Moral Hazard in Active Asset Management: A Negative Consequence of Index Investing*

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Abstract

Closet indexing, in which active managers deceptively replicate index strategies, has increased alongside the explosive growth of explicit index investing. Motivated by these trends, we analyze a model featuring managers who endogenously exert effort to become informed, and investors having passive and active investment options. While increased competition among passive investments lowers fees and improves investors' outside options, active managers' profits are negatively impacted, reducing incentives for costly research. Thus, the growth of passive investing fuels the growth of closet indexing. Empirical tests are consistent with several novel implications of the model. We conclude by considering ways to limit closet indexing, including performance fees and holdings-based measures.

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1 Introduction

Passive investing, through index mutual funds or index ETFs, has increased dramatically in recent decades. Investors may now construct well-diversified portfolios at a fraction of the cost charged by actively managed funds. Despite this benefit, recent research highlights that passive investing has unanticipated consequences for equity markets: (i) increased intraday and daily volatility (Ben-David, Franzono and Moussawi 2014), (ii) increased return comovement (Da and Shive 2013), and (iii) higher trading costs and reduced analyst coverage (Israeli, Lee, Sridharan 2015). However, not all is lost; recent work suggests that these negative effects may be offset by a corresponding increase in active management.[1] Thus, to maintain market efficiency, passive investing and active management must move together.

The elephant in the room is that "active" management is no longer so active. Closet indexing, an agency conflict in which a manager touts an active strategy but passively invests in his fund's benchmark, has increased concurrently with the rise of passive investing (Cremers and Petajisto 2009, and Cremers et al. 2015).[2] Figure 1 shows a strong correlation between the percentage of mutual fund assets held in passive products and the aggregate level of closet indexing (measured as one minus the Cremers and Petajisto (2009) measure). The image is clear — the increase in passive investing is not being offset by more active management. Instead, charlatan funds exacerbate the consequences of passive investing rather than mitigate them. It is important then to understand the link between passive investing and active managers' incentives to be truly active. This is the subject of our paper.

Our main finding is that the advent of passive investing and its unbundling of two services, active asset selection and portfolio management, is inversely related to an active manager's incentive to be truly active. In other words, the rise of closet indexing is largely due to the rise of passive alternatives. The intuition is as follows. Mutual funds are pass through entities and a fund's revenue is determined by fee levels and assets under management (AUM). While an active manager's AUM is primarily driven by investors' beliefs about that manager's effort and skill in asset selection, the fee collected on AUM contains two components: (i) a fee for the portfolio management service (which we coin the *passive fee*) and (ii) a fee for active asset selection (coined the *active fee*). The introduction of passive investing and innovations to it (e.g., ETFs) have driven down the passive fee. Thus, managers now collect lower rents on any given quantity of AUM, decreasing the incentive

[1] Ye (2014) shows that return comovement is removed from stocks with large active institutional ownership.
[2] Furthermore, fees on passive investment products have fallen sharply (French 2008).

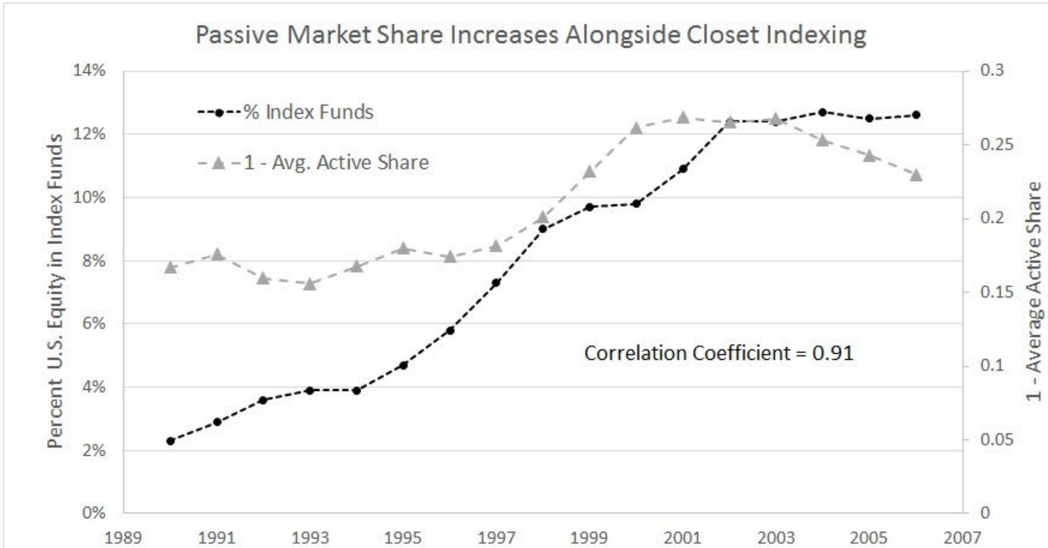


Figure 1: The market share of passively invested products is depicted as the darker line (French 2008). The level of closet indexing in active funds, measured as one minus the average Active Share measure of Cremers and Petajisto (2009), is depicted as the lighter line. The correlation coefficient between the two data series is 0.91334.

to exert costly effort and be truly active.

In the model, a universe of active fund managers supply active asset selection and portfolio management over two periods. Similar to Berk and Green (2004), we presuppose that managers are *ex-ante* heterogenous with respect to skill. However, managers privately observe their types and also choose whether or not to expend costly research effort. Managers who expend effort (truly active funds) expect positive abnormal returns, and expectations are increasing in skill. Managers who shirk (closet indexers) earn zero expected abnormal returns.[3] As in Berk and Green (2004), Pástor and Stambaugh (2012), and Brown and Wu (2014), we also assume funds face decreasing returns to scale and investors competitively allocate capital. Thus, because investors are rational, they collectively invest in each active fund until the expected return (net of fees) on the last dollar is equal to the expected return on their outside option — a passive investment which produces a market return net of the passive fee. This equality uniquely pins down a fund's AUM each period and, importantly, the passive fee cancels out on both sides of the expression. Thus, while the passive fee does not enter the expression for a fund's AUM, it does enter into a manager's revenue

[3]The expression "closet indexing" is not interpreted in the same manner as its use in Berk and Green (2004). As opposed to Berk and Green (2004), closet indexing is an agency conflict in our model and it allows closet indexers to extract rents from truly active managers.

calculation. This is an important insight for understanding why greater passive investing leads to greater closet indexing.

Closet indexing is endogenous in our model. Managers adhere to a threshold rule with respect to their skill; managers above the threshold choose to pursue truly active strategies and managers below it closet index. The threshold itself serves as a sufficient statistic for the severity of the moral hazard conflict. Importantly, the passive fee level is negatively related to the threshold level; as investors' outside opportunities in passive products improve, the rents associated with active management shrink. Lower rents reduce the benefit of effort and lead to greater closet indexing.

The model also provides new insights and testable hypotheses. First, despite exacerbating negative side effects of passive investing and lowering gross expected returns, the model shows that closet indexing does have a positive side. Namely, the presence of moral hazard improves the information content in realized returns. Thus, greater closet indexing allows investors to identify skilled managers more quickly. This insight manifests itself in two empirical predictions: (i) funds' performance-flow sensitivities early in their lives are stronger when closet indexing is more prevalent, and (ii) good funds should grow more quickly (and bad funds should go out of business more quickly) when closet indexing is more prevalent. We provide evidence of correlations consistent with these predictions. A sharp increase in closet indexing (decrease in active share) during the late 1990s was associated with both increased fund-flow sensitivities and increased closure rates for young funds.

While less-costly passive alternatives encourage active managers to closet index, passive investing benefits investors in many ways. Rather than considering limits on passive investing, we analyze possible ways to mitigate closet indexing. First, we consider the effect of a performance-based fee, referred to as a fulcrum fee, on a fund manager's incentive to be truly active. While used rarely in practice, there is precedence for performance-based fees in the mutual fund industry. If the performance-based fee is sufficiently low there is less closet indexing. The intuition is straightforward: by forcing the fund manager to more directly internalize his effort decision, the agency conflict is partially mitigated. However, if the performance-based fee is sufficiently large, there is greater closet indexing. By transferring a large share of excess returns to the manager, investors are crowded out and AUM shrinks. This reduces the benefit of effort and more closet indexing ensues. This result highlights that excessive performance-based fees can actually exacerbate moral hazard in active management.

Second, we consider holdings-based measures of a manager's effort choice: does he hold the benchmark or is he making active bets? This is precisely the intuition behind the Cremers and Petajisto (2009) measure of Active Share which they show to be predictive of future performance.

However, funds may anticipate investors' use of this measure and adjust their own holdings in response. Accordingly, we extend our model to accommodate closet indexers' abilities to choose portfolio variance strategically, i.e., to inject noise into the signal. Intuitively, closet indexers inject noise into their holdings and returns in order to cloud investors' inferences. Noise injection and closet indexing are complements; as the moral hazard problem becomes more severe, closet indexers inject more noise, attenuating investors' abilities to identify skilled managers. Our result suggests that signals of closet indexing based on holdings or tracking error may be less effective in the future as closet indexers adapt their strategies to avoid detection.

Our main contribution analyzes the connection between passive and active investment management. By debundling portfolio management and diversification from active asset selection, the growth of passive investing has driven down fees, reducing the profitability of AUM. Managers have less incentives to increase AUM by outperforming benchmarks, electing instead to reduce costs and become closet indexers. Separating active and passive management fees is an innovation in the literature, and is key in showing that passive indexing begets closet indexing. The feedback effect induced through managers' incentives likely exacerbates the negative effects of passive indexing on market efficiency. Furthermore, we analyze means to mitigate closet indexing, showing that performance-based fees and holdings-based measures may be ineffective.

Our work is closely related to several theoretical papers on asset management. First, our work builds from Berk and Green (2004), inheriting several key assumptions while endogenizing information production, separating fee components, and allowing for costly signaling. We highlight a spillover effect from passive to active management. Huang et. al. (2007) characterizes funds' performance-flow sensitivities in a framework in which investors have heterogenous participation costs. In our setup, performance-flow sensitivities are related to the equilibrium level of closet indexing and not to investor characteristics. Additionally, our work follows the literature examining strategic fee choices and structures. Nanda et. al. (2000) explores the effects of heterogeneous investors on funds' endogenous choice of fees and fee structures. Das and Sundaram (2002) compares fulcrum and incentive fees, showing that fee structures influence managers' incentives to take on risk in their portfolios. We take fees as given and fixed over time, and rather than focusing on welfare implications, focus on their effects on managers' incentives to invest in costly information.

2 Base Model

Consider a two-period model in which there exist a set of actively managed funds and an infinite number of investors. Each fund is run by a risk-neutral manager whose objective is to maximize net payoff. A fund's gross profit is determined through a price channel (management fee) and a quantity channel (assets under management). Actively managed funds supply two services to investors: (i) portfolio management and (ii) active asset selection. Investors are competitive and allocate their capital to equate the expected benefit (i.e., excess return) and cost (management fee).[4] The discount rate between periods is normalized to zero.

Each fund's manager has a private skill type θ that is drawn from the unit continuum $[0, 1]$. The managerial skill types are individually and identically distributed according to a continuous and strictly positive probability density function $g(\tilde{\theta})$,

$$\theta \sim g(\tilde{\theta}), \tag{1}$$

which has the cumulative density function $G(\tilde{\theta})$. A manager's skill corresponds to his fund's ability to earn excess returns; a high draw of θ implies a greater ability to earn excess returns than a low draw. This is expounded on shortly. Skill types are privately observed and cannot be credibly disclosed.

A fund's ability to earn excess returns is also influenced by whether or not the fund engages in costly research. Funds that choose to engage in research incur an incremental effort cost,

$$c > 0, \tag{2}$$

which is not observable. The funds that incur c are defined *active* managed funds (characterized by the subscript A). The funds that do not incur the cost are defined *closet index* funds (characterized by the subscript C).

A fund that engages in costly research achieves a distribution of excess returns (relative to its unmodeled benchmark) that first order stochastically dominates the excess return distribution without research. This feature is captured simply by allowing funds that pay c the chance to earn excess returns (i.e., alpha),

$$\alpha > 0, \tag{3}$$

[4] The assumption of perfectly elastic investor capital is commonplace in the delegated portfolio management literature, e.g. Berk and Green (2004), Pástor and Stambaugh (2012), and Brown and Wu (2014).

in each period.[5] The parameter proxies for the excess returns that are obtainable via active management. All excess returns flow through to investors and the market return r_m is normalized to zero.

An active fund's ability to earn α is influenced by its manager's skill and the elasticity of excess returns relative to assets under management (AUM). The elasticity parameter is denoted

$$\eta > 0. \tag{4}$$

The parameter η captures the notion that active funds experience diminishing-returns-to-scale.[6] A parameter of η near 0 (near ∞), represents almost perfectly inelastic (elastic) excess returns with respect to AUM. The parameter is related to the scalability of a fund's strategy and to the level of competition within the fund's strategy space.[7,8]

Each fund in the active space A earns $\alpha > 0$ in period $t \in \{1, 2\}$ with a probability that is determined by its manger's skill, the fund's elasticity parameter, and the fund's assets under management at t. Namely, the probability of success (S) is given by,

$$\Pr(S|\theta, K_t, \eta, A) \equiv \frac{\theta}{1 + \eta K_t}, \tag{5}$$

where K_t is the fund's AUM at t. The expression in (5) requires that the fund is active, i.e., the fund has incurred c. For simplicity, the probability that a closet index fund earns α is equal to zero but that assumption is relaxed in Section 3. The probability in (5) is increasing in θ, and decreasing in both K_t and η. That is, a high-type fund expects more excess returns *ex-ante* than a low-type fund. Furthermore, the probability of earning excess returns is diminishing in the product of the fund's AUM and the parameter η.

If a fund successfully earns α, investors observe it directly through the fund's performance because all excess returns are captured by the fund's investors. Conversely, if a fund does not earn α, investors cannot distinguish whether the fund was active and failed or if it never undertook costly research in the first place. Investors' inability to observe if a fund incurred c lends plausible deniability to the funds that do not succeed. This model feature incorporates moral hazard through the agency conflict coined closet indexing.

[5] We implicitly assume that effort is long-lived. Our results are qualitatively unchanged so long as effort does not decay entirely in a given period.

[6] Decreasing-returns-to scale is a common assumption in asset management models, e.g. see Berk and Green (2004) and Pástor and Stambaugh (2012). Furthermore, inclusion of the parameter is supported by empirical evidence. See Chen et. al. (2004), Fung et. al (2008), and Pollet and Wilson (2008).

[7] Hoberg et al. (2014) show that funds only experience persistent performance when facing few rivals and Pástor et al. (2014) show decreasing-returns-to-scale at the industry level.

[8] We do not directly model trading costs, however, the parameter η indirectly captures them.

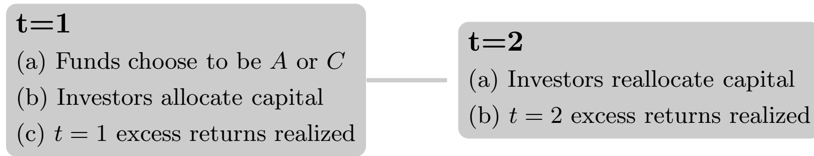


Figure 2: Managed Fund Game Timing

Investors also have access to a universe of passively managed products. Passively managed products (e.g., index funds and ETFs) supply only the portfolio management service to investors, i.e., they allow investors to earn market returns in a well-diversified portfolio. By definition, passive products provide the market return net of a management fee,

$$F_P > 0. \tag{6}$$

Each active fund charges an incremental fee above F_P,

$$F_E > 0, \tag{7}$$

in exchange for active asset selection. As such, the total management fee charged by an active fund is $F_E + F_P$. We refer to F_E as the active fee and F_P as the passive fee. The management fee is sticky and does not change between periods 1 and 2.

Investors are competitive and collectively allocate their capital in each of the two periods so that the expected net benefit of investment in each fund is equal to investing in a passive product. Investors are rational and have accurate beliefs about the funds. As such, at $t = 1$, investors allocate their capital according to their initial beliefs. At the conclusion of that period, excess returns are realized (or not) and passed through to investors. Investors subsequently update their beliefs about each fund and, at the beginning of $t = 2$, reallocate their capital based on their refined beliefs. The model's timing is summarized in Figure 2.

Investors' indifference between investing in an active fund or their outside option of a passive product yields the condition:

$$(1 - F_P) = (1 - F_E - F_P + \alpha \Pr(S_1|K_1, \eta))\,, \tag{8}$$

where $\Pr(S_1|K_t, \eta)$ is the probability of success that investors assess to the fund based on their beliefs. The left-hand side of (8) is the net return on the passive product and the right-hand side is the net return on an actively managed product. Investors allocate capital until the net returns are equal. Similarly, investors reallocate their capital at $t = 2$ based on their refined beliefs,

$$(1 - F_P) = (1 - F_E - F_P + \alpha \Pr(S_2|K_2, \eta, \mathbb{1}_{S_1}))\,, \tag{9}$$

where $\mathbb{1}_{S_1}$ is an indicator function that equals one if the fund succeeded in earning excess returns at $t = 1$ and equals zero otherwise. For clarity, define the following shorthand notation,

$$\Pr(S_1^\star) \equiv E[\theta|A], \tag{10}$$

$$\Pr(S_2^\star|\mathbb{1}_{S_1}) \equiv E[\theta|\mathbb{1}_{S_1}, A]. \tag{11}$$

The preceding notation implies,

$$\Pr(S|K_1, \eta, A) = \frac{\Pr(S_1^\star)}{1 + \eta K_1}, \tag{12}$$

$$\Pr(S|K_2, \eta, A) = \frac{\Pr(S_2^\star|\mathbb{1}_{S_1})}{1 + \eta K_2}. \tag{13}$$

The following lemma utilizes the expressions in (8) and (9) to define the capital allocation rule for the fund in each period.

Lemma 1. *A fund's capital allocations in periods* 1 *and* 2 *are given by,*

$$K_1 = \max\left\{\frac{\alpha \Pr(S_1^\star) - F_E}{\eta F_E}, 0\right\}, \tag{14}$$

$$K_2(\mathbb{1}_{S_1}) = \max\left\{\frac{\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E}{\eta F_E}, 0\right\}. \tag{15}$$

The comparative statics of the capital allocation in Lemma 1 are straightforward: investors allocate more capital to a fund when possible excess returns α are large, when the incremental fee F_E is low, and when the parameter η is small. The lemma also dictates that the capital allocations are increasing in the investors' beliefs about the manager's skill θ. Furthermore, the results of Lemma 1 imply that F_E is restricted to the interval $[0, \alpha]$ without loss of generality; if F_E exceeds α, capital allocations are surely equal to zero.

It is also important to note that F_P does not appear in (14) or (15). Investors pay F_P whether they use an actively managed fund or a passively managed fund. As such, capital allocations are driven entirely by expected excess returns and the incremental fee F_E above F_P. The passive fee F_P, however, does affects fund's net payoff. Active managers extract rents for the portfolio management service even though it does not enter investors' capital allocations.

Each fund's gross profit is determined by the total management fee $F_E + F_P$ and the amount of capital it attracts.[9] Denote a fund's net payoff as Π_j,

$$\Pi_j(\mathrm{a}_j, F_E) = \begin{cases} (F_E + F_P)\,(K_1 + K_2(0)) & \text{if } \mathrm{a}_j = C \\ (F_E + F_P)\,(K_1 + K_2(\mathbb{1}_{S_1})) - c & \text{if } \mathrm{a}_j = A, \end{cases} \tag{16}$$

[9]For tractability, fees are earned on beginning-of-period assets under management.

where a_j represents fund j's choice to be A or C.

The competition among investors implies that each fund captures all expected excess returns (from an investor's perspective). Each fund captures the expected excess returns via fund flows (changes in quantity) because the fee (price) is fixed — this is a point highlighted by both Berk and Green (2004) and Pástor and Stambaugh (2012). As just discussed, each fund's gross profit also includes the rents associated with delegated portfolio management via the passive fee. Consequently, there are two gross profit components: expected excess returns (from an investor's perspective) and the cumulative value of delegated portfolio management. "From an investor's perspective" is emphasized because investors' beliefs about expected excess returns and a fund's beliefs are almost certainly different. The presence of adverse selection, which is discussed in detail shortly, implies that high (low) type funds believe that excess returns will be larger (smaller) than investors do. In Section 2.2, this mismatch of beliefs is addressed by allowing funds to charge a performance-based fee and retain a portion of excess returns.

A fund's choice to be an active fund or closet index fund is endogenous. This is demonstrated by first defining $\Delta(\theta)$ as a benefit-cost function for a fund with type θ,

$$\Delta(\theta) \equiv \Pi_j(A, F_E|\theta) - \Pi_j(C, F_E|\theta), \tag{17}$$

$$= \frac{\theta(F_E + F_P)}{1 + \eta K_1}\left(K_2(1) - K_2(0)\right) - c. \tag{18}$$

The function $\Delta(\theta)$ compares the fund's expected payoffs if it chooses to be an active fund relative to the fund's expected payoffs if it chooses to be a closet indexer. It is clear from (18) that the benefit of being active is increasing with the separation that occurs at $t = 2$, which is the quantity

$$K_2(1) - K_2(0). \tag{19}$$

Highlighting the benefit of separation is useful in understanding the model's tensions. The function in (18) is weakly increasing in θ and $\Delta(0) < 0$. If $\Delta(1) > 0$, there exists some fund type $\theta \in (0, 1)$ for which the fund is indifferent between being an active fund or a closet index fund. The following lemma formalizes the intuition,

Lemma 2. *There exists an equilibrium in which a fund chooses to pay or not pay c according to a threshold,*

$$\theta^* \in (0, 1]. \tag{20}$$

A fund with type $\theta < \theta^*$ chooses to not pay c and is a closet index fund (C). A fund with type $\theta \geq \theta^*$ chooses to pay c and is an active fund (A).

According to Lemma 2, investors face two agency conflicts when they choose a fund manager. The first is adverse selection, which is exogenous in the model. Conditional on choosing a fund that engages in active research, an investor cannot distinguish if the fund is marginal (i.e., a type near θ^*) or superior (i.e., a type near 1). The second is moral hazard, which arises endogenously. Investors cannot distinguish between the funds that are engaging in costly research and the ones that masquerade as if they are. Indeed, an investor choosing a fund will fall prey to moral hazard with probability $G(\theta^*)$. For the remainder of the analysis, the threshold itself is considered, as it is a sufficient statistic for the fraction of funds that are closet indexers and the severity of the moral hazard problem.

Proposition 1. *The threshold θ^* is inversely related to the passive fee.*

Competition and innovation in the passive managed product space has a spillover effect in active management. As the fees associated with products like index funds and ETFs fall, the incentive to actively manage a portfolio also falls. The result of Proposition 1 reconciles the observed empirical trends of closet indexing increasing (Cremers and Petajisto 2009) concurrently with increasing passive investing (French 2008) and decreasing fees on passive products.[10] The analysis implies that the negative consequences of passive investing discussed in our introduction are exacerbated by active management, not mitigated.

Corollary 1.1. *The threshold θ^* is*

(i) increasing in c,

(ii) decreasing in α,

(iii) increasing in η,

The comparative static of θ^* with respect to F_E is equivocal.

Corollary 1.1 provides a full characterization of the threshold. The first two comparative statics in Corollary 1.1 are natural: one would expect the fraction of closet index funds to increase with

[10] French (2008) documents that the annual costs of open-end funds shrunk from 2.19% of AUM in 1980 to 1.00% in 2006. More recently, the Investment Company Institute (2014) reports that expense ratios of actively managed equity funds has fallen from 106 bps to 89 bps from 2000 to 2013. Actively managed bond fund expense ratios have fallen from 78 bps to 65 bps over the same window. The trend is persistent among index funds as well. From 2000 to 2014 expense ratios have fallen from 27 bps to 12 bps for index equity funds and from 21 bps to 11 bps for index bond funds. Front-end load fees have also decreased. Wahal and Wang (2011) provide empirical evidence that, since 1998, the decline in management fees is attributed to an increase in competition in the managed fund space. Increased competition is also reflected through the number of funds. The Investment Company Institute (2014) reports that the number of managed funds has grown more than eightfold between 1982 and 2013.

the cost of being truly active and decrease with potential excess returns. The comparative static with respect to η is less obvious.

Considering η, the fraction of closet index funds increases as excess returns become more sensitive to AUM (due to increased competition or reduced strategy capacity). The result is best understood by considering a fund-flow hedging effect associated with η, and its corresponding effect on the separation that occurs at $t = 2$. If a fund succeeds at $t = 1$, investors update their beliefs upward and more capital flows into the fund. This, however, has an adverse effect on expected excess returns because an increase in AUM decreases the probability of success due to η. Consequently, flows to the fund are dampened. Similarly, if a fund fails at $t = 1$, investors update their beliefs downward and capital flows from the fund. This has a positive effect on expected excess returns because a decrease in AUM increases the probability of success via η. Again, flows from the fund are dampened. Consequently, η hedges fund-flow volatility and necessarily dampens the spread $K_2(1)-K_2(0)$. Larger values of η result in more hedging, decreasing separation at $t = 2$ and the benefit of being truly active.

Finally, the comparative static with respect to F_E is generally consistent with the comparative static with respect to F_P. However, when F_E is sufficiently large, decreasing F_E has a positive effect on AUM which counteracts the loss in fee revenue, flipping the sign on the comparative static. When fees are high and AUM small, decreasing-returns-to-scale are not severe, so lowering fees increases AUM substantially. This effect outweighs the loss of fee revenue which drives the comparative static on F_P.

2.1 Performance-Flow Sensitivity

The model of Berk and Green (2004) demonstrates that the evolution of investors' beliefs is captured through fund flows.[11] Therefore, if performance fund flow sensitivities are more volatile with moral hazard than without, performance must contain more information because investors are reacting more strongly. Define a fund's relative net flow as,

$$\Delta K(\mathbb{1}_{S_1}) \equiv \frac{K_2(\mathbb{1}_{S_1}) - K_1}{K_1}. \tag{21}$$

The expression in (21) immediately extends itself to the following lemma.

Lemma 3. *If $\theta^* < 1$, a fund that succeeds (fails) at $t = 1$ experiences a positive (negative) net fund flow at $t = 2$.*

[11] For empirical work regarding the so-called "fund-flow anomaly", see Ippolito (1992), Chevalier and Ellison (1997), Sirri and Tufano (1998).

Lemma 3 is natural — as long as there is a fraction of funds that are truly active, fund flows are positively related to realized performance. The following corollary characterizes performance-flow sensitivities.

Corollary 1.2. *Performance-flow sensitivities are*

(i) and decreasing in F_P,

(ii) increasing in c,

(iii) decreasing in α,

(iv) increasing in η.

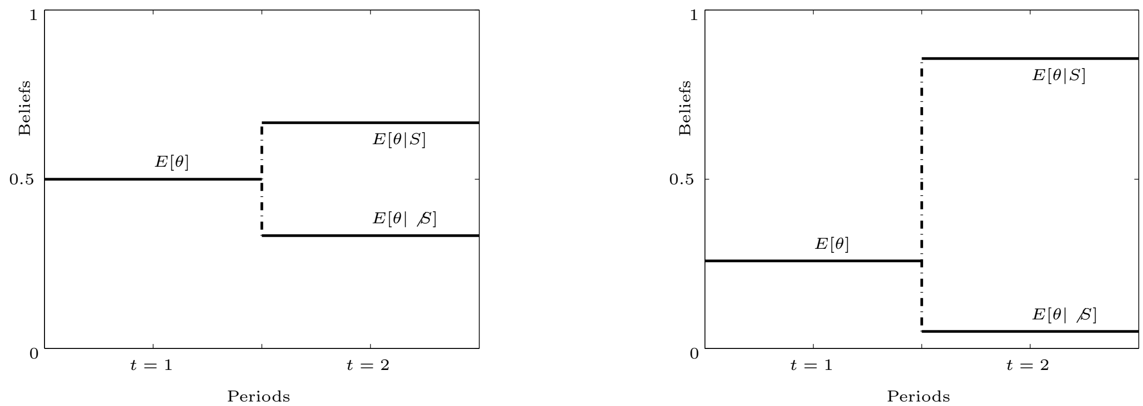
The comparative static with respect to F_E is equivocal.

A quick comparison of Proposition 1 and Corollary 1.1 to Corollary 1.2 provides straightforward intuition: funds' performance-flow sensitivities increase when the fraction of closet index funds increases. In fact, the result suggests that the performance-flow sensitivities in the Berk and Green (2004) model in which there is no moral hazard are dampened relative to a setting in which moral hazard exists. The intuition for the result is that returns provide a signal of both skill and effort. Because fund flows are the channel that communicates investors' beliefs, this implies that a stronger signal-to-noise ratio is associated with greater fund-flow volatility. Therefore, greater closet indexing yields an unanticipated benefit to investors. While investors initially expect lower gross returns with moral hazard versus without, realized performance is more informative and investors are able to identify skilled managers more quickly. Figure 3 diagrams a simple example showing investors' belief evolution with moral hazard and without.

2.2 Performance-Based Fees

Although performance-based fees (i.e., fulcrum fees) are rare in practice, it is natural to think that high powered managerial incentives would partially mitigate the moral hazard conflict of closet indexing.[12] Thus, one might expect less closet indexing in funds that charge performance-based fees versus those that do not. We show that this is not necessarily the case.

[12]The 1970 Amendment to the Investment Advisors Act of 1940 introduced a performance-based fee option for mutual funds. The performance-based fee, however, is only admissible if it a "fulcrum" fee, i.e., the performance component must be symmetric around a predetermined benchmark. The 1970 Amendment implicitly prohibits the standard asymmetric performance-based fees used in other managed products, e.g., the "2-and-20" schedule used by many hedge funds.



(a) Without Moral Hazard

(b) With Moral Hazard

Figure 3: Investor Beliefs without and with Moral Hazard.

In this extension, in addition to charging a management fee of $F_E + F_P$, a fund retains a fraction of excess returns,

$$\lambda > 0. \tag{22}$$

Lemma 4. *A fund's capital allocations in periods* 1 *and* 2 *are given by*

$$K_1^{\lambda} = \max\left\{\frac{\alpha(1-\lambda)\Pr(S_1^{\star}) - F_E}{\eta F_E}, 0\right\}, \tag{23}$$

$$K_2^{\lambda}(\mathbb{1}_{S_1}) = \max\left\{\frac{\alpha(1-\lambda)\Pr(S_2^{\star}|\mathbb{1}_{S_1}) - F_E}{\eta F_E}, 0\right\}. \tag{24}$$

The comparative statics of K_1^{λ} and $K_2^{\lambda}(\mathbb{1}_{S_1})$ with respect to λ are natural: a fund's AUM is diminishing as the fund retains a larger portion of excess returns.

Funds endogenously separate as active funds or as closet indexers depending on those actions' payoffs. Similar to the base model, there exists a unique threshold for which a fund with skill level above the threshold chooses to be an active fund and a fund with a skill level below it chooses to be a closet indexer.

Lemma 5. *In a setup in which funds charge both a management fee and a performance-based fee, there exists a unique equilibrium in which a fund chooses to pay or not pay c according to a threshold,*

$$\theta^{\lambda} \in (0, 1]. \tag{25}$$

A fund with type $\theta < \theta^{\lambda}$ chooses to not pay c and is a closet index fund (C). A fund with type $\theta \geq \theta^{\lambda}$ chooses to pay c and is an active fund (A).

So far, the results of a setup with a performance-based fee are redolent of a setup with only a fixed management fee. However, as Lemma 6 demonstrates, the performance-based fee's level has vacillating implications on the fraction of closet index funds.

Lemma 6. *The comparative static of θ^{λ} is first decreasing and then increasing in λ.*

Introducing a performance-based fee has two opposite effects on funds' incentives. First, a performance fee reduces the expected abnormal returns which are passed on to investors, lowering funds' AUM. As funds' profits depend on AUM directly, reducing AUM is costly and provides less incentive for information production, encouraging more closet indexing. Second, funds internalize a portion of their information-production decision and directly benefit from α. This is very beneficial for highly-skilled funds, as their true type is higher than investors' beliefs. It is also beneficial for all active funds relative to closet indexers. Only active funds have a positive probability of earning α, a portion of which is retained via the performance fee.

When λ is zero, increasing it marginally has a positive impact on information production, reducing closet indexing. From a performance-based fee of zero, AUM is only perturbed slightly. However, the benefit of the performance-based fee is only felt by active funds, making it more costly to closet index. As a result, more funds produce information, and at least initially, performance-based fees help to mitigate the moral hazard problem. However, as λ becomes larger, the funds' own profits are hurt substantially by reduced AUM. With more substantial performance fees, the negative impact on AUM outweighs the benefit of retaining profits internally. This impacts the marginal fund negatively, and as a result closet indexing increases. In the limit, even the most highly-skilled funds are affected and $\lambda = 1$ stops information production altogether. This result highlights that excessive performance-based fees can actually exacerbate moral hazard in asset management.

3 Detecting Closet Indexing

The feedback effect from passive to active investing exacerbates passive investing's negative externalities on market efficiency. Rather than attempting to limit the growth of passive investing, the negative spillover can be limited by reducing funds' abilities to closet index. We turn our attention to ways to detect and potentially deter closet indexing. To date, the most widely used signal of active management is fund holdings. The Cremers and Petajisto (2009) Active Share measure uses a distance measure between fund benchmark and fund holdings to determine what fraction of the portfolio is "active." While the measure has had predictive power in the past, it is not obvious that the strategies of closet indexers will not adapt and incorporate forms of signal jamming, e.g., taking uninformed bets to generate a false sense of active management. We show numerically that signal jamming should be expected as an equilibrium outcome.

The main assumptions from the base model are maintained: each fund manager's skill is individually and identically distributed according to the distribution $g(\tilde{\theta})$ (uniform in the numerical methods), funds that engage in active management pay a fixed cost $c > 0$, and fees contain both a passive management component F_P and an active management component F_E. In this extension, however, an active fund's excess returns $\tilde{\alpha}_{j,t}$ are distributed normally,

$$\tilde{\alpha}_{j,t} \sim N\left(\frac{\alpha\theta}{1+\eta K_t}, \sigma_A^2\right). \tag{26}$$

The additional richness of normally distributed returns comes at the cost of analytic tractability. Consequently, we use numerical methods to solve the model incorporating normally-distributed profits.

A closet index fund's excess returns, $\epsilon_{j,t}$, are zero in expectation, but realized outcomes are noisy,

$$\epsilon_{j,t} \sim N(0, \sigma_C^2), \tag{27}$$

where σ_C^2 is endogenously chosen by the closet funds. A closet index fund's variance choice is achieved by making uninformed bets via over- and under-weighting many of the benchmark's components.[13]

The setup implies that the distribution of returns for a fund that pays c first-order-stochastically dominates the distribution if the fund did not. The setup also better captures the underlying portfolio problem: a highly-skilled manager (θ near 1) will identify many profitable trades and expects to earn abnormal returns. A marginally-skilled manager (θ near θ^*) will be able to identify some profitable trades but will also make some mistakes. Nonetheless, the marginally-skilled manager expects more profitable trades than unprofitable ones. This feature is captured by parameterizing the mean of an active manager's distribution by his type. Furthermore, for consistency with the base model, the mean is scaled by $\frac{1}{1+\eta K_t}$ to reflect diminishing-returns-to-scale.

The setup allows a skilled manager to under-perform; even a highly-skilled manager has positive probability on losing money. Conversely, the setup allows a closet index fund to get lucky and achieve positive abnormal returns. Despite these possibilities, numerical analysis of the model yields similar insights as in Proposition 1 and Corollary 1.1.

[13] Funds can also add noise and increase active share by purchasing out-of-benchmark stocks.

Remark 1. *In a setup with normally distributed excess returns, there exists an equilibrium in which a fund chooses to pay or not pay c according to a threshold,*

$$\theta^* \in (0, 1]. \tag{28}$$

A fund with type $\theta < \theta^*$ chooses to not pay c and is a closet index fund (C). A fund with type $\theta \geq \theta^*$ chooses to pay c and is an active fund (A). The threshold's comparative statics match those in Proposition 1 and Coroillary 1.1. Furthermore, the threshold is increasing in σ_A^2.

Remark 1 reveals that θ^* increases with σ_A^2. The finding is intuitive; the incentive to closet index increases as realized performance becomes a noisier signal of both managerial skill and active management.

Remark 2. *A closet index fund chooses a finite and strictly positive variance σ_C^2. The level of σ_C^2 is positively correlated with θ^*.*

The preceding remark implies that a closet index fund necessarily deviates from its benchmark with uninformed bets. The extent of this "signal-jamming" behavior is positively related to the level of closet indexing, i.e. strategic variance and closet indexing are complements. As more funds closet index, the expected performance gap between active funds and closet index funds increases. The larger gap leads closet index funds to increase variance, directing more probabilistic weight to the expected performance outcomes of truly active funds.[14]

Remark 2 suggests that measures of closet indexing should account for strategic, uninformed deviations from index weightings. Even in a world with risk aversion, we expect funds will add strategic variance, as not adding variance would almost surely identify the fund as a closet indexer. Therefore, as investors attempt to use holdings or tracking error to identify fund skill and effort (or a lack thereof), we expect that closet indexers will counteract these identification methods. In the future, holdings and tracking error may be less effective for identifying skilled managers as closet indexers adapt to wide-spread dissemination of this information.

4 Empirical Evidence

4.1 Performance-Flow Sensitivities

To examine the novel empirical implications of our model, we obtain fund returns and total net assets (TNA) from the CRSP Survivorship-Bias-Free Mutual Fund Database. We follow the mutual

[14] Heinkel and Stoughton (1994) consider a similar problem and emphasize that the variance has an internal maximum. In our context, too much variance results in very high returns being more attributed to closet indexers than to truly active funds.

fund literature in measuring fund flows as:

$$FundFlow_{j,t} = TNA_{j,t} - TNA_{j,t-1}(1 + R_{j,t}) \tag{29}$$

where $R_{j,t}$ is fund j's cumulative 12-month return in year t. We analyze fund flows beginning in the third year of a funds' life (after two years of return and TNA data are available in CRSP) due to the incubation bias documented in Evans (2010). To avoid outliers from driving our results, we follow Coval and Stafford (2007) and require that changes in TNA are not too extreme ($-0.5 < \Delta TNA_{j,t}/TNA_{j,t-1} < 2.0$). We also require TNA_{t-1} is at least \$15M as Elton, Gruber and Blake (2001) shows small funds returns are upwardly biased in the CRSP database (a criterion also used in Amihud and Goyenko 2013 and Simutin 2013).

The 1990s were characterized by increased availability of information and changing investor participation and attention. To focus on investors' learning and mitigate any potential bias introduced by these general trends, we adjust fund flows by the cohort-year average. We measure relative fund flows as:

$$AdjFundFlow_{j,t} = FundFlow_{j,t} - \frac{\sum_{i \in N_j} FundFlow_{i,t}}{|N_j|} \tag{30}$$

where N_j is the set of funds in the same initial cohort as fund j and $|N_j|$ denotes the size of N_j. Adjusting fund flows is similar to using cohort-year fixed effects.

Given the large amount of noise in fund flows, we use coarse data divisions to maximize sample size when comparing funds' performance-flow sensitivities between early and later years. Specifically, we first split funds into below or above median returns in each year (relative to their cohort). We then split funds based on their starting dates between the early 1990s, when closet indexing was just beginning an upward trajectory, and the late 1990s when closet indexing had become prominent. Our final split, allowing us to analyze investor learning, separates funds' adjusted flows in the first two years of our sample (funds' third and fourth years of reporting) from their flows in the following two years (funds' fifth and sixth years of reporting). Using indicator variables to reflect each of these splits, we estimate:

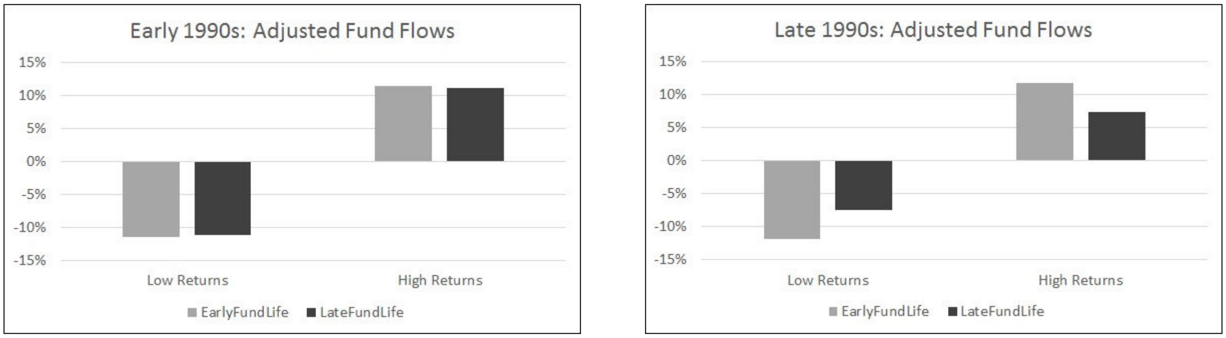
$$\begin{aligned} AdjFundFlow_{j,t} &= \beta_0 + \beta_1 \times \mathit{Early1990s}_{j,t} + \beta_2 \times EarlyFundYears_{j,t} \\ &+\beta_3 \times AboveMedianReturn_{j,t} \\ &+\beta_4 \times \mathit{Early1990s}_{j,t} \times EarlyFundYears_{j,t} \\ &+\beta_5 \times \mathit{Early1990s}_{j,t} \times AboveMedianReturn_{j,t} \\ &+\beta_6 \times AboveMedianReturn_{j,t} \times EarlyFundYears_{j,t} \\ &+\beta_7 \times \mathit{Early1990s}_{j,t} \times AboveMedianReturn_{j,t} \times EarlyFundYears_{j,t} \end{aligned} \tag{31}$$

Early1990s is an indicator variable equal to one for cohorts starting between 1990 and 1994 and zero for cohorts starting between 1995 and 1999. *AboveMedianReturn* is equal to one when a fund's return is higher than the cohort's average that year. *EarlyFundYears* is equal to one in the third and fourth years following funds' inceptions, and equal to zero in funds' fifth and sixth years.

If investors are Bayesians, they will learn more early in funds' lives such that the performance-flow relation may tend to be larger for young funds relative to old funds. In the later period, our split between below- and above-median returns then implies β_2 should be negative, while $\beta_2 + \beta_6$ should be positive. Moreover, if the speed of investor learning increases (due to the increased prominence of closet indexing in the late 1990s (Cremers and Petajisto 2009)), then the contrast in performance-flow relations for young and old funds will increase. As a result, we expect β_4 to be positive (less negative performance-flow sensitivity in the early 1990s following below-median returns) and $\beta_4+\beta_7$ to be negative (smaller performance-flow sensitivity in the early 1990s following above-median returns).

Table 1 presents the estimation results. Investors performance-flow sensitivity is higher for early returns only in the late 1990s. In the late 1990s, the average performance-flow magnitude is 61% larger ($\frac{\beta_2+\beta_6}{\beta_0+\beta_5}$ for above median returns, $\frac{\beta_2}{\beta_0}$ for below median returns, p-values of 0.012 and 0.011) in early years relative to later years. However, in the early 1990s, early years' average performance-flow magnitude is only 3% larger ($\frac{\beta_2+\beta_4}{\beta_0+\beta_1}$ for above median returns, $\frac{\beta_2+\beta_4+\beta_6+\beta_7}{\beta_0+\beta_1+\beta_3+\beta_5}$ for below median returns, p-values of 0.872 and 0.863). Figure 4 displays these differences.

Performance-flow sensitivity differences are statistically different across time periods. For below-median returns, the difference between the early 1990s and late 1990s is represented by β_4 which is positive (p-value 0.093), indicating a larger difference between early and late performance-flow sensitivities in the late 1990s. For above-median returns, the difference between the early 1990s and late 1990s is represented by $\beta_4 + \beta_7$. The sum of the coefficients is in this case negative (p-value



(a) Early 1990s

(b) Late 1990s

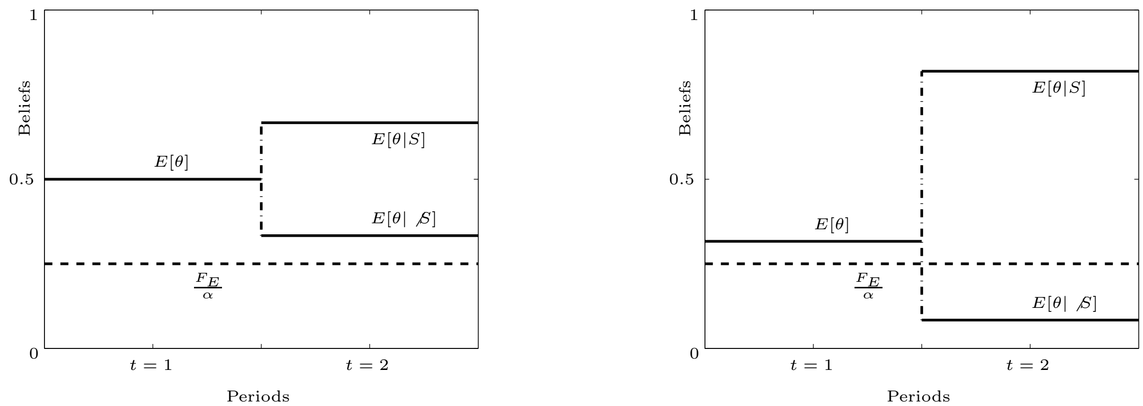
Figure 4: Funds' early-life performance-flow sensitivities are relatively higher in late 1990s.

0.092), but as before, this indicates a larger difference between early and late performance-flow sensitivities in the late 1990s. Throughout this section, two-sided p-values are reported. One-sided tests (as predicted by our theory) are significant at the standard 5% threshold. Finally, a joint test of $\beta_4 = 0$ and $\beta_4 + \beta_7 = 0$ is rejected (p-value 0.059), supporting our model's prediction that investors learn more quickly in the presence of increased moral hazard.

4.2 Fund Closure

Within our model, moral hazard has a time varying effect on manager identification. While clouded initially, identification becomes more clear over time because of enhanced signal-to-noise ratios. Under some parameter settings, belief updating can be severe enough to result in fund closure. Figure 5 shows how introducing moral hazard induces more severe updating, driving beliefs conditional on failure below a critical threshold. Below this threshold, expected abnormal returns are not high enough to justify the excess fee, and funds attract no assets. As moral hazard becomes more severe, more funds generate poor returns, resulting in more fund closures.

Empirically, our model implies that average fund quality should increase over time, and the increase should be more severe with more closet indexing. While moral hazard creates a lower starting level for average skill, performance quickly separates managers and closet indexers subsequently shut down, leaving only highly-skilled managers. Existing evidence suggests that closet indexing and average fund skill have increased together over time. Figure 4 on page 3348 of Cremers and Petajisto (2009) shows that closet indexing has increased since the early 1990s, while Figure 2 on page 38 of Pástor, Stambaugh, and Taylor (2015) shows a similar trend for mean and median fund skill. However, to our knowledge, there is no evidence on whether funds are closing more quickly as closet indexing and average skill increase.



(a) Without Moral Hazard

(b) With Moral Hazard

Figure 5: Fund Closure without and with Moral Hazard.

To address this gap, we perform a simple analysis of how the speed of fund closure has changed over time. We also examine whether a fund's survival rate is tied to its level of closet indexing. We use CRSP for funds' inception dates and, where applicable, funds' termination dates. We join this data with the fund-level Active Share data from Cremers and Petajisto (2009) (extended through 2012). The Active Share measure is inversely related to closet indexing.[15] Using fund-level inception dates, termination dates, and initial level of Active Share we estimate a Cox proportional hazard model based on a fund's initial Active Share level and inception-year fixed effects to control for fund cohorts. Including year fixed effects proxies for the aggregate level of closet indexing at a fund's inception. Table 2 presents the estimation results.

Using 1990 as the benchmark year, the relative hazard ratios over the time series are displayed in Figure 6. The relative hazard ratio is calculated as,

$$\Delta\lambda(1990, t) = \exp^{FA_{1990} - FA_t}, \tag{32}$$

where $\Delta\lambda(1990, t) - 1$ represents the relative probability increase of closure for a fund opened in year t relative to a fund opened in 1990. For example, the relative hazard ratio in 2000 is approximately 2.5, implying that a fund opened in 2000 has a 150% greater probability of closing relative to a fund opened in 1990. The relative hazard ratios are low in the early 1980s, but then steadily increase through the end of the 1990s. The relative hazard ratios then level off in the 2000s. The similarity in trends among closeting indexing, average managerial skill and speed of fund closure lend supporting evidence that moral hazard plays a role in reconciling the concurrent trends of

[15]See Cremers and Petajisto (2009) for a discussion of the measure.

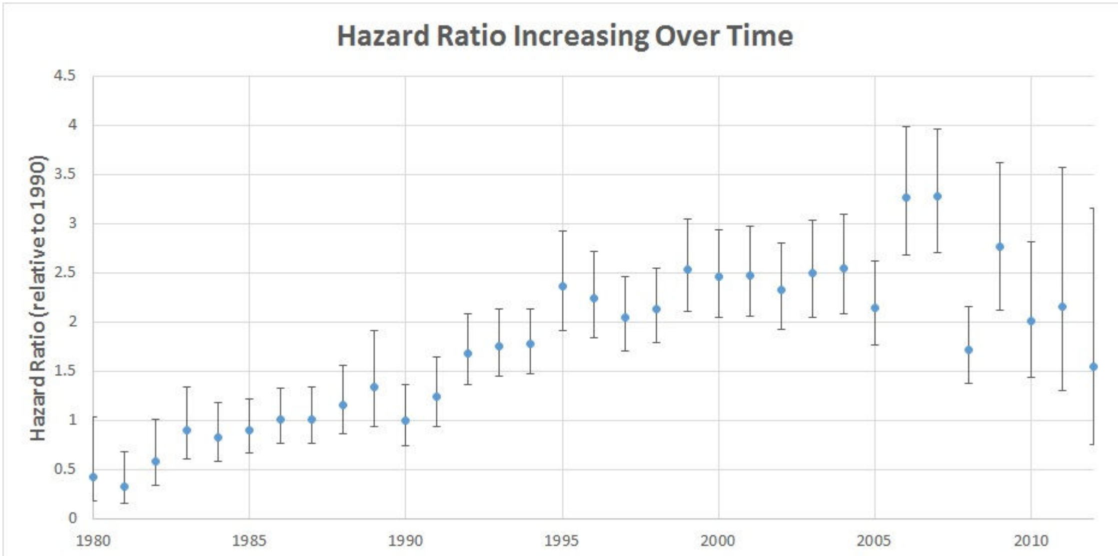


Figure 6: Annual hazard rates estimated via Cox proportional hazard model. Hazard rates are reported relative to 1990 and are displayed with 95% confidence intervals.

increasing skill in the presence of increasing closet indexing.

Our theory of moral hazard in active management is further supported by evidence relating closet indexing and speed of fund closure. The coefficient associated with a fund's initial level of closet indexing is statistically significant at the 1% level. The coefficient's value of -0.234 is also economically significant. It implies that a fund in the 10^{th} percentile is 10.7% more likely to close in a given year than a fund in the 90^{th} percentile. Similarly, a fund in the 25^{th} percentile is 5.2% more likely to close in a given year than a fund in the 75^{th} percentile.[16] This too is suggestive that closet indexing, as measured by active share, is negatively related to fund survival. This suggests that active share measures moral-hazard-based closet indexing, rather than closet indexing motivated by decreasing-returns-to-scale as in Berk and Green (2004). Our results also suggest that a fund's survival is tied to its level of closet indexing, so overall increases in managerial skill are related to increases in moral hazard.

5 Conclusion

The wide-spread adoption of index investing has been viewed as a boon for investors. Lower fees and expanded investment options have led to tremendous growth for passively managed products.

[16]The calculation is performed by taking the Active Share (AS) for the x^{th} and y^{th} percentiles and putting them into the expression $\exp^{-0.234(AS_x - AS_y)} - 1$.

However, passive investing is now being associated with reduced market efficiency. and passively-managed active funds (closet indexers) may be exacerbating such effects. Our analysis links the growth of passive investing to active managers' incentives, showing that lower fees lead to reduced information production and increased closet indexing. Rather than counteracting the negative effects of increased passive investing, truly active management decreases due to increased moral hazard. Mitigating the negative spillover effects due to closet indexing is not trivial. Popular holdings-based measures may not be effective as funds can engage in signal-jamming to appear active.

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Appendix A

Proof of Lemma 1:

The expressions in (8) and (9) are rewritten as,

$$F_E = \alpha \Pr(S_1 | K_1, \eta), \tag{A1}$$

$$F_E = \alpha \Pr(S_2 | K_2, \eta, \mathbb{1}_{S_1}). \tag{A2}$$

The expressions are expanded to incorporate (5) while using (10) and (11) to simplify notation,

$$F_E = \frac{\alpha \Pr(S_1^\star)}{1 + \eta K_{1,t}}, \tag{A3}$$

$$F_E = \frac{\alpha \Pr(S_2^\star | \mathbb{1}_{S_1})}{1 + \eta K_{2,t}}. \tag{A4}$$

The non-negative solutions to the preceding expressions are given by,

$$K_1 = \max\left\{ \frac{\alpha \Pr(S_1^\star) - F_E}{\eta F_E}, 0 \right\}, \tag{A5}$$

$$K_2(\mathbb{1}_{S_1}) = \max\left\{ \frac{\alpha \Pr(S_2^\star | \mathbb{1}_{S_1}) - F_E}{\eta F_E}, 0 \right\}. \tag{A6}$$

■

Proof of Lemma 2:

The profit functions in (16) are rewritten using (14) and (15),

$$\Pi_j(\mathrm{a}_j, F_E) = \begin{cases} (F_E + F_P)\left(\max\left\{\frac{\alpha \Pr(S_1^\star) - F_E}{\eta F_E}, 0\right\} + \max\left\{\frac{\alpha \Pr(S_2^\star|0) - F_E}{\eta F_E}, 0\right\}\right) & \text{if a} = C, \\ (F_E + F_P)\left(\max\left\{\frac{\alpha \Pr(S_1^\star) - F_E}{\eta F_E}, 0\right\} + \max\left\{\frac{\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E}{\eta F_E}, 0\right\}\right) - c & \text{if a} = A. \end{cases} \tag{A7}$$

The profit functions in A7 yield an explicit form of $\Delta(\theta)$. The function $\Delta(\theta)$ defined in (18), is explicitly given by,

$$\Delta(\theta) = \frac{\theta(F_E + F_P)}{1 + \eta K_1}\left(\max\left\{\frac{\alpha \Pr(S_2^\star|1) - F_E}{\eta F_E}, 0\right\} - \max\left\{\frac{\alpha \Pr(S_2^\star|0) - F_E}{\eta F_E}, 0\right\}\right) - c. \tag{A8}$$

$\Delta(\theta)$ is clearly increasing in θ because $\Pr(S_2^\star|1)$ and $\Pr(S_2^\star|0)$ do not depend on θ and an individual fund's decision does not impact θ^*. Furthermore, $\Delta(0) < 0$ for $c > 0$ and $Max(\Delta(\theta))$ is achieved at $\theta = 1$. By the intermediate value theorem, if there is an internal solution (θ^*) it must be that $\theta^* \in (0, 1]$.

Note that if an internal value of θ satisfies $\Delta(\theta) = 0$, the following equality implicitly defines θ^*,

$$\theta^* = \frac{c\left(1 + \eta \max\left\{\frac{\alpha \Pr(S_1^\star) - F_E}{\eta F_E}, 0\right\}\right)}{(F_E + F_P)\left(\max\left\{\frac{\alpha \Pr(S_2^\star|1) - F_E}{\eta F_E}, 0\right\} - \max\left\{\frac{\alpha \Pr(S_2^\star|0) - F_E}{\eta F_E}, 0\right\}\right)}, \tag{A9}$$

Focusing on the non-trivial case in which capital at $t = 1$ is positive, the expression simplifies to,

$$\theta^* = \frac{c\eta\alpha \Pr(S_1^\star)}{(F_E + F_P)\left(\alpha \Pr(S_2^\star|1) - F_E - \max\left\{\alpha \Pr(S_2^\star|0) - F_E, 0\right\}\right)}, \tag{A10}$$

■

Proof of Proposition 1:

Define

$$\Psi(\hat{\theta}) \equiv \hat{\theta} - \frac{c\eta\alpha \Pr(S_1^\star|\hat{\theta})}{(F_E + F_P)\left(\alpha \Pr(S_2^\star|1, \hat{\theta}) - F_E - \max\left\{\alpha \Pr(S_2^\star|0, \hat{\theta}) - F_E, 0\right\}\right)}. \tag{A11}$$

Note that $\Psi(\theta^*) = 0$. The function $\Psi(\hat{\theta})$ can be utilized with the implicit function theorem to characterize θ^*,

$$\frac{\partial \theta^*}{\partial \omega} = - \left. \frac{\partial \Psi / \partial \omega}{\partial \Psi / \partial \hat{\theta}} \right|_{\hat{\theta}=\theta^*}, \tag{A12}$$

for $\omega \in \{c, F_P, \alpha, \eta, F_E\}$. In the comparative statics studied hereafter, there are two cases to consider:

(i) $\max \left\{ \alpha \Pr(S_2^\star | 0, \hat{\theta}) - F_E, 0 \right\} = \alpha \Pr(S_2^\star | 0, \hat{\theta}) - F_E$ and,

(ii) $\max \left\{ \alpha \Pr(S_2^\star | 0, \hat{\theta}) - F_E, 0 \right\} = 0$.

Respectively, define $\Psi(\hat{\theta})$ in each of these two cases as,

$$\Psi^{\not{0}}(\hat{\theta}) \equiv \hat{\theta} - \frac{c\eta \Pr(S_1^\star | \hat{\theta})}{(F_E + F_P) \Big(\Pr(S_2^\star | 1, \hat{\theta}) - \Pr(S_2^\star | 0, \hat{\theta}) \Big)}, \tag{A13}$$

$$\Psi^{0}(\hat{\theta}) \equiv \hat{\theta} - \frac{c\eta\alpha \Pr(S_1^\star | \hat{\theta})}{(F_E + F_P) \Big(\alpha \Pr(S_2^\star | 1, \hat{\theta}) - F_E \Big)}. \tag{A14}$$

Before signing the partial derivativs, we derive explicitly the probabilities of success and their derivatives. Rearranging (10) and (10) gives

$$\Pr(S_1^\star) \equiv \Pr(S_1 | K_1, \eta)(1 + \eta K_1), \tag{A15}$$

$$\Pr(S_2^\star | \mathbb{1}_{S_1}) \equiv \Pr(S_2 | K_2, \eta, \mathbb{1}_{S_1})(1 + \eta K_2). \tag{A16}$$

The explicit forms of $\Pr(S_1^\star)$, $\Pr(S_2^\star|1)$, and $\Pr(S_2^\star|0)$ are now derived, starting with $\Pr(S_1^\star)$,

$$\Pr(S_1^\star) = \Pr(S_1^\star | A) \Pr(A) + \Pr(S_1^\star | C) \Pr(C) \tag{A17}$$

$$= \Pr(S_1^\star | A) \Pr(A) \tag{A18}$$

$$= \frac{\int_{\theta^*}^1 \theta g(\theta) \, \mathrm{d}\theta}{\int_{\theta^*}^1 g(\theta) \, \mathrm{d}\theta} \left(\int_{\theta^*}^1 g(\theta) \, \mathrm{d}\theta \right) \tag{A19}$$

$$= \int_{\theta^*}^1 \theta g(\theta) \, \mathrm{d}\theta. \tag{A20}$$

Now, consider $\Pr(S_2^\star|1)$,

$$\Pr(S_2^\star | 1) = \Pr(S_2^\star | A, 1) \Pr(A|1) + \Pr(S_2^\star | C, 1) \Pr(C|1) \tag{A21}$$

$$= \Pr(S_2^\star | A, 1) \Pr(A|1) \tag{A22}$$

$$= \frac{\Pr(S_2^\star \cap 1 | A)}{\Pr(1|A)} \Pr(A|1) \tag{A23}$$

$$= \frac{\int_{\theta^*}^1 \theta^2 g(\theta) \, \mathrm{d}\theta}{\int_{\theta^*}^1 \theta g(\theta) \, \mathrm{d}\theta}. \tag{A24}$$

Now, consider $\Pr(S_2^\star|0)$,

$$\begin{aligned}
\Pr(S_2^\star|0) &= \Pr(S_2^\star|A,0)\Pr(A|0) + \Pr(S_2^\star|C,1)\Pr(C|0) && \text{(A25)}\\
&= \Pr(S_2^\star|A,0)\Pr(A|0) && \text{(A26)}\\
&= \Pr(S_2^\star|A,0)\frac{\Pr(A\cap 0)}{\Pr(0)} && \text{(A27)}\\
&= \frac{\Pr(S_2^\star\cap 0|A)}{\Pr(0|A)}\frac{\Pr(0|A)\Pr(A)}{\Pr(0|A)\Pr(A)+\Pr(0|C)\Pr(C)} && \text{(A28)}\\
&= \left(\frac{\frac{\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{1-G(\theta^*)}}{\frac{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta}{1-G(\theta^*)}}\right)\left(\frac{(1-G(\theta^*))\frac{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta}{(1-G(\theta^*))}}{(1-G(\theta^*))\frac{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta}{(1-G(\theta^*))}+G(\theta^*)}\right) && \text{(A29)}\\
&= \left(\frac{\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta}\right)\left(\frac{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta+G(\theta^*)}\right) && \text{(A30)}\\
&= \frac{\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta+G(\theta^*)} && \text{(A31)}
\end{aligned}$$

Note that $\Pr(S_1^\star)$ is decreasing in θ^*, $\Pr(S_2^\star|1)$ is increasing in θ^*, and $\Pr(S_2^\star|0)$ is decreasing in θ^*,

$$\begin{aligned}
\frac{\partial\Pr(S_1^\star)}{\partial\theta^*} &= -\theta^* g(\theta^*) && \text{(A32)}\\
&\leq 0. && \text{(A33)}\\
\frac{\partial\Pr(S_2^\star|1)}{\partial\theta^*} &= \frac{\theta^* g(\theta^*)}{\int_{\theta^*}^1 \theta g(\theta)\,\mathrm{d}\theta}\left(\Pr(S_2^\star|1)-\theta^*\right), && \text{(A34)}
\end{aligned}$$

which is positive because $\Pr(S_2^\star|1) = E[\theta|1,A] \geq \theta^*$,

$$\begin{aligned}
&\geq 0. && \text{(A35)}\\
\frac{\partial\Pr(S_2^\star|0)}{\partial\theta^*} &= \frac{-\theta^*(1-\theta^*)g(\theta^*)\left(\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta+G(\theta^*)\right)-\theta^* g(\theta^*)\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\left(\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta+G(\theta^*)\right)^2} && \text{(A36)}\\
&= -g(\theta^*)\theta^*\frac{(1-\theta^*)\left(\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta+G(\theta^*)\right)+\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\left(\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta+G(\theta^*)\right)^2} && \text{(A37)}\\
&\leq 0. && \text{(A38)}
\end{aligned}$$

Using the signs given by (A32), (A34) and (A37), we sign the partial derivatives, beginning with $\partial\Psi^{\alpha}/\partial\hat{\theta}$

and $\partial\Psi^0/\partial\hat{\theta}$,

$$\frac{\partial\Psi^{\not 0}}{\partial\hat{\theta}} = 1 - \frac{c\eta\frac{\partial \Pr(S_1^\star|\hat{\theta})}{\partial\hat{\theta}}}{(F_E+F_P)\left(\Pr(S_2^\star|1,\hat{\theta}) - \Pr(S_2^\star|0,\hat{\theta})\right)} + \frac{c\eta\Pr(S_1^\star|\hat{\theta})\left(\frac{\partial \Pr(S_2^\star|1,\hat{\theta})}{\partial\hat{\theta}} - \frac{\partial \Pr(S_2^\star|0,\hat{\theta})}{\partial\hat{\theta}}\right)}{(F_E+F_P)\left(\Pr(S_2^\star|1,\hat{\theta}) - \Pr(S_2^\star|0,\hat{\theta})\right)^2} \tag{A39}$$

$$\geq 0, \tag{A40}$$

$$\frac{\partial\Psi^{0}}{\partial\hat{\theta}} = 1 - \frac{c\eta\alpha\frac{\partial \Pr(S_1^\star|\hat{\theta})}{\partial\hat{\theta}}}{(F_E+F_P)\left(\alpha\Pr(S_2^\star|1,\hat{\theta}) - F_E\right)} + \frac{c\eta\alpha^2\Pr(S_1^\star|\hat{\theta})\frac{\partial \Pr(S_2^\star|1,\hat{\theta})}{\partial\hat{\theta}}}{(F_E+F_P)\left(\alpha\Pr(S_2^\star|1,\hat{\theta}) - F_E\right)^2} \tag{A41}$$

$$\geq 0. \tag{A42}$$

Now, we consider $\partial\Psi^{\not 0}/\partial c$ and $\partial\Psi^0/\partial c$,

$$\frac{\partial\Psi^{\not 0}}{\partial c} = -\frac{\eta\Pr(S_1^\star|\hat{\theta})}{(F_E+F_P)\left(\Pr(S_2^\star|1,\hat{\theta}) - \Pr(S_2^\star|0,\hat{\theta})\right)} \tag{A43}$$

$$\leq 0, \tag{A44}$$

$$\frac{\partial\Psi^{0}}{\partial c} = -\frac{\eta\alpha\Pr(S_1^\star|\hat{\theta})}{(F_E+F_P)\left(\alpha\Pr(S_2^\star|1,\hat{\theta}) - F_E\right)} \tag{A45}$$

$$\leq 0. \tag{A46}$$

Now, we consider $\partial\Psi^{\not 0}/\partial F_P$ and $\partial\Psi^0/\partial F_P$,

$$\frac{\partial\Psi^{\not 0}}{\partial F_P} = \frac{c\eta\Pr(S_1^\star|\hat{\theta})}{(F_E+F_P)^2\left(\Pr(S_2^\star|1,\hat{\theta}) - \Pr(S_2^\star|0,\hat{\theta})\right)} \tag{A47}$$

$$\geq 0, \tag{A48}$$

$$\frac{\partial\Psi^{0}}{\partial F_P} = \frac{c\eta\alpha\Pr(S_1^\star|\hat{\theta})}{(F_E+F_P)^2\left(\alpha\Pr(S_2^\star|1,\hat{\theta}) - F_E\right)} \tag{A49}$$

$$\geq 0. \tag{A50}$$

Now, we consider $\partial\Psi^{\not 0}/\partial\alpha$ and $\partial\Psi^0/\partial\alpha$,

$$\frac{\partial\Psi^{\not 0}}{\partial\alpha} = 0 \tag{A51}$$

$$= 0, \tag{A52}$$

$$\frac{\partial\Psi^{0}}{\partial\alpha} = \frac{c\eta\alpha\Pr(S_1^\star|\hat{\theta})\Pr(S_2^\star|1,\hat{\theta})}{(F_E+F_P)\left(\alpha\Pr(S_2^\star|1,\hat{\theta}) - F_E\right)^2} \tag{A53}$$

$$\geq 0. \tag{A54}$$

Now, we consider $\partial\Psi^{\not 0}/\partial\eta$ and $\partial\Psi^{0}/\partial\eta$,

$$\frac{\partial\Psi^{\not 0}}{\partial\eta} = -\frac{c\Pr(S_1^\star|\hat{\theta})}{(F_E+F_P)\left(\Pr(S_2^\star|1,\hat{\theta})-\Pr(S_2^\star|0,\hat{\theta})\right)} \tag{A55}$$

$$\leq 0, \tag{A56}$$

$$\frac{\partial\Psi^{0}}{\partial\eta} = -\frac{c\alpha\Pr(S_1^\star|\hat{\theta})}{(F_E+F_P)\left(\alpha\Pr(S_2^\star|1,\hat{\theta})-F_E\right)} \tag{A57}$$

$$\leq 0. \tag{A58}$$

Now, we consider $\partial\Psi^{\not 0}/\partial F_E$ and $\partial\Psi^{0}/\partial F_E$. The sign on the derivative is equivocal,

$$\frac{\partial\Psi^{\not 0}}{\partial F_E} = \frac{c\eta\Pr(S_1^\star|\hat{\theta})}{(F_E+F_P)^2\left(\Pr(S_2^\star|1,\hat{\theta})-\Pr(S_2^\star|0,\hat{\theta})\right)} \tag{A59}$$

$$\geq 0, \tag{A60}$$

$$\frac{\partial\Psi^{0}}{\partial F_E} = \frac{c\eta\alpha\Pr(S_1^\star|\hat{\theta})}{(F_E+F_P)^2\left(\alpha\Pr(S_2^\star|1,\hat{\theta})-F_E\right)} - \frac{c\eta\alpha\Pr(S_1^\star|\hat{\theta})}{(F_E+F_P)\left(\alpha\Pr(S_2^\star|1,\hat{\theta})-F_E\right)^2} \tag{A61}$$

$$\gtreqless 0, \tag{A62}$$

The sign is weakly positive if and only if,

$$F_E \leq \frac{\alpha\Pr(S_2^\star|1,\hat{\theta})-F_P}{2}. \tag{A63}$$

The following summarizes the comparative statics,

$$\frac{\partial\theta^*}{\partial c} = -\left.\frac{\partial\Psi/\partial c}{\partial\Psi/\partial\hat{\theta}}\right|_{\hat{\theta}=\theta^*} \tag{A64}$$

$$\geq 0. \tag{A65}$$

$$\frac{\partial\theta^*}{\partial F_P} = -\left.\frac{\partial\Psi/\partial F_P}{\partial\Psi/\partial\hat{\theta}}\right|_{\hat{\theta}=\theta^*} \tag{A66}$$

$$\leq 0. \tag{A67}$$

$$\frac{\partial\theta^*}{\partial \alpha} = -\left.\frac{\partial\Psi/\partial \alpha}{\partial\Psi/\partial\hat{\theta}}\right|_{\hat{\theta}=\theta^*} \tag{A68}$$

$$\leq 0. \tag{A69}$$

$$\frac{\partial\theta^*}{\partial \eta} = -\left.\frac{\partial\Psi/\partial \eta}{\partial\Psi/\partial\hat{\theta}}\right|_{\hat{\theta}=\theta^*} \tag{A70}$$

$$\geq 0. \tag{A71}$$

$$\frac{\partial\theta^*}{\partial F_E} = -\left.\frac{\partial\Psi/\partial F_E}{\partial\Psi/\partial\hat{\theta}}\right|_{\hat{\theta}=\theta^*} \tag{A72}$$

$$\gtreqless 0 \text{ the comparative static is equivocal.} \tag{A73}$$

■

Proof of Corollary 1.1:

The analysis is in the proof of Proposition 1

■

Proof of Lemma 3:

An explicit form for $\Delta K(\mathbb{1}_{S_1})$ is given by,

$$\Delta K(\mathbb{1}_{S_1}) = \frac{\max\{\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E, 0\}}{\alpha \Pr(S_1^\star) - F_E} - 1. \tag{A74}$$

Now, consider that a fund succeeds. According to (A74), the fund's net fund flow is given by,

$$\Delta K(1) = \frac{\alpha \Pr(S_2^\star|1) - F_E}{\alpha \Pr(S_1^\star) - F_E} - 1, \tag{A75}$$

which is strictly positive because $\Pr(S_2^\star|1) > \Pr(S_1^\star)$,

$$> 0. \tag{A76}$$

A similar analysis demonstrates that $\Delta K(0) < 0$.

■

Proof of Corollary 1.2: First, note that the necessary and sufficient condition for,

$$\left(\frac{\frac{\partial \Pr(S_2^\star|0)}{\partial\theta^*}}{\alpha \Pr(S_1^\star) - F_E} - \frac{\frac{\partial \Pr(S_1^\star)}{\partial\theta^*}(\alpha \Pr(S_2^\star|0) - F_E)}{(\alpha \Pr(S_1^\star) - F_E)^2}\right) \tag{A77}$$

to be negative is,

$$\frac{\frac{\partial \Pr(S_2^\star|0)}{\partial\theta^*}}{\frac{\partial \Pr(S_1^\star)}{\partial\theta^*}} \geq \frac{\alpha \Pr(S_2^\star|0) - F_E}{\alpha \Pr(S_1^\star) - F_E}. \tag{A78}$$

The region of non-trivial parameters is $F_E \in [0, \alpha \Pr(S_2^\star|0)]$. Within the set of non-trivial parameters, the inequality on the right-hand side of (A78) is maximized at $F_E = 0$. Therefore, if the inequality holds at,

$$\frac{\frac{\partial \Pr(S_2^\star|0)}{\partial \theta^*}}{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}} \geq \frac{\Pr(S_2^\star|0)}{\Pr(S_1^\star)}, \tag{A79}$$

it holds for all $F_E \in [0, \alpha \Pr(S_2^\star|0)]$. The expression is expanded using (A20), (A31), (A32), and (A37),

$$\frac{-g(\theta^*)\theta^* \frac{(1-\theta^*)\left(\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)\right) + \int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\left(\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)\right)^2}}{-\theta^* g(\theta^*)} \geq \frac{\frac{\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)}}{\int_{\theta^*}^1 \theta g(\theta)\,\mathrm{d}\theta}, \tag{A80}$$

which simplifies to,

$$\frac{(1-\theta^*)\left(\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)\right) + \int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)} \geq \frac{\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta}{\int_{\theta^*}^1 \theta g(\theta)\,\mathrm{d}\theta}. \tag{A81}$$

The inequality expands to,

$$\begin{aligned}
&\int_{\theta^*}^1 \theta(1-\theta^*)g(\theta)\,\mathrm{d}\theta \int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)\int_{\theta^*}^1 \theta(1-\theta^*)g(\theta)\,\mathrm{d}\theta + \int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta \int_{\theta^*}^1 \theta g(\theta)\,\mathrm{d}\theta \\
&\geq \int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta \int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta + G(\theta^*)\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta.
\end{aligned} \tag{A82}$$

The inequality holds because,

$$\int_{\theta^*}^1 \theta(1-\theta^*)g(\theta)\,\mathrm{d}\theta \int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta \geq \int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta \int_{\theta^*}^1 (1-\theta)g(\theta)\,\mathrm{d}\theta, \tag{A83}$$

and,

$$G(\theta^*)\int_{\theta^*}^1 \theta(1-\theta^*)g(\theta)\,\mathrm{d}\theta \geq G(\theta^*)\int_{\theta^*}^1 \theta(1-\theta)g(\theta)\,\mathrm{d}\theta. \tag{A84}$$

Therefore, the expression in (A77) is negative within the non-trivial parameter set.

Now, consider the comparative statics of a fund's net fund flow with respect to $\{c, F_P, \alpha, \eta, F_E\}$. If $\max\{\alpha \Pr(S_2^\star|0) - F_E, 0\} = 0$ then the fund flow sensitivities are constant when a fund fails to achieve excess returns. Consequently, for this analysis we focus on the case in which

$$\max\{\alpha \Pr(S_2^\star|0) - F_E, 0\} = \alpha \Pr(S_2^\star|0) - F_E \tag{A85}$$

First, consider the comparative static with respect to c,

$$\begin{aligned}
\frac{\partial \Delta K(\mathbb{1}_{S_1})}{\partial c} &= \frac{\partial}{\partial c}\left(\frac{\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E}{\alpha \Pr(S_1^\star) - F_E} - 1\right) && \text{(A86)} \\
&= \alpha \frac{\partial \theta^*}{\partial c}\left(\frac{\frac{\partial \Pr(S_2^\star|\mathbb{1}_{S_1})}{\partial \theta^*}}{\alpha \Pr(S_1^\star) - F_E} - \frac{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}(\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E)}{(\alpha \Pr(S_1^\star) - F_E)^2}\right) && \text{(A87)} \\
&= \begin{cases} \geq 0 & \text{if } \mathbb{1}_{S_1} = 1, \\ \leq 0 & \text{if } \mathbb{1}_{S_1} = 0. \end{cases} && \text{(A88)}
\end{aligned}$$

Now, consider the comparative static with respect to F_P,

$$\frac{\partial \Delta K(\mathbb{1}_{S_1})}{\partial F_P} = \frac{\partial}{\partial F_P}\left(\frac{\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E}{\alpha \Pr(S_1^\star) - F_E} - 1\right) \tag{A89}$$

$$= \alpha \frac{\partial \theta^*}{\partial F_P}\left(\frac{\frac{\partial \Pr(S_2^\star|\mathbb{1}_{S_1})}{\partial \theta^*}}{\alpha \Pr(S_1^\star) - F_E} - \frac{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}\left(\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E\right)}{(\alpha \Pr(S_1^\star) - F_E)^2}\right) \tag{A90}$$

$$= \begin{cases} \leq 0 & \text{if } \mathbb{1}_{S_1} = 1, \\ \geq 0 & \text{if } \mathbb{1}_{S_1} = 0. \end{cases} \tag{A91}$$

Now, consider the comparative static with respect to α,

$$\frac{\partial \Delta K(\mathbb{1}_{S_1})}{\partial \alpha} = \frac{\partial}{\partial \alpha}\left(\frac{\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E}{\alpha \Pr(S_1^\star) - F_E} - 1\right) \tag{A92}$$

$$= \alpha \frac{\partial \theta^*}{\partial \alpha}\left(\frac{\frac{\partial \Pr(S_2^\star|\mathbb{1}_{S_1})}{\partial \theta^*}}{\alpha \Pr(S_1^\star) - F_E} - \frac{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}\left(\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E\right)}{(\alpha \Pr(S_1^\star) - F_E)^2}\right)$$
$$+ \left(\frac{\Pr(S_2^\star|\mathbb{1}_{S_1})}{\alpha \Pr(S_1^\star) - F_E}\right) - \left(\frac{\Pr(S_1^\star)(\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E)}{(\alpha \Pr(S_1^\star) - F_E)^2}\right) \tag{A93}$$

$$= \begin{cases} \leq 0 & \text{if } \mathbb{1}_{S_1} = 1, \\ \geq 0 & \text{if } \mathbb{1}_{S_1} = 0, \end{cases} \tag{A94}$$

because the sign of

$$\left(\frac{\Pr(S_2^\star|\mathbb{1}_{S_1})}{\alpha \Pr(S_1^\star) - F_E}\right) - \left(\frac{\Pr(S_1^\star)(\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E)}{(\alpha \Pr(S_1^\star) - F_E)^2}\right) \tag{A95}$$

has the same sign as

$$- F_E \left(Pr(S_2^\star|\mathbb{1}_{S_1}) - Pr(S_1^\star)\right). \tag{A96}$$

Now, consider the comparative static with respect to η,

$$\frac{\partial \Delta K(\mathbb{1}_{S_1})}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\frac{\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E}{\alpha \Pr(S_1^\star) - F_E} - 1\right) \tag{A97}$$

$$= \alpha \frac{\partial \theta^*}{\partial \eta}\left(\frac{\frac{\partial \Pr(S_2^\star|\mathbb{1}_{S_1})}{\partial \theta^*}}{\alpha \Pr(S_1^\star) - F_E} - \frac{\frac{\partial \Pr(S_1^\star)}{\partial \theta^*}\left(\alpha \Pr(S_2^\star|\mathbb{1}_{S_1}) - F_E\right)}{(\alpha \Pr(S_1^\star) - F_E)^2}\right) \tag{A98}$$

$$= \begin{cases} \geq 0 & \text{if } \mathbb{1}_{S_1} = 1, \\ \leq 0 & \text{if } \mathbb{1}_{S_1} = 0. \end{cases} \tag{A99}$$

■

Proof of Lemma 4:

The proof follows the proof of Lemma 1.

■

Proof of Lemma 5:

The profit function for a fund with type θ is given by,

$$\Pi(\mathrm{a}_j, \theta_j) = \begin{cases} (F_E + F_P)\left(K_1 + K_2(0)\right) & \text{if } \mathrm{a}_j = C \\ K_1\left(F_E + F_P + \frac{\theta\lambda\alpha}{1+\eta K_1}\right) + \frac{K_2(1)\theta}{1+\eta K_1}\left(F_E + F_P + \frac{\theta\lambda\alpha}{1+\eta K_2(1)}\right) & \\ +K_2(0)\left(1 - \frac{\theta}{1+\eta K_1}\right)\left(F_E + F_P + \frac{\theta\lambda\alpha}{1+\eta K_2(0)}\right) - c & \text{if } \mathrm{a}_j = A. \end{cases} \tag{A100}$$

Using (A100), define $\Delta^\lambda(\theta)$ as a benefit-cost function for a fund with type θ,

$$\Delta^\lambda(\theta_j) \equiv \Pi(A,\theta_j) - \Pi(C,\theta_j) \tag{A101}$$

$$= K_1 \frac{\theta\lambda\alpha}{1+\eta K_1} + \frac{K_2(1)\theta}{1+\eta K_1}\left(\frac{\theta\lambda\alpha}{1+\eta K_2(1)}\right) + \frac{\theta(K_2(1)-K_2(0))(F_E+F_P)}{1+\eta K_1}$$
$$+ K_2(0)\left(1-\frac{\theta}{1+\eta K_1}\right)\left(\frac{\theta\lambda\alpha}{1+\eta K_2(0)}\right) - c, \tag{A102}$$

If an internal threshold exists, it is implicitly defined by $\Delta^\lambda(\theta^\lambda) = 0$, otherwise $\theta^\lambda = 1$. Consider that θ^λ is an internal value. The expression in (A102) equals zero at,

$$0 = K_1 \frac{\theta^\lambda\lambda\alpha}{1+\eta K_1} + \frac{K_2(1)\theta^\lambda}{1+\eta K_1}\left(\frac{\theta^\lambda\lambda\alpha}{1+\eta K_2(1)}\right) + \frac{\theta^\lambda(K_2(1)-K_2(0))(F_E+F_P)}{1+\eta K_1}$$
$$+ K_2(0)\left(1-\frac{\theta^\lambda}{1+\eta K_1}\right)\left(\frac{\theta^\lambda\lambda\alpha}{1+\eta K_2(0)}\right) - c. \tag{A103}$$

Define

$$x \equiv \frac{1}{1+\eta K_1} \tag{A104}$$
$$y \equiv \frac{1}{1+\eta K_2(1)} \tag{A105}$$
$$z \equiv \frac{1}{1+\eta K_2(0)} \tag{A106}$$

And note that $y \le x \le z$. The expression becomes,

$$0 = \theta^\lambda x K_1 \lambda\alpha + {\theta^\lambda}^2 x K_2(1) y \lambda\alpha + \theta^\lambda x (K_2(1)-K_2(0))(F_E+F_P)$$
$$+ \theta^\lambda z \lambda\alpha K_2(0) - {\theta^\lambda}^2 x K_2(0) z \lambda\alpha - c. \tag{A107}$$

The preceding expression is reordered as,

$$0 = {\theta^\lambda}^2 \underbrace{x\lambda\alpha\left(yK_2(1) - zK_2(0)\right)}_{a} + \theta^\lambda \underbrace{\left((F_E+F_P)x\left(K_2(1)-K_2(0)\right) + \lambda\alpha x K_1 + \lambda\alpha z K_2(0)\right)}_{b} - \underbrace{c}_{c}. \tag{A108}$$

The quadratic formula yields,

$$\theta^\lambda = -\frac{(F_E+F_P)x\left(K_2(1)-K_2(0)\right) + \lambda\alpha x K_1 + \lambda\alpha z K_2(0)}{2x\lambda\alpha\left(yK_2(1)-zK_2(0)\right)}$$
$$\pm \frac{\sqrt{\left((F_E+F_P)x\left(K_2(1)-K_2(0)\right) + \lambda\alpha x K_1 + \lambda\alpha z K_2(0)\right)^2 + 4x\lambda\alpha\left(yK_2(1)-zK_2(0)\right)c}}{2x\lambda\alpha\left(yK_2(1)-zK_2(0)\right)}. \tag{A109}$$

Consider the first term in the quadratic formula above (and notice the negative sign drops when one switches the term of the denominator to $(zK_2(0) - yK_2(1))$ instead of $(yK_2(1) - zK_2(0))$),

$$\frac{(F_E + F_P)x\,(K_2(1) - K_2(0)) + \lambda\alpha x K_1 + \lambda\alpha z K_2(0)}{2x\lambda\alpha\,(zK_2(0) - yK_2(1))} = \underbrace{\frac{(F_E + F_P)\,(K_2(1) - K_2(0))}{2\lambda\alpha\,(zK_2(0) - yK_2(1))}}_{\leq 0} + \underbrace{\frac{K_1}{2\,(zK_2(0) - yK_2(1))}}_{\leq 0} + \underbrace{\frac{zK_2(0)}{2x\,(zK_2(0) - yK_2(1))}}_{\leq 0} \tag{A110}$$

$$\leq 0, \tag{A111}$$

because

$$zK_2(0) - yK_2(1) = \frac{K_2(0)}{1 + \eta K_2(0)} - \frac{K_2(1)}{1 + \eta K_2(1)} \tag{A112}$$

$$\leq 0 \tag{A113}$$

Therefore, if an internal solution exists, it is unique and it must be implicitly defined by,

$$\theta^{\lambda} = -\frac{(F_E + F_P)x\,(K_2(1) - K_2(0)) + \lambda\alpha x K_1 + \lambda\alpha z K_2(0)}{2x\lambda\alpha\,(yK_2(1) - zK_2(0))} + \frac{\sqrt{((F_E + F_P)x\,(K_2(1) - K_2(0)) + \lambda\alpha x K_1 + \lambda\alpha z K_2(0))^2 + 4x\lambda\alpha\,(yK_2(1) - zK_2(0))\,c}}{2x\lambda\alpha\,(yK_2(1) - zK_2(0))}. \tag{A114}$$

■

Proof of Lemma 6:

To prove the lemma, we consider the local change in θ^{λ} in the limiting case when $\lambda = 0$. Define

$$\Psi^{\lambda}(\hat{\theta}) \equiv \frac{K_1(\hat{\theta})\hat{\theta}\lambda\alpha}{1 + \eta K_1(\hat{\theta})} + \frac{K_2(1,\hat{\theta})\hat{\theta}}{1 + \eta K_1(\hat{\theta})}\left(\frac{\hat{\theta}\lambda\alpha}{1 + \eta K_2(1,\hat{\theta})}\right) + \frac{\hat{\theta}(K_2(1,\hat{\theta}) - K_2(0,\hat{\theta}))(F_E + F_P)}{1 + \eta K_1(\hat{\theta})} + K_2(0,\hat{\theta})\left(1 - \frac{\hat{\theta}}{1 + \eta K_1(\hat{\theta})}\right)\left(\frac{\hat{\theta}\lambda\alpha}{1 + \eta K_2(0,\hat{\theta})}\right) - c, \tag{A115}$$

where $\Psi^{\lambda}(\theta^{\lambda}) = 0$. The change in θ^{λ} with respect to a change in λ is solved for using the implicit function theorem and Ψ^{λ},

$$\frac{\partial\theta^{\lambda}}{\partial\lambda} = -\left.\frac{\partial\Psi^{\lambda}/\partial\lambda}{\partial\Psi^{\lambda}/\partial\hat{\theta}}\right|_{\hat{\theta}=\theta^{\lambda}}. \tag{A116}$$

We consider the limiting case in which $\lambda = 0$. First consider $\partial\Psi^\lambda/\partial\hat{\theta}$,

$$\begin{aligned}
\left.\frac{\partial\Psi^\lambda}{\partial\hat{\theta}}\right|_{\lambda=0} &= \lambda\frac{\partial}{\partial\hat{\theta}}\left(\frac{K_1(\hat{\theta})\hat{\theta}\alpha}{1+\eta K_1(\hat{\theta})}\right) + \lambda\frac{\partial}{\partial\hat{\theta}}\left(\frac{K_2(1,\hat{\theta})\hat{\theta}}{1+\eta K_1(\hat{\theta})}\left(\frac{\hat{\theta}\alpha}{1+\eta K_2(1,\hat{\theta})}\right)\right) \\
&+ \frac{\partial}{\partial\hat{\theta}}\left(\frac{\hat{\theta}(K_2(1,\hat{\theta})-K_2(0,\hat{\theta}))(F_E+F_P)}{1+\eta K_1(\hat{\theta})}\right) \\
&+ \lambda\frac{\partial}{\partial\hat{\theta}}\left(K_2(0,\hat{\theta})\left(1-\frac{\hat{\theta}}{1+\eta K_1(\hat{\theta})}\right)\left(\frac{\hat{\theta}\alpha}{1+\eta K_2(0,\hat{\theta})}\right)\right) && \text{(A117)} \\
&= \frac{\partial}{\partial\hat{\theta}}\left(\frac{\hat{\theta}(K_2(1,\hat{\theta})-K_2(0,\hat{\theta}))(F_E+F_P)}{1+\eta K_1(\hat{\theta})}\right) && \text{(A118)} \\
&= \frac{(K_2(1,\hat{\theta})-K_2(0,\hat{\theta}))(F_E+F_P)}{1+\eta K_1(\hat{\theta})} + \frac{\hat{\theta}\left(\frac{\partial K_2(1,\hat{\theta})}{\partial\hat{\theta}} - \frac{\partial K_2(0,\hat{\theta})}{\partial\hat{\theta}}\right)(F_E+F_P)}{1+\eta K_1(\hat{\theta})} \\
&- \frac{\hat{\theta}\eta\frac{\partial K_1(\hat{\theta})}{\partial\hat{\theta}}(K_2(1,\hat{\theta})-K_2(0,\hat{\theta}))(F_E+F_P)}{(1+\eta K_1(\hat{\theta}))^2} && \text{(A119)} \\
&\geq 0. && \text{(A120)}
\end{aligned}$$

Now, consider $\partial\Psi^\lambda/\partial\lambda$

$$\begin{aligned}
\left.\frac{\partial\Psi^\lambda}{\partial\lambda}\right|_{\lambda=0} &= \lambda\frac{\partial}{\partial\lambda}\left(\frac{K_1(\lambda)\hat{\theta}\alpha}{1+\eta K_1(\lambda)}\right) + \lambda\frac{\partial}{\partial\lambda}\left(\frac{K_2(1,\lambda)\hat{\theta}}{1+\eta K_1(\lambda)}\left(\frac{\hat{\theta}\alpha}{1+\eta K_2(1,\lambda)}\right)\right) \\
&+ \frac{\partial}{\partial\lambda}\left(\frac{\hat{\theta}(K_2(1,\lambda)-K_2(0,\lambda))(F_E+F_P)}{1+\eta K_1(\lambda)}\right) \\
&+ \lambda\frac{\partial}{\partial\lambda}\left(K_2(0,\lambda)\left(1-\frac{\hat{\theta}}{1+\eta K_1(\lambda)}\right)\left(\frac{\hat{\theta}\alpha}{1+\eta K_2(0,\lambda)}\right)\right) && \text{(A121)} \\
&+ \frac{K_1(\lambda)\hat{\theta}\alpha}{1+\eta K_1(\lambda)} + \frac{K_2(1,\lambda)\hat{\theta}}{1+\eta K_1(\lambda)}\left(\frac{\hat{\theta}\alpha}{1+\eta K_2(1,\lambda)}\right) \\
&+ K_2(0,\lambda)\left(1-\frac{\hat{\theta}}{1+\eta K_1(\lambda)}\right)\left(\frac{\hat{\theta}\alpha}{1+\eta K_2(0,\lambda)}\right) && \text{(A122)} \\
&= \frac{\hat{\theta}\left(\frac{K_2(1,\lambda)}{\partial\lambda} - \frac{K_2(0,\lambda)}{\partial\lambda}\right)(F_E+F_P)}{1+\eta K_1(\lambda)} - \frac{\hat{\theta}\eta\frac{\partial K_1(\lambda)}{\partial\lambda}(K_2(1,\lambda)-K_2(0,\lambda))(F_E+F_P)}{(1+\eta K_1(\lambda))^2} \\
&+ \frac{K_1(\lambda)\hat{\theta}\alpha}{1+\eta K_1(\lambda)} + \frac{K_2(1,\lambda)\hat{\theta}}{1+\eta K_1(\lambda)}\left(\frac{\hat{\theta}\alpha}{1+\eta K_2(1,\lambda)}\right) \\
&+ K_2(0,\lambda)\left(1-\frac{\hat{\theta}}{1+\eta K_1(\lambda)}\right)\left(\frac{\hat{\theta}\alpha}{1+\eta K_2(0,\lambda)}\right). && \text{(A123)}
\end{aligned}$$

The preceding expression simplifies to,

$$
\begin{aligned}
&= \hat{\theta}\left(\frac{K_1(\lambda)\alpha}{1+\eta K_1(\lambda)} + \frac{K_2(1,\lambda)\hat{\theta}}{1+\eta K_1(\lambda)}\left(\frac{\alpha}{1+\eta K_2(1,\lambda)}\right)\right. \\
&\quad \left. + K_2(0,\lambda)\left(1-\frac{\hat{\theta}}{1+\eta K_1(\lambda)}\right)\left(\frac{\alpha}{1+\eta K_2(0,\lambda)}\right)\right) && \text{(A124)} \\
&\geq 0. && \text{(A125)}
\end{aligned}
$$

Therefore at $\lambda = 0$,

$$\left.\frac{\partial \theta^{\lambda}}{\partial \lambda}\right|_{\lambda=0} \leq 0. \tag{A126}$$

Furthermore, at $\lambda = 1$, $K_1 = K_2(0) = K_2(0) = 0$ and $1 = \theta^{\lambda}\big|_{\lambda=1} \geq \theta^{\lambda}\big|_{\lambda=0}$. Thus, because the functions are continuous in λ, the comparative static may be non-monotonic in λ.

■

Table 1: Analysis of relative flow-performance sensitivities. $Early1990s$ is equal to one for cohorts started in the years 1990 - 1994, and zero for cohorts started in the years 1995 - 1999. $AboveMedianReturn$ is an indicator based on cohort-year median returns. $EarlyFundYears$ is equal to one for the third and fourth years of a fund's life, and equal to zero for the fifth and sixth years. t-statistics are reported in parentheses. ***, **, * indicate significance at the 1%, 5% and 10% levels.

	AdjFundFlow
	(1)
Early1990s	-0.037**
	(-2.268)
EarlyFundYears	-0.045**
	(-2.534)
AboveMedianReturn	0.148***
	(8.874)
Early1990s × *EarlyFundYears*	0.042*
	(1.681)
Early1990s × *AboveMedianReturn*	0.075***
	(3.204)
AboveMedianReturn × *EarlyFundYears*	0.090***
	(3.579)
Early1990s × *AboveMedianReturn* × *EarlyFundYears*	-0.084**
	(-2.379)
Constant	-0.074***
	(-6.283)
Observations	6,659

Table 2: Estimation results from Cox proportional hazard model. t-statistics are reported in parentheses. ***, **, * indicate significance at the 1%, 5% and 10% levels.

	(1)		
ActiveShare	-0.234***		
	(-4.648)		
StartYear=1980	-0.342	StartYear=1997	1.229***
	(-0.755)		(13.138)
StartYear=1981	-0.620	StartYear=1998	1.271***
	(-1.612)		(14.077)
StartYear=1982	-0.020	StartYear=1999	1.444***
	(-0.072)		(15.443)
StartYear=1983	0.409**	StartYear=2000	1.412***
	(2.046)		(15.307)
StartYear=1984	0.322*	StartYear=2001	1.420***
	(1.782)		(15.033)
StartYear=1985	0.406***	StartYear=2002	1.356***
	(2.672)		(14.220)
StartYear=1986	0.519***	StartYear=2003	1.429***
	(3.634)		(14.264)
StartYear=1987	0.527***	StartYear=2004	1.447***
	(3.737)		(14.425)
StartYear=1988	0.659***	StartYear=2005	1.278***
	(4.321)		(12.562)
StartYear=1989	0.807***	StartYear=2006	1.699***
	(4.460)		(16.848)
StartYear=1990	0.515***	StartYear=2007	1.700***
	(3.291)		(17.374)
StartYear=1991	0.728***	StartYear=2008	1.055***
	(5.000)		(9.134)
StartYear=1992	1.033***	StartYear=2009	1.533***
	(9.509)		(11.266)
StartYear=1993	1.075***	StartYear=2010	1.213***
	(10.807)		(7.029)
StartYear=1994	1.088***	StartYear=2011	1.279***
	(11.523)		(4.956)
StartYear=1995	1.374***	StartYear=2012	0.946***
	(12.577)		(2.585)
StartYear=1996	1.318***		
	(13.234)		
Observations	17,313		