alpha)-\ln(y\xi_T)+K_{2T}}{K_{3T}}\right),
\end{aligned} \tag{A9}
$$

where y is defined implicitly by

$$
\begin{aligned}
W_{ET} &= E_T[\xi_\tau(y\xi_\tau)^{-1/\gamma_E}\mathbb{1}_{\{y\xi_\tau>B/\alpha\}}]/\xi_T + E_T[\xi_\tau((y\xi_\tau)^{-1/\gamma_E}+\alpha)\mathbb{1}_{\{y\xi_\tau\leq B/\alpha\}}]/\xi_T \\
&= (y\xi_T)^{-1/\gamma_E}K_{1T} + \alpha N\left(\frac{\ln(B/\alpha)-\ln(y\xi_T)-K_{2T}}{K_{3T}}\right).
\end{aligned} \tag{A10}
$$

In what follows, we will also need the expression for the marginal indirect utility function $v_E'(\cdot)$. Equations (A9) and (A10) define the indirect utility as a composite function $v_E(y(W_{ET}))$, which means that

$$
\frac{dv_E}{dW_{ET}} = \frac{dv_E}{dy}\frac{dy}{dW_{ET}}. \tag{A11}
$$

Computing the two derivatives on the right-hand side of (A11) from (A9) and (A10), respectively, we get

$$
\frac{dv_E}{dy} = -K_{1T}y^{-1/\gamma_E}\xi_T^{(\gamma_E-1)/\gamma_E}/\gamma_E - B*n\left(\frac{\ln(B/\alpha)-\ln(y\xi_T)+K_{2T}}{K_{3T}}\right)/(yK_{3T}), \tag{A12}
$$

$$
\frac{dy}{dW_{ET}} = \left(-K_{1T}\xi_T^{-1/\gamma_E}y^{-1-1/\gamma_E}/\gamma_E - \alpha n\left(\frac{\ln(B/\alpha)-\ln(y\xi_T)-K_{2T}}{K_{3T}}\right)/(yK_{3T})\right)^{-1}. \tag{A13}
$$

Substituting (A12) and (A13) into (A11) and rearranging yields the marginal indirect utility

$$
\frac{dv_E}{dW_{ET}} = \frac{K_{1T}K_{3T}(y\xi_T)^{(\gamma_E-1)/\gamma_E} + \gamma_E B*n\left(\frac{\ln(B/\alpha)-\ln(y\xi_T)+K_{2T}}{K_{3T}}\right)}{K_{1T}K_{3T}(y\xi_T)^{-1/\gamma_E} + \gamma_E \alpha n\left(\frac{\ln(B/\alpha)-\ln(y\xi_T)-K_{2T}}{K_{3T}}\right)}. \tag{A14}
$$

Though it is straightforward to prove that $v_E(\cdot)$ is increasing, establishing that $v_E(\cdot)$ is concave appears daunting given its complicated functional form. However, we have verified the concavity for a large number of model calibrations. We proceed by treating $v_E(\cdot)$ as an increasing concave function, with $v_E'(\cdot)>0$ and $v_E''(\cdot)<0$.

Taking into account that the entrepreneur's wealth at time T, after she pays the financier, is $W_{ET} = V_T - W_{FT}$, we can equivalently write the entrepreneur's optimization problem

(provided in Definition 1) as

$$\max_{\phi_t, W_{FT}} E[v_E(V_T - W_{FT})] \tag{A15}$$

$$\text{subject to} \quad dV_t = V_t\phi_t\mu dt + V_t\phi_t\sigma d\omega_t, \tag{A16}$$

$$E[u_F(W_{FT})] \geq \bar{u}_F. \tag{A17}$$

The first-order conditions for (A15), given the financier's utility function (3), are

$$v'_E(V^*_T - W^*_{FT}) = z\xi_T, \tag{A18}$$

$$-v'_E(V^*_T - W^*_{FT}) = z_1(W^*_{FT})^{-\gamma_F}, \tag{A19}$$

where z and z_1 are the Lagrange multipliers associated with the constraints (A16) and (A17), respectively. From (A18) and (A19), we obtain

$$W^*_{FT} = (-z\xi_T/z_1)^{-1/\gamma_F}. \tag{A20}$$

Substituting (A20) into (A17) yields

$$\bar{u}_F = \frac{(-z/z_1)^{1-1/\gamma_F}}{1-\gamma_F}E[\xi_T^{1-1/\gamma_F}] = \frac{(-z/z_1)^{1-1/\gamma_F}}{1-\gamma_F}e^{(1-\gamma_F)\mu^2 T/(2\gamma_F^2\sigma^2)}. \tag{A21}$$

From (A21), the ratio $-z/z_1$ is

$$-z/z_1 = (\bar{u}_F(1-\gamma_F))^{\gamma_F/(\gamma_F-1)}e^{\mu^2 T/(2\gamma_F\sigma^2)}. \tag{A22}$$

Substituting (A22) into (A20) yields

$$W^*_{FT}(\xi_T) = (\bar{u}_F(1-\gamma_F))^{-1/(\gamma_F-1)}e^{-\mu^2/(2\gamma_F^2\sigma^2)}\xi_T^{-1/\gamma_F}. \tag{A23}$$

The optimal security specifies the entrepreneur's payoff as a function of the project value V^*_T, and so for each state of the world ξ_T we need to compute the corresponding project value

$V_T^*(\xi_T)$. From (A18), (A10) and (A23), V_T^* is given by

$$V_T^*(\xi_T) = K_{1T}(y\xi_T)^{-1/\gamma_E} + \alpha N \left(\frac{\ln(B/\alpha) - \ln(y\xi_T) - K_{2T}}{K_{3T}} \right) + (\bar{u}_F(1 - \gamma_F))^{-1/(\gamma_F - 1)} e^{-\mu^2/(2\gamma_F^2 \sigma^2)} \xi_T^{-1/\gamma_F},$$

(A24)

where y is computed from (A14) by equating the right-hand side of (A14) to $z\xi_T$. The Lagrange multiplier z is such that the time-T project value $V_T^*(\xi_T)$ is feasible given the initial value V_0:

$$E[\xi_T V_T^*(\xi_T)] = V_0.$$

(A25)

Equations (A23) and (A24) then provide the parametric characterization of the optimal security $W_{FT}^*(V_T)$, as stated in Proposition 2.

Q.E.D.

Appendix B: Alternative specifications of status concerns

B1. Multiplicative Status Specification

From numerous studies in finance and economics, it is understood that, when one modifies a standard utility function to account for a certain aspect of human behavior, model implications may well differ depending on whether the modification is additive or multiplicative. A prominent example is the extensive habit formation literature, in which models with multiplicative habits (e.g., Abel (1990)) often generate different predictions from those with additive habits (e.g., Campbell and Cochrane (1999)). Given this, one may wonder whether our main predictions are robust to an alternative multiplicative status specification. In this Section, we examine this question and demonstrate that our main results and insights remain equally valid.

We consider a setting that is the same as that described in Section 2, the only change being that the entrepreneur's utility function $u_E(\cdot)$ is now given by

$$
U_E(W_{E\tau}) = \begin{cases} \frac{W_{E\tau}^{1-\gamma_E}}{1-\gamma_E} & W_{E\tau} < L, \\ \alpha \frac{W_{E\tau}^{1-\gamma_E}}{1-\gamma_E} + B & W_{E\tau} \geq L, \end{cases} \tag{B1}
$$

where $B = (1-\alpha)L^{1-\gamma_E}/(1-\gamma_E)$. All the parameters in (B1) have the same interpretations as they have in (2). The utility function (B1) is concave-convex-concave when $\alpha > 1$; its shape is similar to that of (2) (see Figure 1). The case of $\alpha = 1$ corresponds to the (globally concave) CRRA utility function with no status concerns. Proposition B1 characterizes the optimal security; the proof is provided at the end of this subsection.

Proposition B1. *The optimal security $W_{FT}^*(V_T)$ for the multiplicative status specification (B1) is given parametrically through a pair of functions $(W_{FT}(x), V_T(x))$, where x varies from*

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0 to +∞. The two functions are

$$W_{FT}(x) = (\bar{u}_F(1 - \gamma_F))^{-1/(\gamma_F - 1)} e^{-\mu^2/(2\gamma_F^2 \sigma^2)} x^{-1/\gamma_F}, \tag{B2}$$

$$V_T(x) = g(x)^{-1/\gamma_E} K_{1T} N\left(\frac{-\ln b + \ln g(x)) - K_4}{K_{3T}}\right) + \alpha^{1/\gamma_E} g(x)^{-1/\gamma_E} K_{1T} N\left(\frac{\ln b - \ln g(x) + K_4}{K_{3T}}\right)$$

$$+ (\bar{u}_F(1 - \gamma_F))^{-1/(\gamma_F - 1)} e^{-\mu^2/(2\gamma_F^2 \sigma^2)} x^{-1/\gamma_F}, \tag{B3}$$

where the function $g(x)$ is implicitly given by

$$\frac{g(x)^{(\gamma_E - 1)/\gamma_E} K_{1T} K_{3T} C(g(x), 1 - \gamma_E) - B * n\left((\ln b - \ln g(x) + K_{2T})/K_{3T}\right)}{g(x)^{-1/\gamma_E} K_{1T} K_{3T} C(g(x), 1)} = zx, \tag{B4}$$

In the above, $C(g(x))$ is

$$C(g(x), \beta) = -\frac{N\left((-\ln b + \ln g(x) - K_4)/K_{3T}\right)}{\gamma_E} + \frac{n\left((-\ln b + \ln g(x) - K_4)/K_{3T}\right)}{\beta K_{3T}}$$

$$- \alpha^{1/\gamma_E} \frac{N\left((\ln b - \ln g(x) + K_4)/K_{3T}\right)}{\gamma_E} - \alpha^{1/\gamma_E} \frac{n\left((\ln b - \ln g(x) + K_4)/K_{3T}\right)}{\beta K_{3T}}, \tag{B5}$$

$N(\cdot)$ and $n(\cdot)$ are the standard normal cumulative distribution function and probability density function, respectively, B is as in equation (B1), b is given in equation (B8), the constant K_4 is given by

$$K_4 \equiv (1 - 2\gamma_E)(\tau - T)\mu^2/(2\gamma_E \sigma^2), \tag{B6}$$

K_{1T}, K_{2T}, and K_{3T} are as given in Proposition 1, and z is given by $E[\xi_T V_T^*(z\xi_T)] = V_0$.

Figure B1 depicts the optimal security (panel (a)), and examines its properties with respect to the project volatility (panel (b)), entrepreneur's risk aversion (panel (c)), and financier's risk aversion (panel (d)). We see that the results are analogous to those obtained under the additive status specification—see Figures 3, 4, 5(a), and 5(b), respectively.

(a) Optimal security

(b) Varying project volatility

(c) Varying entrepreneur's risk aversion

(d) Varying financier's risk aversion
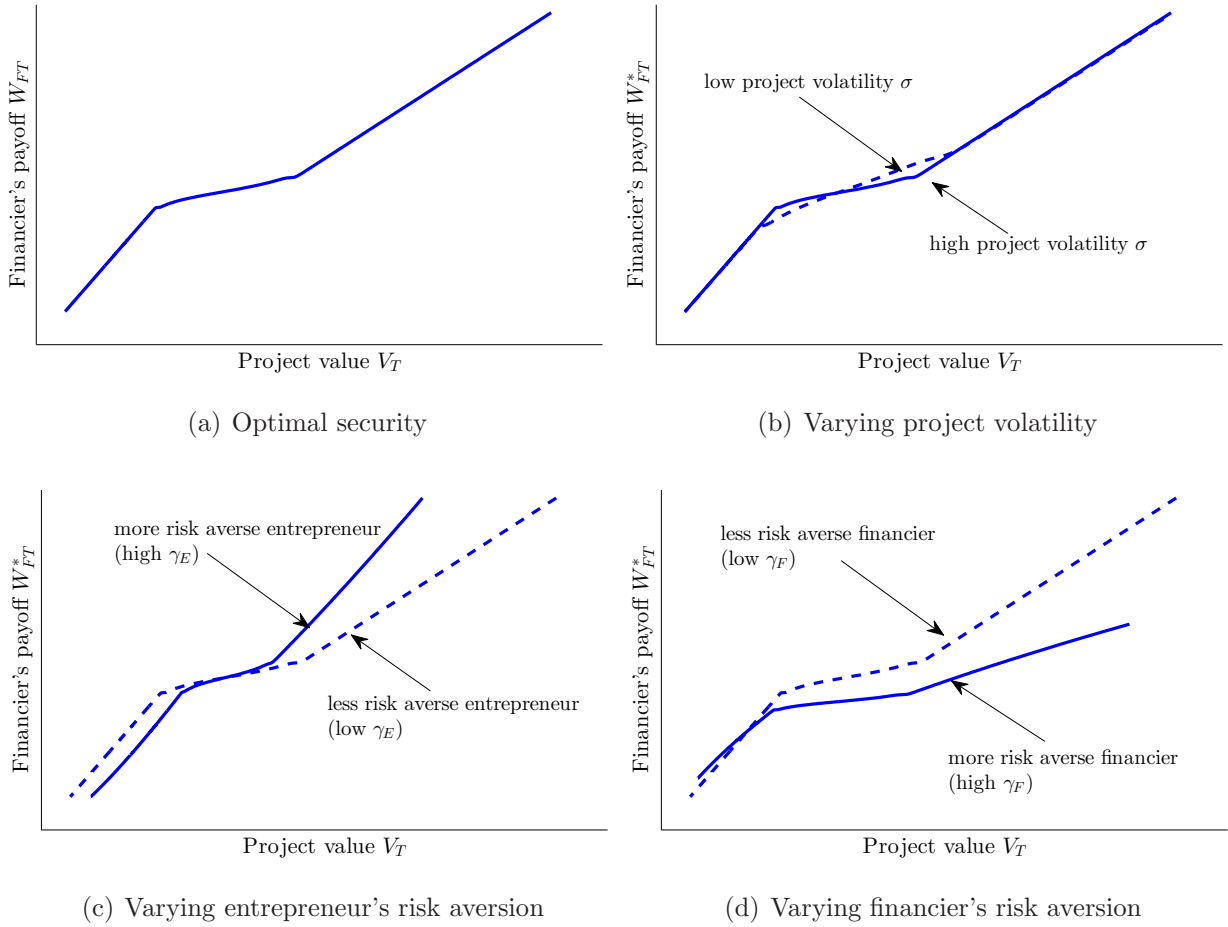
Figure B1: Optimal Security and its Properties under Multiplicative Status. Panel (a) depicts the optimal security for the multiplicative status specification. Panel (b) depicts the optimal security for relatively high project volatility (solid line) and relatively low project volatility (dashed line). Panel (c) depicts the optimal security when the entrepreneur is relatively more risk averse (solid line) and relatively less risk averse (dashed line). Panel (d) depicts the optimal security when the financier is relatively more risk averse (solid line) and relatively less risk averse (dashed line).[20]

Proof of Proposition B1. Because the steps of the proof are similar to those used in the proofs of Propositions 1 and 2, we provide only brief elaborations throughout the proof below.

Parameters a and b of the concavifying line $a + b * W_{E\tau}$ and the tangency points $\underline{L}$ and $\overline{L}$ are computed from the system

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[20]The parameter values are $\gamma_E = 3$, $\alpha = 15$, $L = 2$, $B = 1.74$, $V_0 = 3$, $\mu = 0.1$, $\sigma = 0.8$, $T = 3.5$, $\tau = 4$, $\gamma_F = 3$, and $u_F = -0.5$. In panel (b), $\sigma = 0.2$ for the dashed line and $\sigma = 0.8$ for the solid line. In panel (c), $\gamma_E = 3$ for the dashed line and $\gamma_E = 5$ for the solid line. In panel (d), $\gamma_F = 3$ for the dashed line and $\gamma_F = 5$ for the solid line.

$$\frac{\underline{L}^{1-\gamma_E}}{1-\gamma_E} = a + b\underline{L},$$

$$\frac{\overline{L}^{1-\gamma_E}}{1-\gamma_E}\alpha + B = a + b\overline{L}, \tag{B7}$$

$$\underline{L}^{-\gamma_E} = b,$$

$$\alpha\overline{L}^{-\gamma_E} = b,$$

solving which yields

$$a = \frac{B}{1-\alpha^{1/\gamma_E}}, \; b = \left(\frac{(\gamma_E-1)B}{\gamma_E(\alpha^{1/\gamma_E}-1)}\right)^{\gamma_E/(\gamma_E-1)},$$

$$\underline{L} = \left(\frac{(\gamma_E-1)B}{\gamma_E(\alpha^{1/\gamma_E}-1)}\right)^{1/(1-\gamma_E)}, \; \overline{L} = \alpha^{1/\gamma_E}\left(\frac{(\gamma_E-1)B}{\gamma_E(\alpha^{1/\gamma_E}-1)}\right)^{1/(1-\gamma_E)}. \tag{B8}$$

The concavified utility function of the entrepreneur is

$$u_E(W_{E\tau}) = \begin{cases} \dfrac{(W_{E\tau})^{1-\gamma_E}}{1-\gamma_E} & W_{E\tau} < \underline{L}, \\ a + b * W_{E\tau} & \underline{L} \le W_{E\tau} \le \overline{L}, \\ \alpha\dfrac{(W_{E\tau})^{1-\gamma_E}}{1-\gamma_E} + B & W_{E\tau} > \overline{L}. \end{cases} \tag{B9}$$

Using the first-order condition

$$V_\tau^* = \begin{cases} (y\xi_\tau)^{-1/\gamma_E} & y\xi_\tau > b, \\ (y\xi_\tau/\alpha)^{-1/\gamma_E} & y\xi_\tau \le b, \end{cases} \tag{B10}$$

in which y satisfies $E_T[\xi_\tau V_\tau^*] = \xi_T W_{ET}$, we compute the indirect utility function $v_E(W_{ET}) = E_T[u_e(V_\tau^*)]$:

$$\begin{aligned} v_E(W_{ET}) = {} & \frac{(y\xi_T)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E}K_{1T}N\left(\frac{-\ln b + \ln(y\xi_T) - K_4}{K_{3T}}\right) \\ & + \frac{\alpha^{1/\gamma_E}(y\xi_T)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E}K_{1T}N\left(\frac{\ln b - \ln(y\xi_T) + K_4}{K_{3T}}\right) + B * N\left(\frac{\ln b - \ln(y\xi_T) + K_{2T}}{K_{3T}}\right), \end{aligned} \tag{B11}$$

where $y\xi_T$ is given by

$$W_{ET} = (y\xi_T)^{-1/\gamma_E} K_{1T} N \left(\frac{-\ln b + \ln(y\xi_T) - K_4}{K_{3T}} \right) + \alpha^{1/\gamma_E} (y\xi_T)^{-1/\gamma_E} K_{1T} N \left(\frac{\ln b - \ln(y\xi_T) + K_4}{K_{3T}} \right).$$

(B12)

Differentiating (B11) and (B12) and rearranging, we obtain, respectively,

$$\frac{dv_E}{d(y\xi_T)} = (y\xi_T)^{-1/\gamma_E} K_{1T} C(y\xi_T, 1 - \gamma_E) - \frac{B * n \left((\ln b - \ln(y\xi_T) + K_{2T})/K_{3T} \right)}{y\xi_T K_{3T}}, \quad (B13)$$

$$\frac{d(y\xi_T)}{W_{ET}} = \left((y\xi_T)^{-1-1/\gamma_E} K_{1T} C(y\xi_T, 1) \right)^{-1}. \quad (B14)$$

where $C(\cdot, \cdot)$ is given in (B5). Multiplying (B13) and (B14) yields, after some simple algebra, the marginal indirect utility

$$\frac{dv_E}{dW_{ET}} = \frac{(y\xi_T)^{(\gamma_E - 1)/\gamma_E} K_{1T} K_{3T} C(y\xi_T, 1 - \gamma_E) - B * n \left((\ln b - \ln(y\xi_T) + K_{2T})/K_{3T} \right)}{(y\xi_T)^{-1/\gamma_E} K_{1T} K_{3T} C(y\xi_T, 1)}.$$

(B15)

As is the case for the indirect utility function (A9), establishing analytically that $v_E(\cdot)$ given in (B15) is an increasing concave function does not appear possible. Therefore, we have verified numerically that this is the case for a large number of model calibrations.

The entrepreneur solves the optimization problem (A15) in which the indirect utility $v_E(\cdot)$ is now given by (B15). Modifying the solution of this problem presented in Proposition 2 appropriately so as to account for the different $v_E(\cdot)$ yields the optimal security presented in Proposition B1.

$Q.E.D.$

B2. Different High and Low-Status Risk Aversions

In this Section, we extend our main setting to incorporate different attitudes towards risk depending on status. This analysis is motivated by recent works in which an individual's risk aversion differs between low and high wealth levels. Examples are Ait-Sahalia, Parker, and Yogo (2004) and Wachter and Yogo (2010). These studies show that allowing for differences in risk aversion is important when fitting the data.[21]

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[21] Ait-Sahalia, Parker, and Yogo (2004) and Wachter and Yogo (2010) consider preferences defined over two goods, basic and luxury, where each type of good is associated with its own risk aversion parameter. In addition to controlling risk aversion, the relation between these parameters determines whether the utility function is homothetic or not. The two works show that the nonhomotheticity of the utility function—which obtains when the risk aversions are different—improves the fit to the data.

Accordingly, we assume that the entrepreneur's utility function $u_E(W_{E\tau})$ is

$$
u_E(W_{E\tau}) = \begin{cases} \dfrac{(W_{E\tau})^{1-\gamma_{E\ell}}}{1-\gamma_{E\ell}} & W_{E\tau} < L, \\[4mm] \dfrac{(W_{E\tau}-\alpha)^{1-\gamma_{Eh}}}{1-\gamma_{Eh}} + B & W_{E\tau} \geq L, \end{cases} \tag{B16}
$$

where $\alpha \in [L - L^{\gamma_{E\ell}/\gamma_{Eh}}, L)$, $\gamma_{E\ell}, \gamma_{Eh}, L > 0$, and $B = L^{1-\gamma_{E\ell}}/(1-\gamma_{E\ell}) - (L-\alpha)^{1-\gamma_{Eh}}/(1-\gamma_{Eh})$. As compared to specification (2), the novelty here is the presence of two risk aversion parameters, $\gamma_{E\ell}$ and γ_{Eh}, acting in low and high-status regions, respectively. The special case of $\alpha = L - L^{\gamma_{E\ell}/\gamma_{Eh}}$ corresponds to (globally) concave preferences without status concerns, whereas $\alpha > L - L^{\gamma_{E\ell}/\gamma_{Eh}}$ corresponds to a concave-convex-concave utility function capturing status concerns. The other parameters in (B16) have the same interpretations as the corresponding parameters in (2).

Proposition B2. *The optimal security $W_{FT}^*(V_T)$ under specification (B16) is given parametrically through a pair of functions $(W_{FT}(x), V_T(x))$, where the parameter x varies from 0 to $+\infty$. The two functions are*

$$
W_{FT}(x) = (\bar{u}_F(1-\gamma_F))^{-1/(\gamma_F-1)} e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F}, \tag{B17}
$$

$$
V_T(x) = g(x)^{-1/\gamma_{E\ell}} K_{1l} \left(1 - N\left(\frac{\ln b - \ln g(x)z + K_{4l}}{K_{3T}} \right) \right) + g(x)^{-1/\gamma_{Eh}} K_{1h} N\left(\frac{\ln b - \ln g(x) + K_4}{K_{3T}} \right)
$$

$$
+ \alpha N\left(\frac{\ln b - \ln g(x) - K_{2T}}{K_{3T}} \right) + (\bar{u}_F(1-\gamma_F))^{-1/(\gamma_F-1)} e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F}, \tag{B18}
$$

where $g(x)$ is implicitly given by

$$\left[-g(x)^{\frac{\gamma_{E\ell}-1}{\gamma_{E\ell}}} \frac{K_{1l}}{\gamma_{E\ell}} \left(1 - N\left(\frac{\ln b - \ln g(x) + K_{4l}}{K_{3T}} \right) \right) + \frac{g(x)^{\frac{\gamma_{E\ell}-1}{\gamma_{E\ell}}}}{1-\gamma_{E\ell}} \frac{K_{1l}}{K_{3T}} n\left(\frac{\ln b - \ln g(x) + K_{4l}}{K_{3T}} \right) \right.$$

$$-g(x)^{\frac{\gamma_{Eh}-1}{\gamma_{Eh}}} \frac{K_{1h}}{\gamma_{Eh}} N\left(\frac{\ln b - \ln g(x) + K_{4h}}{K_{3T}} \right) - \frac{g(x)^{\frac{\gamma_{Eh}-1}{\gamma_{Eh}}}}{1-\gamma_{Eh}} \frac{K_{1h}}{K_{3T}} n\left(\frac{\ln b - \ln g(x) + K_{4h}}{K_{3T}} \right)$$

$$\left. - \frac{B}{K_{3T}} n\left(\frac{\ln b - \ln g(x) + K_{2T}}{K_{3T}} \right) \right] * \left[-g(x)^{-\frac{1}{\gamma_{E\ell}}} \frac{K_{1l}}{\gamma_{E\ell}} \left(1 - N\left(\frac{\ln b - \ln g(x) + K_{4l}}{K_{3T}} \right) \right) \right.$$

$$+ g(x)^{-\frac{1}{\gamma_{E\ell}}} \frac{K_{1l}}{K_{3T}} n\left(\frac{\ln b - \ln g(x) + K_{4l}}{K_{3T}} \right) - g(x)^{-\frac{1}{\gamma_{Eh}}} \frac{K_{1h}}{\gamma_{Eh}} N\left(\frac{\ln b - \ln g(x) + K_{4h}}{K_{3T}} \right)$$

$$\left. - g(x)^{-\frac{1}{\gamma_{Eh}}} \frac{K_{1h}}{K_{3T}} n\left(\frac{\ln b - \ln g(x) + K_{4h}}{K_{3T}} \right) - \frac{\alpha}{K_{3T}} n\left(\frac{\ln b - \ln g(x) - K_{2T}}{K_{3T}} \right) \right]^{-1} = zx. \quad (B19)$$

In the above, the constant b is implicitly given by

$$\frac{\gamma_{Eh} b^{(\gamma_{Eh}-1)/\gamma_{Eh}}}{1-\gamma_{Eh}} - \frac{\gamma_{E\ell} b^{(\gamma_{E\ell}-1)/\gamma_{E\ell}}}{1-\gamma_{E\ell}} - \alpha b + B = 0, \quad (B20)$$

K_{2T} and K_{3T} are as given in Proposition 1, $K_{1\ell}$, K_{1h}, $K_{4\ell}$, and K_{4h} are given by

$$K_{1\ell} \equiv e^{(1-\gamma_{E\ell})(\tau-T)\mu^2/(2\gamma_{E\ell}^2\sigma^2)}, \ K_{1h} \equiv e^{(1-\gamma_{Eh})(\tau-T)\mu^2/(2\gamma_{Eh}^2\sigma^2)},$$

$$K_{4\ell} \equiv (1-2\gamma_{E\ell})(\tau-T)\mu^2/(2\gamma_{E\ell}\sigma^2), \ K_{4h} \equiv (1-2\gamma_{Eh})(\tau-T)\mu^2/(2\gamma_{Eh}\sigma^2), \quad (B21)$$

and z is given by $E[\xi_T V_T^*(z\xi_T)] = V_0$.

The proof of Proposition B2 is provided at the end of this subsection. Figure B2 examines how varying the high-status risk aversion affects the optimal security. We see that the most pronounced effect is the changing slope of the right-most segment of the optimal security. As discussed in the main text, this slope reflects the conversion ratio implied by a convertible security. Hence, the existence of convertible securities with different conversion ratios can be explained by the differences in high-status risk aversion across entrepreneurs. When the entrepreneur's high-status risk aversion is relatively high, and when she has high status, she makes the financier's payoff for high project values more sensitive to the project value so as to reduce the sensitivity of her own wealth. This is why the slope of the right-most segment increases in high-status risk aversion. The behavior of the optimal security with respect to the other parameters is analogous to that examined in the main paper, and omitted here.
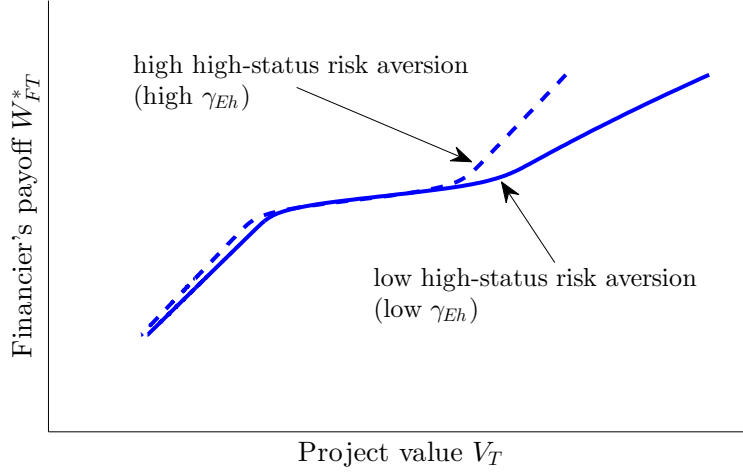
Figure B2: Effect of High-status Risk Aversion on the Optimal Security. The figure depicts the optimal security for relatively low high-status risk aversion $\gamma_{Eh} = 3$ (solid line) and relatively high high-status risk aversion $\gamma_{Eh} = 5$. The other parameter values are as in Figure 3.

Proof of Proposition B2. As in the proof of Proposition B1, we comment on the derivations only briefly because the logic of the proof is analogous to that in the proofs of Propositions 1 and 2.

Parameters a and b of the concavifying line $a + b * W_{E\tau}$ and the tangency points $\underline{L}$ and $\overline{L}$ are given by

$$
\frac{\underline{L}^{1-\overline{\gamma}_E}}{1 - \gamma_{E\ell}} = a + b\underline{L},
$$
$$
\frac{(\overline{L} - \alpha)^{1-\gamma_{Eh}}}{1 - \gamma_{Eh}} + B = a + b\overline{L}, \tag{B22}
$$
$$
\underline{L}^{-\gamma_{E\ell}} = b,
$$
$$
(\overline{L} - \alpha)^{-\gamma_{Eh}} = b.
$$

Solving this system of equations, we find that the slope b is given by (B20). The first-order condition is

$$
V_\tau^* = \begin{cases} (y\xi_\tau)^{-1/\gamma_{E\ell}} & y\xi_\tau > b, \\ (y\xi_\tau)^{-1/\gamma_{Eh}} + \alpha & y\xi_\tau \le b, \end{cases} \tag{B23}
$$

where y satisfies $E_T[\xi_\tau V_\tau^*] = \xi_T W_{ET}$. Using (B23), we compute the indirect utility function

$v_E(W_{ET}) = E_T[u_E(V^*_\tau)]$ as

$$
v_E(W_{ET}) = \frac{(y\xi_T)^{(\gamma_{E\ell}-1)/\gamma_{E\ell}}}{1-\gamma_{E\ell}} K_{1l} \left(1 - N\left(\frac{\ln b - \ln(y\xi_T) + K_{4l}}{K_{3T}} \right) \right)
$$
$$
+ \frac{(y\xi_T)^{(\gamma_{Eh}-1)/\gamma_{Eh}}}{1-\gamma_{Eh}} K_{1h} * N\left(\frac{\ln b - \ln(y\xi_T) + K_{4h}}{K_{3T}} \right) + B * N\left(\frac{\ln b - \ln(y\xi_T) + K_{2T}}{K_{3T}} \right),
$$

$$\text{(B24)}$$

and $y\xi_T$ is defined implicitly by

$$
W_{ET} = (y\xi_T)^{-1/\gamma_{E\ell}} K_{1l} \left(1 - N\left(\frac{\ln b - \ln(y\xi_T) + K_{4l}}{K_{3T}} \right) \right) + (y\xi_T)^{-1/\gamma_{Eh}} K_{1h} N\left(\frac{\ln b - \ln(y\xi_T) + K_{4h}}{K_{3T}} \right)
$$
$$
+ \alpha N\left(\frac{\ln b - \ln(y\xi_T) - K_{2T}}{K_{3T}} \right). \qquad\qquad \text{(B25)}
$$

Computing from (B24) and (B25) the derivatives $\frac{dv_E}{d(y\xi_T)}$ and $\frac{d(y\xi_T)}{W_{ET}}$, respectively, and multiplying them yields the marginal indirect utility that appears on the left-hand side of equation (B19). Modifying the solution of the entrepreneur's optimization problem (A15) presented in Proposition 2 appropriately to account for the different indirect utility—(B24) instead of (A9)—yields the optimal security presented in Proposition B2. $\qquad$ Q.E.D.

References

Abel, A., 1990, "Asset Prices under Habit Formation and Catching up with the Joneses," *American Economic Review Papers and Proceedings*, 80, 38–42.

Aghion, P., and J. Stein, 2008, "Growth vs. Margins: Destabilizing Consequences of Giving the Stock Market What it Wants," *Journal of Finance*, 63, 1025–1058.

Ait-Sahalia, Y., J. Parker, and M. Yogo, 2004, "Luxury Goods and the Equity Premium," *Journal of Finance*, 59, 2959–3004.

Allen, F., and A. Winton, 1995, "Corporate Financial Structure, Incentives, and Optimal Contracting," in *Handbooks in Operations Research and Management Science*, eds. R. Jarrow, V. Maksimovic, and W. Ziemba, vol. 9, North-Holland.

Auriol, E., and R. Renault, 2008, "Status and Incentives," *RAND Journal of Economics*, 39, 305–326.

Ball, S., C. Eckel, P. Grossman, and W. Zame, 2001, "Status in Markets," *Quarterly Journal of Economics*, 116, 161–188.

Basak, S., A. Pavlova, and A. Shapiro, 2007, "Optimal Asset Allocation and Risk Shifting in Money Management," *Review of Financial Studies*, 20, 1583–1621.

Becker, G., K. Murphy, and I. Werning, 2005, "The Equilibrium Distribution of Income and the Market for Status," *Journal of Political Economy*, 113, 282–310.

Begley, T., and W. Tan, 2001, "The Socio-Cultural Environment for Entrepreneurship: A Comparison between East Asian and Anglo-Saxon Countries," *Journal of International Business Studies*, 32, 537–553.

Besley, T., and M. Ghatak, 2008, "Status Incentives," *American Economic Review*, 98, 206–211.

Biais, B., T. Mariotti, and J. C. Rochet, 2013, "Dynamic Financial Contracting" in *Advances in Economics and Econometrics*, Tenth World Congress, eds. Daron Acemoglu, Manuel Arellano, and Eddie Dekel, vol. 1, Cambridge University Press, 125–171.

Bolton P., and C. Harris, 2013, "The Dynamics of Optimal Risk Sharing," working paper, Columbia University.

Brealey, R., S. Myers, and F. Allen, 2010, *Principles of Corporate Finance*, 10th edition, New York: McGraw-Hill/Irwin.

Brennan, M., and E. Schwartz, 1988, "The Case for Convertibles," *Journal of Applied Corporate Finance*, 1, 55–64.

Cadenillas, A., J. Cvitanic, and F. Zapatero, 2007, "Optimal Risk-Sharing with Effort and Project Choice," *Journal of Economic Theory*, 133, 403–440.

Campbell, J., and J. Cochrane, 1999, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205–251.

Carpenter, J., 2000, "Does Option Compensation Increase Managerial Risk Appetite?" *Journal of Finance*, 55, 2311–2331.

Chakraborty, A., and B. Yilmaz, 2011, "Adverse Selection and Convertible Bonds," *Review of Economic Studies*, 78, 148–175.

Constantinides, G., and B. Grundy, 1989, "Optimal Investment with Stock Repurchase and Financing as Signals," *Review of Financial Studies*, 2, 445–465.

Cornelli, F., and O. Yosha, 2003, "Financing and the Role of Convertible Securities," *Review of Economic Studies*, 70, 1–32.

Dijk, O., M. Holmen, and M. Kirchler, 2014, "Rank Matters—The Impact of Social Competition on Portfolio Choice," *European Economic Review*, 97–110.

Frank, R., 1985, *Choosing the Right Pond*, New York: Oxford University Press.

Friedman, M., and L. Savage, 1948, "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy*, 56, 279–304.

Georgarakos, D., M. Haliassos, and G. Pasini, 2014, "Household Debt and Social Interactions," *Review of Financial Studies*, 27, 1404–1433.

Goel, A., and A. Thakor, 2010, "Do Envious CEOs Cause Merger Waves?" *Review of Financial Studies*, 23, 487–517.

Gompers, P., 1999, "Ownership and Control in Entrepreneurial Firms: An Examination of Convertible Securities in Venture Capital Investment," working paper, Harvard Business School.

Green, R., 1984, "Investment Incentives, Debt, and Warrants," *Journal of Financial Economics*, 13, 115–136.

Greene, W., 2011, *Econometric Analysis*, 7th edition, Prentice Hall.

Gregory, N., 1980, "Relative Wealth and Risk Taking: A Short Note on the Friedman-Savage Utility Function," *Journal of Political Economy*, 88, 1226–1230.

Hart, O., 2001, "Financial Contracting," *Journal of Economic Literature*, 39, 1079–1100.

Heffetz, O., and R. Frank, 2011, "Preferences for Status: Evidence and Economic Implications," in *Handbook of Social Economics*, eds. Jess Benhabib, Matthew O. Jackson, and Alberto Bisin, vol. 1A, The Netherlands: North-Holland, 69–91.

Hellmann, T., 2006, "IPOs, Acquisitions and the Use of Convertible Securities in Venture Capital," *Journal of Financial Economics*, 81, 649–679.

Hong, H., W. Jiang, N. Wang, and B. Zhao, 2014, "Trading for Status," *Review of Financial Studies*, forthcoming.

Kaplan, S., and P. Stromberg, 2003, "Financial Contracting Theory Meets the Real World: An Empirical Analysis of Venture Capital Contracts," *Review of Economic Studies*, 70, 281–315.

Lyandres, E., and A. Zhdanov, 2014, "Convertible Debt and Investment Timing," *Journal of Corporate Finance*, 24, 21–37.

Malmendier, U., and G. Tate, 2009, "Superstar CEOs," *Quarterly Journal of Economics*, 124, 1593–1638.

Maug, E., A. Niessen-Ruenzi, and A. Zhivotova, 2013, "Pride and Prestige: Why Some Firms Pay Their CEOs Less," working paper, University of Mannheim.

Moldovanu, B., A. Sela, and X. Shi, 2007, "Contests for Status," *Journal of Political Economy*, 115, 338–363.

Noddings, T., S. Christoph, and J. Noddings, 2001, *The International Handbook of Convertible Securities: A Global Guide to the Convertible Market*, 2nd edition, Chicago: Glenlake Publishing Company.

Patel, N., and M. Subrahmanyam, 1978, "Utility Theory and Participation in Unfair Lotteries," *Journal of Economic Theory*, 19, 555–557.

Repullo, R., and J. Suarez, 2004, "Venture Capital Finance: A Security Design Approach," *Review of Finance*, 8, 75–108.

Robson, A., 1992, "Status, the Distribution of Wealth, Private and Social Attitudes to Risk," *Econometrica*, 60, 837-857.

Roussanov, N., 2010, "Diversification and its Discontents: Idiosyncratic and Entrepreneurial Risk in the Quest for Social Status," *Journal of Finance*, 65, 1755–1788.

Sahlman, W., 1990, "The Structure and Governance of Venture Capital Organizations," *Journal of Financial Economics*, 27, 473–521.

Sannikov, Y., 2012, "Dynamic Security Design and Corporate Financing," in *Handbook of Economics and Finance*, vol. 2, Elsevier-Northholland.

Schmidt, K., 2003, "Convertible Securities and Venture Capital Finance," *Journal of Finance*, 58, 1139–1166.

Shemesh, J., 2014, "CEO Social Status and Risk Taking," working paper, University of Southern California.

Stein, J., 1992, "Convertible Bonds as Backdoor Equity Financing," *Journal of Financial Economics*, 32, 3–21.

Wachter, J., and M. Yogo, 2010, "Why Do Household Portfolio Shares Rise in Wealth?" *Review of Financial Studies*, 2, 3929–3965.

Wade, J., J. Porac, T. Pollock, and S. Graffin, 2006, "The Burden of Celebrity: The Impact of CEO Certification Contests on CEO Pay and Performance," *Academy of Management Journal*, 49, 643–660.

Wendner, R., and L. Goulder, 2008, "Status Effects, Public Goods Provision, and the Excess Burden," *Journal of Public Economics*, 92, 1968–1985.