Collateralization, Leverage, and Systemic Risk

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Abstract

We describe a general equilibrium model with a banking system, with the deposit bank collecting deposits from households and the merchant bank providing funds to firms. The merchant bank borrows collateralized short-term funds from the deposit bank. In a financial downturn, as the value of collateral goes down, the merchant bank has to sell with short notice, reinforcing the crisis, and may default if its cash buffer is insufficient. The deposit bank suffers from the loss due to the depreciated assets. If the value of its assets is insufficient to cover deposits, it also defaults. Deposits are insured by the government and the premium paid by the deposit bank is the expected loss on the deposits. Systemic risk is then defined as the expected loss on deposits under stress, i.e., in a financial crisis. We calibrate the model on the U.S. economy and show how this measure behaves under stress.

Keywords: Real business cycle model, Systemic risk, Collateral, Leverage.
JEL classification: C11, E44, G11.

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1 Introduction

In the recent financial crisis, financial institutions have been affected in different ways. Some institutions (such as security brokers and dealers or hedge funds) have lost part of their access to the credit market. They have been forced to sell part of their assets, resulting in a severe deleveraging. For other institutions (such as deposit banks), the fall in the value of their assets has implied a fall in their equity, resulting in an initial increase in their leverage. We illustrate this contrast by computing the correlation between the leverage of the different types of institutions and variables reflecting the economic or financial cycles. Correlations with the leverage are computed over the 2007-2011 period and reported in Table 1.[1] For security brokers and dealers, leverage is positively correlated with GDP growth and the S&P500 return (61% and 42%, respectively) and negatively correlated with the VIX change (−45%), suggesting a procyclical dynamics. In contrast, for commercial banks, leverage is negatively correlated with GDP growth and the S&P500 return (−22% and −39%, respectively) and positively correlated with the VIX change (58%), indicating a counter-cyclical dynamics. More specifically, in the fourth quarter of 2008, the leverage of security brokers and dealers experienced a severe fall, from 26.9 to 17.5, while, at the very same time, the leverage of commercial banks was still going up, from 8.8 to 9.1.[2]

Clearly these estimates suggest that banks face different situations depending on their financing structure. These questions have been addressed, from a theoretical perspective, in a set of recent papers. Security brokers and dealers have limited access to debt, as in the models proposed by Geanakoplos and Fostel (2008), Adrian and Shin (2010, 2014), or Brunnermeier and Pedersen (2009). In contrast, commercial banks have limited access to equity financing, as in the models of He and Krishnamurthy (2012, 2013) or Brunnermeier and Sannikov (2014). By relying on these two main mechanisms, He,

[1]Leverage is measured as the book value of the debt divided by the book value of the equity, using data from the Flow of Funds of the United States. The economic cycle is measured by the real GDP growth, whereas the financial cycle is measured by the quarterly growth of the S&P500 index and the change in the VIX index.

[2]He, Khang, and Krishnamurthy (2010) provide several arguments pointing in favor of a much larger increase in the leverage of commercial banks in the crisis. In particular, it is very likely that the initial fall in the value of their assets has been dramatically under-estimated, so that the marked-to-market value of equity was probably much lower than reported.

Khang, and Krishnamurthy (2010) have proposed a rationalization of why in the recent financial crisis the leverage of security brokers and dealers has been procyclical, i.e., decreasing in the crisis, whereas the leverage of commercial banks has been countercyclical.

Our paper builds on these different blocks. We consider two types of banks, whose balance sheets are driven by two contrasting mechnisms: the "deposit bank " shares features of the commercial banks and the "merchant bank " share features of the security brokers and dealers. The deposit bank receives short-term deposits from households and lends to the merchant bank on the interbank market. The merchant bank borrows from deposit bank and buys a long-term claim on nonfinancial firms' assets. For both banks, liabilities are short-term and insensitive to market fluctuations, because the bank commits itself to repay the face value of its deposits or interbank debt plus an interest. In contrast, merchant bank assets are long-term and fluctuate with the state of the economy. We do not consider the usual firm/bank agency problem as in Bernanke, Gertler, and Gilchrist (1999), because the merchant bank is the shareholder of the firm. Instead, we focus on the agency problem on the interbank market. The merchant bank has to post some assets as collateral for its interbank loan. The haircut on the collateral is optimally determined by the deposit bank to maximize its expected return on equity.[3] In a downturn, as the value of the firms' assets goes down, the merchant bank has to post more collateral. If it does not have sufficient cash, the bank has to sell depreciated assets with short notice (fire sales) and/or delever. This process amplifies the downturn. If the downturn is sufficiently severe, the merchant bank defaults. The deposit bank then seizes and liquidates the assets left by the merchant bank. It therefore incurs a loss on its own assets. If the loss is too large and the bank does no have sufficient cash, the marked-to-market value of the assets does not cover the face value of deposits, and the bank defaults. As this scenario shows, two main mechanisms are at play: (1) the value of the risky assets may be affected by the state of the economy, whereas the liabilities

[3]The interbank loan with collateral can be viewed as a repurchase agreement (repo) or an asset-backed commercial paper: The borrower sells securities and repurchases the securities at maturity at a given price including an interest fixed in advance. Security repos represent approximately 60% of the liabilities of security brokers and dealers over the 2007-11 period (Flow of Funds data).

are not contingent. (2) If the bank does not have sufficient cash to cover its liabilities, it defaults and the lender liquidates the remaining assets.

We introduce in the model a deposit insurance mechanism. Deposits are guaranteed by the government, which asks for an insurance premium to be paid by the deposit bank and equal to the expected loss on deposits in case of a default.[4] The premium corresponds to the expected difference between the face value of the deposits and the market value of the deposit bank assets. The latter is likely to be severely affected in a downturn, whereas the former is not. This difference in the valuation of the assets and the deposits (marked-to-market versus book value) generates the capital shortfall of the deposit bank and potentially triggers its default.

We then investigate more specifically how a fall in the value of the nonfinancial firms' equity can general systemic crisis. We define systemic risk as the expected loss on the deposits in a stress scenario and call this measure SEL (for stressed expected loss). In an economic downturn, which we define as a fall in the return on securities issued by the firms, the merchant bank may be forced to default, resulting in a loss on the assets of the deposit bank. The market value of these assets may be lower than the face value of deposits. The difference (the capital shortfall) would have to be covered by the government as a result of the deposit insurance mechanism. Our simplified description of the financial system allows us to derive a simple expression for the SEL, which incorporates the main elements discussed above. The value of the deposit bank assets under the crisis reflects the loss on the interbank loans in case of a merchant bank default. It therefore depends on the merchant bank strategy (amount of free cash, cost of fire sales) and on the deposit bank strategy (amount of free cash, level of the haircut, cost of liquidation). This explicit link with the key variables driving banks' strategy clearly opens the door to identifying policy measures that could help mitigate systemic risk.

[4]In the U.S., deposits are guaranteed by the FDIC up to a stated limit. The FDIC is funded by the premiums paid by banks for the deposit insurance coverage. We could alternatively consider that depositors require a risk premium over and above the risk-free rate to cover the expected loss in case of a deposit bank default.

We calibrate the model to match key features of the U.S. economy.[5] For macro parameters and shock dynamics, we rely on Gertler and Kiyotaki (2010) and Christiano, Motto, and Rostagno (2010). For financial parameters and financial returns, we use data over the recent period. We then describe the equilibrium implied by this parametrization, in particular on financial intermediaries. The optimal values we obtain for the cash held by the merchant bank (21% of its equity) and the haircut on collateral (30% of the value of the assets posted as collateral) are in the ballpark of actual numbers. We also find that, in absence of regulation, the deposit bank optimally does not hold cash. This result is not surprising in our model as the benefit from holding cash does not compensate its opportunity cost. An important implication of this result is that the deposit bank is highly sensitive to a merchant bank default. The fall in securities return that triggers a deposit bank default is only slightly larger than the fall that triggers a merchant bank default. This clearly suggests that imposing a minimum buffer of cash is a way to mitigate deposit bank default.

Finally, we consider the case of a market crash, which we design as a fall in the return of firm's securities, and we determine the expected loss of the deposit bank under a stress scenario. We obtain that, with our baseline parameterization, the SEL would represent approximately 18% of the assets of the deposit bank (or 80% of its equity) in a 40% market crash. We show that changing the value of the key parameters (cost of fire sales and liquidation cost) would easily increase the SEL but hardly decrease it. We then evaluate the effect of imposing a minimum amount of cash to the deposit bank. For instance, imposing the bank to hold 60% of its deposits in cash would reduce the SEL from 18% to 3.5% of the assets in a market crash. The SEL is a model-consistent and operational measure of systemic risk. We assume that the crisis is triggered by a negative shock on the expected return of investment projects in the economy. This concept is relatively close to the market return and, therefore, could be associated to a market crash. However, we could alternatively assume a crash on the sovereign bond market or

[5]Our model has several non-linearities due to the option-like payoff structure of the banks. The (merchant and deposit) banks benefit from the increase in the value of their assets above the face value of their debt, and default otherwise. For this reason, we solve and simulate our model in its nonlinear form. We let the estimation of the key parameter of the model for further research.

on the real estate market. Other ways to define a crisis can be introduced provided the macroeconomic model is adapted to this stress scenario.

Our paper is closely related to Acharya et al. (2012) and He and Krishnamurthy (2014) in that we define systemic risk in a model-consistent way. We share with He and Krishnamurthy (2014) the construction of a general equilibrium model of the economy, in which nonlinearities contribute to systemic risk. In Acharya et al. (2012), the externality that generates systemic risk is the propensity of a financial institution to be undercapitalized in a crisis, i.e., when the financial system as a whole is undercapitalized. In this context, there are likely to be few financial institutions willing to absorb liabilities and acquire the failing firms. They define the systemic risk (SRISK) as the expected difference between a fraction of the assets and the bank's equity in a crisis.[6]

The rest of the paper is organized as follows. In Section 2, we describe the model and its main mechanisms. In Section 3, we provide details on the calibration of the model and its implications in terms of impulse response functions. In Section 4, we provide an analytical and quantitative analysis of the SEL as a measure of systemic risk and compare this measure to the SRISK measure of Acharya et al. (2012) and Brownlees and Engle (2012).

2 Model

This section briefly describes the main aspects of the theoretical model, focusing on the mechanisms at stake in the finance side of the economy. Details are provided in the Appendix. The model is composed of households, final goods firms, capital producers, merchant banks, deposit banks, and authorities. As in Gertler and Kiyotaki (2010), we restrict our attention to a real economy. We do not introduce nominal rigidities in the model because it is not necessary to exhibit the main results. Introducing nominal rigidities would be straightforward in this framework.[7] As a consequence, the risk-free

[6]Brownlees and Engle (2012) provide an empirical evaluation of this measure.

[7]We have developed a more complete model with inflation dynamics and a central bank determining the evolution of the risk-free rate. This extension clearly changes the macro dynamics, but does not alter the main mechanisms driving the finance side of the economy. Therefore, we keep the simple model for exposition purpose.

short-term rate is determined by the equilibrium between the number of bonds issued by the government and the number of bonds bought by private agents.

The real side of the model is kept as simple as possible: households supply labor used by firms to produce final goods. Firms also use capital, produced by capital producers and are financed by merchant banks. Households consume and save money on their account at the deposit banks. There are two types of banks, namely merchant banks and deposit banks, to clearly identify the two steps of intermediation: short-term deposits are collected by deposit banks from households and long-term financing is provided by merchant banks to firms. In this model, interest rates on interbank loans and on deposits are determined optimally at the equilibrium, so that the risk premium over the risk-free rate exactly compensates the risk of default of the merchant bank and deposit bank, respectively. All the sources of risk in the model are therefore priced at the equilibrium, so that all markets clear.

2.1 Households

There is a continuum of identical households, indexed by j, who derive utility from consumption C_t and disutility from labor H_t, defined as the number of hours worked. The representative household maximizes the expected value of the future flows of utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t v_t \left[\frac{(C_t - \gamma C_{t-1})^{1-\sigma}}{1-\sigma} - \eta \frac{(H_t)^{1+\zeta}}{1+\zeta} \right], \tag{1}$$

where β is the discount factor, σ is the (inverse of the) elasticity of intertemporal substitution (and the relative risk aversion), ζ is the (inverse of the) elasticity of the labor supply, γ is the habit formation parameter, η is the weight on leisure in the utility function, and v_t is the preference shock. The specification of v_t and all the other shocks will be discussed in Section 3.1.

Households save money in the form of deposits, Dep_t, in deposit banks and invest in bank equity. They receive dividends and have to fund new banks when some banks default. We assume that both inflows (dividends from existing banks) and outflows (financing of new banks) exactly compensate each other, so that revenues from bank capital do not show up in the budget constraint (see Section 2.8).[8] We do not consider nominal rigidities in the model so that wages and interest rates are expressed in real terms.

The budget constraint is:

$$(1+\tau_c)C_t + Dep_t \leq (1-\tau_s)W_t H_t + (1+R_{F,t-1})Dep_{t-1} + T_t, \tag{2}$$

where W_t is the real wage per hour worked, T_t is the net lump-sum transfer from the government, and τ_c and τ_s denote the tax rates on consumption and on labor income, respectively. In the model, the deposit bank can default but the deposits are insured by the government: The deposit bank pays an insurance premium, IP_{t-1}, to the government. Therefore, the deposit rate received by households is the real risk-free rate, $R_{F,t-1}$, which is determined one period in advance, and the deposit rate paid by the bank at date t is $R_{Dep,t-1} = R_{F,t-1} + IP_{t-1}$. Section 2.7 describes how the insurance premium is determined.

The optimality conditions associated with consumption, labor, and deposits are, respectively:

$$(1+\tau_c)\lambda_t = v_t(C_t - \gamma C_{t-1})^{-\sigma} - \beta E_t[v_{t+1}(C_{t+1} - \gamma C_t)^{-\sigma}], \tag{3}$$

$$(1-\tau_s)\lambda_t W_t = v_t \eta (H_t)^{\zeta}, \tag{4}$$

$$1 = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1+R_{F,t})\right] = E_t\left[\Lambda_{t,t+1}(1+R_{F,t})\right]. \tag{5}$$

The first equation defines λ_t the Lagrange multiplier of the budget constraint (marginal utility of consumption). The second equation defines the real wage. The third equation is the pricing kernel, which determines the risk-free rate. The variable $\Lambda_{t,t+1} = \beta\lambda_{t+1}/\lambda_t$ denotes the household's stochastic discount factor.

[8] Alternatively, it may be assumed that all profits of banks are reinvested in accumulating bank capital (zero-dividend policy). However, given that banks can default in our model, we need to design a mechanism by which new banks are created and financed by households. See Gerali et al. (2010) for additional details.

2.2 Firms

Perfectly competitive firms combine labor supplied by households and entrepreneurial capital to produce final goods with constant return to scale. The production function of firm i in period t is (see Nuño and Thomas, 2013):

$$Y_t(i) = Z_t[\varepsilon_t(i)K_t(i)]^\alpha L_t(i)^{1-\alpha} - \Phi, \tag{6}$$

with K_t is the capital available at the beginning of the period, L_t is the labor, and Z_t is the aggregate total factor productivity, common to all firms, which follows a stationary AR(1) process. Φ is the fixed cost of production, determined such that firms make no economic profit at the equilibrium. The shock $\varepsilon_t(i)$ is specific to firm i, affecting the amount of effective capital, from $K_t(i)$ to $\varepsilon_t(i)K_t(i)$. This process is iid over time and across firms, with $E[\varepsilon_t(i)] = 1$ for all i. We assume that $\log \varepsilon_t(i)$ is normally distributed.

We adopt the following timing: at the end of period $t-1$, the firm determines the optimal stock $K_t(i)$ of physical capital for the next period. Then, at the beginning of period t, idiosyncratic shock $\varepsilon_t(i)$ realizes and randomly changes the quality of the capital to $\varepsilon_t(i)K_t(i)$. The firm maximizes its operating profit $Y_t(i) - W_t L_t(i)$ with respect to labor and production takes place. The realization of the idiosyncratic shocks will differentiate firms across firms.

The firm maximizes its operating profit with respect to labor $L_t(i)$ subject to the production function constraint, with capital taken as given. The first-order condition is:

$$W_t = (1-\alpha)Z_t \left(\frac{\varepsilon_t(i)K_t(i)}{L_t(i)}\right)^\alpha .$$

As labor is free to move across firms at no cost, real wage is equalized across firms, so that the effective capital-labor ratio is also equalized across firms:

$$\frac{\varepsilon_t(i)K_t(i)}{L_t(i)} = \left(\frac{W_t}{(1-\alpha)Z_t}\right)^{1/\alpha} .$$

This gives the following expression for the operating profit:

$$Y_t(i) - (1-\alpha)Z_t\left(\frac{\varepsilon_t(i)K_t(i)}{L_t(i)}\right)^{\alpha} L_t(i) = \alpha Y_t(i).$$

As there is no economic profit at the equilibrium, the operating profit is equal to the cost of effective capital. The return on effective capital is therefore:

$$R_{K,t} = \frac{Y_t(i) - W_t L_t(i)}{\varepsilon_t(i)K_t(i)} = \alpha Z_t\left(\frac{(1-\alpha)Z_t}{W_t}\right)^{(1-\alpha)/\alpha},$$

which is also independent from i.

At the end of period t, the cash flow from the investment project is equal to the sum of the operating profit, $R_{K,t}(\varepsilon_t(i)K_t(i))$, and the value of the depreciated capital, $Q_{K,t+1}(1-\delta)(\varepsilon_t(i)K_t(i))$, where $Q_{K,t+1}$ denotes the price of capital at the end of the period and δ is the depreciation rate of capital. Therefore, the total cash flow is: $[R_{K,t} + (1-\delta)Q_{K,t+1}](\varepsilon_t(i)K_t(i))$, so that the investment made at the end of $t-1$ in the firm's project, $Q_{K,t}K_t(i)$, has an ex-post return in period t given by:

$$1 + R_{S,t}(i) = \frac{R_{K,t} + (1-\delta)Q_{K,t+1}}{Q_{K,t}}\ \varepsilon_t(i). \tag{7}$$

At the end of period t, the firm buys at price $Q_{K,t+1}$ a new stock of capital that will be used in period $t+1$.[9]

The investment of the firm is financed by a merchant bank. The bank invests in only one project. The firm sells a claim on its future cash flows to the bank. The claim is equal to the value of the capital units acquired by the firm at the beginning of period t, i.e., $S_t(i) = Q_{K,t}K_t(i)$. Therefore, the gross rate of return on the bank's investment made at date t is $(1 + R_{S,t}(i))$, known at the end of period t. As the investment is financed one period in advance, before the realization of the firm-specific shock, $R_{S,t}(i)$ is not known at the time the bank makes its decision.

[9] In fact, the firm does not need to resale the old capital and buy new capital every period. We use this mechanism to measure the return on investment for the bank due to the increase in the price of capital.

2.3 Capital Producers

Physical capital used by the firm, K_t, is produced by the capital producer and bought by the firm using funds from the merchant bank. The technology uses the old capital units, K_{t-1}, and a fraction of the final goods purchased from the firm as investment goods, I_{t-1}, to produce new physical capital, K_t. At the end of date $t-1$, the new capital is sold at price $Q_{K,t}$ to the firm to produce the final goods in t.

As in Christiano, Motto, and Rostagno (2010) and Dib (2010), the optimization program is:

$$\max_{\{I_t\}} \quad E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left\{ Q_{K,t} \left[I_t - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 I_t \right] - I_t \right\},$$

where I_t denotes the investment in units of final goods, and κ is the investment adjustment cost parameter. The term in squared brackets is the equipment produced with the new investment I_t.

The optimality condition gives:

$$\begin{aligned} 1 \quad &= \quad Q_{K,t} \left[1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] \\ &\quad + \beta \kappa E_t \left[\left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 Q_{K,t+1} \frac{\lambda_{t+1}}{\lambda_t} \right]. \end{aligned}$$

2.4 Merchant Banks

We now consider the merchant bank financing firm i and will omit the notation "(i)" for exposition purpose. The return of its investment in the assets of the firm is $R_{S,t}$. As it is known at the end of date t only, we will use from now on the notation $\tilde{R}_{S,t} = R_{S,t}(i)$ to avoid confusion, where the tilde means that a return defined between t and $t+1$ is known at the end of the period only. As the firm-specific shock $\varepsilon_t(i)$ is log-normally distributed, so is $\tilde{R}_{S,t}$. We define $\mu_{S,t} = E_t[\tilde{R}_{S,t}]$ and $\sigma_S^2 = V_t[\tilde{R}_{S,t}]$.

The merchant bank holds cash and the assets of the firm. It is funded by equity and interbank debt. Interbank debt is collateralized, meaning that a fraction of the securities has to be posted to cover the potential loss of the lender (the deposit bank). The margin rate (haircut) is optimally determined by the deposit bank.

In the balance sheet below, the items are written in terms of the net worth (or equity) of the bank, $N_{M,t}$, because the objective of the bank is to maximize its expected equity. In fact, the balance sheet is determined as follows: The merchant bank faces a demand for funds from the firm (S_t). It defines its optimal leverage through the amount of collateral used for raising interbank debt so as to maximize its expected return on equity. Then, given the fraction of equity used as collateral (a_t), it defines the amount of debt raised used to buy the securities issued by the firm.

Balance sheet of a merchant bank

	Assets	**Liabilities**	
$(R_{F,t})$	Cash: $C_{M,t} = N_{M,t}(1 - a_t)$	Debt: $D_t = N_{M,t}a_t/m_t$	$(R_{D,t})$
$(\tilde{R}_{S,t})$	Securities: $S_t = N_{M,t}a_t(1 + m_t)/m_t$	Equity: $N_{M,t}$	$(\tilde{R}_{N_M,t})$

The margin rate (or haircut) on the interbank debt, denoted by m_t, is exogenously given for the moment.[10] If the value of the securities falls, the merchant bank has to post more collateral (margin calls). When the bank has enough cash, it can use available liquidity to cover the margin call. When return on securities, $\tilde{R}_{S,t}$, is below a given threshold $\overline{R}_{S,t}$, the bank does not hold enough cash and has to sell a fraction of its securities to reduce its debt and satisfy the haircut (fire sale). When $\tilde{R}_{S,t}$ is below another given threshold $\underline{R}_{S,t}$, cash and securities are not sufficient to cover margin calls and the merchant bank defaults.

The securities bought by the merchant bank are the shares issued by the firms. Therefore, $\tilde{R}_{S,t}$ corresponds to the return of the firm's project. The balance sheet structure implies that:

$$(1 + R_{F,t})C_{M,t} + (1 + \tilde{R}_{S,t})S_t = (1 + R_{D,t})D_t + (1 + \tilde{R}_{N_M,t})N_{M,t},$$

so that the return on merchant bank equity is:

$$(1 + \tilde{R}_{N_M,t}) = (1 + \tilde{R}_{S,t})\frac{S_t}{N_{M,t}} + (1 + R_{F,t})\frac{C_{M,t}}{N_{M,t}} - (1 + R_{D,t})\frac{D_t}{N_{M,t}}.$$

[10] In the following, the deposit bank will determine the optimal haircut so as to maximize its expected return. Alternatively, the haircut could be determined by the deposit bank to cover potential losses.

The risk-free rate ($R_{F,t}$) and the interest rate on interbank debt ($R_{D,t}$) are fixed one period in advance for the merchant bank. In contrast, the return on securities ($\tilde{R}_{S,t}$) and return on bank's equity ($\tilde{R}_{N_M,t}$) are observed at the end of the period.

The merchant bank uses $a_t N_{M,t}$ of its equity as collateral to finance its position in securities. The collateral is invested in securities. Some unencumbered cash (free cash) is kept by the bank as a buffer in case of margin calls. The importance of holding a buffer of liquidity for leverage-constraints banks has been studied by Allen and Gale (1994) and Holmström and Tirole (1998). Margin calls work as follows: At the beginning of date $t+1$, the value of the securities held by the bank is $(1+\tilde{R}_{S,t})S_t$. We assume that interbank debt does not change until the bank invests in a new project ($D_{t+1} = D_t$). Therefore, the bank has to manage its collateral to ensure that, at the end of $t+1$, the value of the securities held satisfies: $S_{t+1} = (1+m_t)D_{t+1} = (1+m_t)D_t$.

As described in the Appendix 6.1, three cases can arise. In the first case (normal time), the return on securities, $\tilde{R}_{S,t}$, is sufficiently large and the merchant bank does not have to put more collateral or can use its free cash for posting more collateral. In the second case (fire sale), the merchant bank does not have sufficient cash and must sell some assets to reduce its need for collateral. As the bank sells securities with short notice, it incurs a cost, proportional to the value of securities sold, denoted by ϕ. Duarte and Eisenbach (2013) study of size of the cost of fire sales for different categories of financial institutions. We discuss the calibration of this parameter in Section 3.1. [11] In the last case (default), even by selling all its assets, the merchant bank is not able to meet the request for collateral and defaults. The following proposition provides the expression for the expected return on equity of the merchant bank.

Proposition 1 *The expected equity of the merchant bank is:*

$$E_t[N_{M,t+1}] = E_t[N^{(NT)}_{M,t+1}] \times \Pr[\tilde{R}_{S,t} > \overline{R}_{S,t}] + E_t[N^{(FS)}_{M,t+1}] \times \Pr[\tilde{R}_{S,t} \in [\underline{R}_{S,t}, \overline{R}_{S,t}]],$$

where the expected equity return under the different regimes are:

[11] To avoid additional complications in the model, we do not explicitly identify the buyers of the assets sold by the merchant bank with short notice. We could assume that the merchant bank liquidates the assets of the firm, which are then sold to the capital producers.

- *Normal time*

$$
\begin{aligned}
E_t[N^{(NT)}_{M,t+1}] &= E_t[N_{M,t+1} \mid \tilde{R}_{S,t} > \overline{R}_{S,t}] \\
&= \left[(1+\mu^{(NT)}_{S,t})\frac{a_t(1+m_t)}{m_t} + (1+R_{F,t})(1-a_t) - (1+R_{D,t})\frac{a_t}{m_t}\right] N_{M,t},
\end{aligned}
$$

- *Merchant bank fire sales*

$$
\begin{aligned}
E_t[N^{(FS)}_{M,t+1}] &= E_t[N_{M,t+1} \mid \tilde{R}_{S,t} \in [\underline{R}_{S,t}; \overline{R}_{S,t}]] \\
&= \Psi_t \left[(1+\mu^{(FS)}_{S,t})(1-\phi)a_t + (1+R_{F,t})\frac{m_t(1-a_t)}{1+m_t} - (1+R_{D,t})\frac{a_t}{1+m_t}\right] N_{M,t},
\end{aligned}
$$

- *Merchant bank default*

$$
E_t[N^{(MBD)}_{M,t+1}] = E_t[N_{M,t+1} \mid \tilde{R}_{S,t}\underline{R}_{S,t}] = 0,
$$

with $\Psi_t = (1+m_t)/((1+m_t)(1-\phi)-1)$*,* $\mu^{(NT)}_{S,t} = E_t[\tilde{R}_{S,t} \mid \textit{Normal time}\ ]$*, and* $\mu^{(FS)}_{S,t} = E_t[\tilde{R}_{S,t} \mid \textit{Fire sales}\ ]$*. Thresholds between the different regimes are:*

$$
\begin{aligned}
1+\overline{R}_{S,t} &= \frac{m_t}{1+m_t} + \frac{1}{1+m_t}\left((1+R_{D,t}) - (1+R_{F,t})\frac{m_t(1-a_t)}{a_t}\right), \\
1+\underline{R}_{S,t} &= \frac{1}{(1-\phi)(1+m_t)}\left((1+R_{D,t}) - (1+R_{F,t})\frac{m_t(1-a_t)}{a_t}\right).
\end{aligned}
$$

Proof: Appendix 6.1. The table below displays the different situations that a merchant bank can meet, depending on the value of the return on securities at the end of t. $\overline{R}_{S,t}$ denotes the threshold between the regular margin call case and the fire sales case; $\underline{R}_{S,t}$ denotes the threshold between the fire sales case and the default case.

$\underline{R}_{S,t}$	$\overline{R}_{S,t}$	
Default	Fire sales	Regular margin call
cash = 0	cash = 0	cash ≥ 0
securities = 0	securities ≥ 0	securities ≥ 0
(Case 3)	(Case 2)	(Case 1)

The bank takes $R_{F,t}$ and $R_{D,t}$ as given and knows the distribution of $\tilde{R}_{S,t}$ (with parameters $\mu_{S,t}$ and σ_S), the haircut (m_t), and the cost of fire sales (ϕ) and determines the optimal fraction of equity used as collateral a_t that maximizes its expected equity:

$$\max_{\{a_t\}} \quad E_t[N_{M,t+1}/N_{M,t}] = E_t[1 + \tilde{R}_{N_M,t}].$$

Once the optimal fraction a_t^* is determined, we compute the optimal leverage of the bank as $L_{M,t}^* = (C_{M,t} + S_t)/N_{M,t} = 1 + a_t^*/m_t$ and the resulting optimal equity. The optimal equity covers the demand for securities S_t and satisfies the optimal leverage $L_{M,t}^*$. Once a_t^* is determined, we deduce what should be the optimal equity at date t that will cover the demand for securities S_t and the optimal leverage a_t^*.

We notice that the leverage has an ambiguous effect on the return on equity of the merchant bank. On the one hand, the default threshold, $\underline{R}_{S,t}$, increases with leverage, so that a higher leverage is accompanied by a higher probability of default of the merchant bank. On the other hand, the expected equity return in normal time is increasing with leverage provided $\mu_{S,t}^{(NT)} > R_{D,t}$.

2.5 Equilibrium Interest Rate on Interbank Debt

So far, we have assumed that the interbank interest rate, $R_{D,t}$, is given at the beginning of period t. Yet, as the merchant bank can default, interbank loans are in fact risky for the deposit bank. In the case of a default of the merchant bank, the deposit bank seizes and liquidates the assets of the merchant bank. We assume that there is a liquidation cost, which we measure by $\theta \in [0;1]$. The deposit bank should take this potential loss into account and require a risk premium. The risk premium, which is determined ex ante, has to cover the expected loss on the interbank loan due to the possible default of the merchant bank.

The expected return on the interbank loans for the deposit bank is given by:

$$E_t[D_{t+1}] = D_t \times \Pr[\text{No default } (\tilde{R}_{S,t} > \underline{R}_{S,t})] + E_t[D_{t+1}^{(MBD)}] \times \Pr[\text{Default } (\tilde{R}_{S,t} \leq \underline{R}_{S,t})],$$

where $E_t[D_{t+1}^{(MBD)}]$ corresponds to the expected value of the assets of the merchant bank in case of a default, after payment of the interest on loans, i.e.:

$$\begin{aligned} E_t[D_{t+1}^{(MBD)}] &= E_t[D_{t+1} \mid \text{Merchant bank default }] \\ &= (1-\theta)(1+\mu_{S,t}^{(MBD)})S_t + (1+R_{F,t})C_{M,t} - R_{D,t}D_t, \end{aligned}$$

where $\mu_{S,t}^{(MBD)} = E_t[\tilde{R}_{S,t} \mid \tilde{R}_{S,t} \leq \underline{R}_{S,t}]$. Therefore, the expected value of the interbank loan is:

$$E_t[D_{t+1}] = D_t - \left[(1+R_{D,t})D_t - (1-\theta)(1+\mu_{S,t}^{(MBD)})S_t - (1+R_{F,t})C_{M,t}\right] \Phi_S(\underline{R}_{S,t}).$$

In case of a merchant bank default, the value of the interbank loan is reduced by the loss affecting the assets of the merchant bank.

Given the possibility of an ex-post loss on the interbank loan, the deposit bank requires an ex-ante interest rate $R_{D,t}$ that takes the expected loss into account. Therefore, we have: $R_{D,t} = R_{F,t} + RP_{D,t}$, where $RP_{D,t}$ corresponds to the risk premium over the risk-free rate $R_{F,t}$. The risk premium is defined such that it covers the expected loss in case of default:

$$RP_{D,t} = -E_t\left[\frac{D_{t+1} - D_t}{D_t}\right] = \mu_{D,t}^{(MBD)} \Phi_S(\underline{R}_{S,t}),$$

where $\mu_{D,t}^{(MBD)} = (1-\theta)(1+m_t)(1+\mu_{S,t}^{(MBD)}) + \frac{m_t(1-a_t)}{a_t}(1+R_{F,t}) - (1+R_{D,t})$ denotes the expected return on the interbank loan in case of a merchant bank default. The risk premium increases with leverage through two channels: both the probability of default of the merchant bank and the loss on interbank bank in case of default increase with leverage.

At the equilibrium, the optimal decision of the merchant bank on the amount of collateral (a_t^*) has to be consistent with the equilibrium interest rate on interbank loans, i.e., $R_{D,t}$ should satisfy:

$$R_{D,t} = R_{F,t} + \left[(1+R_{D,t}) - (1-\theta)(1+m_t)(1+\mu_{S,t}^{(MBD)}) - \frac{m_t(1-a_t^*)}{a_t^*}(1+R_{F,t})\right] \Phi_S(\underline{R}_{S,t}).$$

Therefore, the equilibrium interbank rate, $R_{D,t}$, is determined as a fixed point of this equation. We note that $\mu_{S,t}^{(MBD)}$, $\underline{R}_{S,t}$, and a_t^* depend on $R_{D,t}$ at the equilibrium. So the optimal leverage of the merchant bank (through a_t^*) and the equilibrium interbank rate ($R_{D,t}$) are determined simultaneously for a given haircut m_t and a given risk-free rate $R_{F,t}$.

Eventually, the ex-post return on the interbank loan is the face value interest rate on the loan (received in cash by the deposit bank from the merchant bank) less the ex-post loss due to the change in the value of the debt if the merchant bank defaults:

$$\tilde{R}_{D,t} = R_{D,t} + \frac{D_{t+1} - D_t}{D_t}.$$

2.6 Deposit Banks

If the merchant bank builds excessive leverage, its default is more likely. Such a default would incur a loss on the interbank loan held by the deposit bank. This potential loss is compensated at the equilibrium by the risk premium paid by the merchant bank on its interbank loan. If the deposit bank has too much exposure to interbank loans, it may be unable to pay back deposits to depositors, inducing a default. We assume that the value of deposits (Dep_t) is determined by household savings decisions, whereas the value of the interbank loan is determined by investment needs of the firm and the haircut chosen by the deposit bank ($D_t = S_t/(1 + m_t)$). The deposit bank then determines the optimal leverage (the value of its equity) and therefore the optimal value of free cash to hold. The level of cash lies between two extreme cases. In the first case, the bank does not hold cash and its equity is equal to $N_{D,t} = D_t - Dep_t$. In the opposite case, the bank holds all its deposits in cash and its equity is equal to $N_{D,t} = D_t$. We therefore define $N_{D,t}$ as $D_t - b_t Dep_t$, where b_t measures the fraction of deposits invested in interbank loans ($b_t \in [0; 1]$). The cash held by the deposit bank is thus $C_{D,t} = (1 - b_t)Dep_t$. The simplified balance sheet of the deposit bank is represented below.

Balance Sheet of a Deposit Bank

	Assets	Liabilities	
$(R_{F,t})$	Cash: $C_{D,t} = (1-b_t)Dep_t$	Deposit: Dep_t	$(R_{Dep,t})$
$(\tilde{R}_{D,t})$	Interbank loan: $D_t = S_t/(1+m_t)$	Equity: $N_{D,t} = D_t - b_t Dep_t$	$(\tilde{R}_{N_D,t})$

We notice that the interest rate on deposits, $R_{Dep,t} = R_{F,t} + IP_t$, is higher than the interest received by the households, because $R_{Dep,t}$ incorporates the insurance premium paid to the government (determined in Section 2.7). In principle, even if the expected return of cash $(R_{F,t})$ is lower than the cost of deposits $(R_{Dep,t})$, the deposit bank may have some incentive to hold cash. As for the merchant bank, cash might play the role of a buffer in case of a decrease in the value of the other assets, i.e., the interbank loan.[12] The leverage of the deposit bank is defined as $L_{D,t} = 1 + Dep_t/(S_t/(1+m_t) - b_t Dep_t)$. It increases with the haircut m_t. As the haircut typically increases in the crisis, we expect the leverage of the deposit bank to have a counter-cyclical dynamics, as suggested by empirical evidence. Leverage also increases with the fraction of deposits invested by the bank in the interbank loan.

The value of the assets of the deposit bank at the end of date t is equal to the value of liabilities:

$$(1+\tilde{R}_{D,t})D_t + (1+R_{F,t})(1-b_t)Dep_t = (1+R_{Dep,t})Dep_t + (1+\tilde{R}_{N_D,t})(D_t - b_t Dep_t).$$

The objective of the deposit bank is to find the haircut (m_t) and fraction of its assets (b_t) to lend to the merchant bank that maximize its expected equity $E_t[N_{D,t+1}]$, which is given by:

$$\begin{aligned} N_{D,t+1} &= (1+\tilde{R}_{N_D,t})N_{D,t} \\ &= (1+\tilde{R}_{D,t})D_t + (1+R_{F,t})(1-b_t)Dep_t - (1+R_{Dep,t})Dep_t \\ &= \left[(1+R_{F,t}) + (\tilde{R}_{D,t} - R_{F,t})\frac{D_t}{D_t - b_t Dep_t} - (R_{Dep,t} - R_{F,t})\frac{Dep_t}{D_t - b_t Dep_t}\right] N_{D,t}. \end{aligned}$$

[12]It turns out that, in our parametrization, the deposit bank does not hold free cash, although it may not always be the case. The reason for not holding cash is that the benefit from holding cash (cost of defaulting) is smaller than the opportunity cost of not lending to the merchant bank. We could provide more incentive to the deposit bank to hold cash, for instance by introducing a quadratic cost of default, but we keep this extension for future research.

Three situations can happen depending on the realization of the return on interbank loan $\tilde{R}_{D,t}$, i.e., depending on the possible default of the merchant bank. In the first case (no default of the merchant bank), the interbank loan is fully repaid and the deposit bank is solvent. In the second case, the merchant bank defaults and the deposit bank liquidates the assets of the merchant bank. It incurs a liquidation cost, denoted by θ, but the liquidated assets are enough to cover the deposits and the deposit bank remains solvent. In the third case, the loss on the interbank loan is so large that the deposit bank itself defaults. The following proposition provides the expression for the expected return on equity of the deposit bank.

Proposition 2 *The expected value of equity of the deposit bank is:*

$$\begin{aligned} E_t[N_{D,t+1}] &= E_t[N_{D,t+1}^{(NT)}] \times \Pr[\tilde{R}_{S,t} > \underline{R}_{S,t}] \\ &\quad + E_t[N_{D,t+1}^{(MBD)}] \times \Pr[\tilde{R}_{S,t} \in [\underline{R}_{SD,t}; \underline{R}_{S,t}]] + 0 \times \Pr[\tilde{R}_{S,t} \leq \underline{R}_{SD,t}], \end{aligned}$$

where the expected equity return under the different regimes are:

- *Normal time*

$$\begin{aligned} E_t[N_{D,t+1}^{(NT)}] &= E_t[N_{D,t+1} \mid \tilde{R}_{S,t} > \underline{R}_{S,t}] \\ &= \left[1 + R_{F,t} + (R_{D,t} - R_{F,t})\frac{D_t}{N_{D,t}} - (R_{Dep,t} - R_{F,t})\frac{Dep_t}{N_{D,t}}\right] N_{D,t}, \end{aligned}$$

- *Merchant bank default*

$$\begin{aligned} E_t[N_{D,t+1}^{(MBD)}] &= E_t[N_{D,t+1} \mid \tilde{R}_{S,t} \in [\underline{R}_{SD,t}; \underline{R}_{S,t}]] \\ &= \left[1 + R_{F,t} + (\mu_{D,t}^{(MBD)} - R_{F,t})\frac{D_t}{N_{D,t}} - (R_{Dep,t} - R_{F,t})\frac{Dep_t}{N_{D,t}}\right] N_{D,t}, \end{aligned}$$

- *Deposit bank default*

$$E_t[N_{D,t+1}^{(DBD)}] = E_t[N_{D,t+1} \mid \tilde{R}_{S,t} \leq \underline{R}_{SD,t}] = 0.$$

The threshold between the merchant bank default and the deposit bank default regimes is:

$$1+\underline{R}_{SD,t} = \frac{1}{(1+m_t)(1-\theta)}\left[(1+R_{Dep,t})\frac{Dep_t}{D_t} - (1+R_{F,t})\left(\frac{(1-b_t)Dep_t}{D_t} + \frac{m_t(1-a_t)}{a_t}\right)\right]$$

when $\underline{R}_{SD,t} \leq \underline{R}_{S,t}$, *and* $(1+\underline{R}_{S,t})$ *when* $\underline{R}_{SD,t} > \underline{R}_{S,t}$.

Proof: Appendix 6.2. The table below displays the different situations that the deposit bank can meet, depending on the value of the return on securities at the end of period t.

$\underline{R}_{SD,t}$	$\underline{R}_{S,t}$	
Default	Loss	Normal time
Equity ≤ 0	Interbank loan < 0	Cash ≥ 0
		Interbank loan $= D_t$
(Case 3)	(Case 2)	(Case 1)

The optimization program of the deposit bank is to maximize its expected return on equity with respect to the amount of cash (b_t) and the haircut (m_t):

$$\max_{\{b_t,m_t\}} \quad E_t[N_{D,t+1}/N_{D,t}] = E_t[1+\tilde{R}_{N_D,t}].$$

2.7 Equilibrium Interest Rate on Deposits

Assume that the merchant bank defaults ($\tilde{R}_{S,t} \leq \underline{R}_{S,t}$) and that the deposit bank cannot repay all deposits ($\tilde{R}_{S,t} \leq \underline{R}_{SD,t}$). In such a case, the deposit bank defaults and the value of deposits that can be repaid at $t+1$ is:

$$\begin{aligned} Dep_{t+1} &= (1+\tilde{R}_{D,t})D_t + (1+R_{F,t})(1-b_t)Dep_t \\ &= \left[(1-\theta)(1+\tilde{R}_{S,t}) + \frac{m_t(1-a_t)}{a_t}\frac{1+R_{F,t}}{1+m_t} - \frac{R_{D,t}}{1+m_t}\right]S_t \\ &\quad +(1-b_t)(1+R_{F,t})Dep_t. \end{aligned}$$

Therefore, the expected value of deposits in case of a deposit bank default is:

$$\begin{aligned} E_t[Dep_{t+1}^{(DBD)}] &= E_t[Dep_{t+1} \mid \text{Deposit bank default }] \\ &= \left[(1-\theta)(1+\mu_{S,t}^{(DBD)}) + \frac{m_t(1-a_t)}{a_t}\frac{1+R_{F,t}}{1+m_t} - \frac{R_{D,t}}{1+m_t}\right] S_t \\ &\quad +(1-b_t)(1+R_{F,t})Dep_t, \end{aligned}$$

where $\mu_{S,t}^{(DBD)} = E_t[\tilde{R}_{S,t} \mid \text{Deposit bank default }] = E_t[\tilde{R}_{S,t} \mid \tilde{R}_{S,t} \leq \underline{R}_{SD,t}]$.

The expected value of deposits at the end of period t is therefore:

$$E_t[Dep_{t+1}] = (1+R_{Dep,t})Dep_t \times \Pr[\tilde{R}_{S,t} > \underline{R}_{SD,t}] + E_t[Dep_{t+1}^{(DBD)}] \times \Pr[\tilde{R}_{S,t} \leq \underline{R}_{SD,t}].$$

Depositors benefit from a deposit guarantee from the government. The deposit bank has to pay an insurance premium to the government that covers the expected loss (after payment of the interest on deposits) in case of default. Therefore, we have $R_{Dep,t} = R_{F,t} + IP_{Dep,t}$, where the insurance premium is defined as:

$$\begin{aligned} IP_{Dep,t} &= -E_t\left[\frac{Dep_{t+1}}{Dep_t} - (1+R_{Dep,t})\right] = -E_t\left[\frac{Dep_{t+1}^{(DBD)}}{Dep_t} - (1+R_{Dep,t})\right]\Phi_S(\underline{R}_{SD,t}) \\ &= -\Bigg\{\left[(1-\theta)(1+\mu_{S,t}^{(DBD)}) + \frac{m_t(1-a_t)}{a_t}\frac{1+R_{F,t}}{1+m_t} - \frac{R_{D,t}}{1+m_t}\right]\frac{S_t}{Dep_t} \\ &\quad +(1-b_t)(1+R_{F,t}) - (1+R_{Dep,t})\Bigg\}\Phi_S(\underline{R}_{SD,t}). \end{aligned} \tag{8}$$

It should be noticed that, once deposits are guaranteed by the government, the value of the guaranteed deposits at $t+1$ will be ultimately $Dep_{t+1}^{(guar)} = (1+R_{Dep,t})Dep_t$.

At the equilibrium, the dposit bank optimally determines the amount of cash (b_t^*) and the haircut (m_t^*), which have to be consistent with the equilibrium deposit rate, i.e., $R_{Dep,t}$ should satisfy:

$$R_{Dep,t} = R_{F,t} + IP_{Dep,t} \tag{9}$$

$$\begin{aligned} &= R_{F,t} - \Bigg\{\left[(1-\theta)(1+\mu_{S,t}^{(DBD)}) + \frac{m_t^*(1-a_t^*)}{a_t^*}\frac{1+R_{F,t}}{1+m_t^*} - \frac{R_{D,t}}{1+m_t^*}\right]\frac{S_t}{Dep_t} \\ &\quad +(1-b_t^*)(1+R_{F,t}) - (1+R_{Dep,t})\Bigg\}\Phi_S(\underline{R}_{SD,t}). \end{aligned} \tag{10}$$

Therefore, the equilibrium deposit rate $R_{Dep,t}$ is determined as a fixed point of this equation. Thus, the optimal leverage of the deposit bank (through b_t^*), the haircut (m_t^*), and the equilibrium deposit rate are determined simultaneously for a given risk-free rate ($R_{F,t}$).

2.8 Dynamics of Bank's Equity

As discussed above, banks can default. This raises the issue of the dynamics of the number of banks and of the aggregate equity of the banks. In principle, the firms do not default, as they have no debt. If a merchant bank defaults, its assets are seized by the deposit bank. As the deposit bank has no particular skill at monitoring firms, we assume that it liquidates the assets of the firm at a liquidation cost θ.

We assume a stationary equilibrium, in which defaulting banks are replaced by new ones. As stockholders, households finance the new banks. The proportion of merchant banks that default is $\psi_{M,t} \equiv \Pr[\tilde{R}_{S,t} \leq \underline{R}_{S,t}]$ and the proportion of deposit banks that default is $\psi_{D,t} \equiv \Pr[\tilde{R}_{S,t} \leq \underline{R}_{SD,t}]$. We also denote $\delta_{M,t}$ and $\delta_{D,t}$ the dividend payout of the merchant banks and deposit banks, respectively. See Gertler and Karadi (2011).

We also assume that the new banks are funded with the same equity as the continuing ones. The aggregate value of the equity of the continuing banks is, generically for merchant and deposit banks:

$$N_{t+1}^{Cont} = (1 - \psi_t)[(1 - \delta_t)(1 + \tilde{R}_{N,t})N_t],$$

where ψ_t and δ_t denote the aggregate default rate and dividend payout. The aggregate value of the equity of the new banks is therefore:

$$N_{t+1}^{New} = \psi_t[(1 - \delta_t)(1 + \tilde{R}_{N,t})N_t],$$

so that the aggregate value of the equity of all banks is:

$$N_{t+1} = N_{t+1}^{Cont} + N_{t+1}^{New} = (1 - \delta_t)(1 + \tilde{R}_{N,t})N_t.$$

Households will receive $(1-\psi_t)\delta_t(1+\tilde{R}_{N,t})N_t$ in dividends from the continuing banks and will invest $\psi_t(1-\delta_t)(1+\tilde{R}_{N,t})N_t$ in the new banks equity, so that their net revenue is $(\delta_t-\psi_t)(1+\tilde{R}_{N,t})N_t$. For simplification, we assume that $\delta_t=\psi_t$. In other words, households are not expected to receive dividends after financing new banks. To have a balanced growth of merchant bank and deposit bank equity, we assume that $(1-\delta_{M,t})(1+E[\tilde{R}_{N_M,t}])=(1-\delta_{D,t})(1+E[\tilde{R}_{N_D,t}])$.

2.9 Authorities, Regulation, and Market Clearing

At time t, the government buys a fraction of the final goods, G_t. It reimburses the principal of its debt from the previous period, B_{t-1}, and issues new bonds with one-period maturity. Government bonds are held by merchant banks and deposit banks as cash ($B_t=C_{M,t}+C_{D,t}$). As government debt is risk-free, it yields interest rate $R_{F,t-1}$. The budget constraint in period t is given by:

$$G_t+T_t+(1+R_{F,t-1})B_{t-1}+NCDI_t\leq[\tau_c C_t+\tau_s W_t H_t]+B_t,$$

where the dynamics of the government spending ratio, $g_t=G_t/Y_t$, is a stationary AR(1) process. $NCDI_t$ denotes the net cost of deposit insurance, defined as:

$$NCDI_t=-(Dep_{t+1}^{(DBD)}-(1+R_{Dep,t})Dep_t)\mathbb{1}_{\{\tilde{R}_{S,t}<\underline{R}_{SD,t}\}}-(IP_{Dep,t}Dep_t).$$

The first term corresponds to the expenses in case of a default of the deposit bank, whereas the second term is the insurance premium received by the government each period.

In our model, as inflation does not play any role, it is not necessary to have a monetary policy rule that would target the inflation rate. The risk-free rate ($R_{F,t}$) is in fact determined by the equilibrium between short-term bonds issued by the government and bought by the bank. Even if the model does not rely on conventional monetary policy, the central bank may play a key role in the crisis by injecting liquidity. For instance, the central bank may inject credit in response to movements in credit spreads, as in Gertler and Karadi (2011). Another aspect, which we are going to focus on, is regulation, i.e., in our context, how to mitigate systemic risk. Among the instruments available to the central bank to reduce risks in the banking system, we may mention: imposing a minimum of equity (capital requirement), a minimum of cash (liquidity constraint), or a maximum leverage (leverage constraint). We will consider this issue in Section 4.3.

At equilibrium, all the markets clear, resulting in the following relations:

- Final goods market-clearing condition: $Y_t = C_t + G_t + I_t$.
- Labor market-clearing condition: $H_t = L_t$.
- Government bond market-clearing condition: $B_t = C_{M,t} + C_{D,t}$.

3 Model Evaluation

3.1 Calibration

Our parameter set consists of structural parameters, which describe the behavior of the various agents, and quantities, which describe the balance sheet of the agents. Most of the structural parameters are related to the real side of the model, reflecting decisions of households, firms, and authorities. All calibrated parameters are summarized in Table 2. Our calibration of macro parameters is based on previous literature, in particular Gertler and Kiyotaki (2010) and Christiano, Rotto, and Rostagno (2010). Parameters describing households and firms are all drawn from Gertler and Kiyotaki (2010). In particular, we assume that the elasticity of substitution of consumption and labor supply are $\sigma = 1$ and $\zeta = 0.333$, respectively, and the habit formation parameter is $\gamma = 0.81$. Also the effective share of capital is equal to $\alpha = 0.33$ and the steady-state depreciation rate is $\delta = 2.5\%$ per quarter. As regards capital producers, we take the adjustment cost parameter $\kappa = 26.64$ from Christiano, Rotto, and Rostagno (2010). Tax rates on consumption and labour income (equal to $\tau_c = 5\%$ and $\tau_s = 24\%$, respectively) are from Christiano, Rotto, and Rostagno (2010). The steady-state share of government expenditures ($\bar{g} = 20\%$) is common to Gertler and Kiyotaki (2010) and Christiano, Rotto, and Rostagno (2010).

[Insert Table 2 here]

As we calibrate the model, we introduce a limited number of shocks in the model, related to the household's preference (v_t), the total factor productivity (Z_t), the government spending ratio (g_t), and the firm-specific quality shock ($\varepsilon_t(i)$). The first three variables have an autoregressive representation in logs:

$$\begin{aligned} \log v_t &= \rho_v \log v_{t-1} + \eta_{v,t}, \\ \log Z_t &= \rho_Z \log Z_{t-1} + \eta_{z,t}, \\ \log g_t &= (1-\rho_g)\log \bar{g} + \rho_g \log g_{t-1} + \eta_{g,t}, \end{aligned}$$

where the shocks are iid and normally distributed: $\eta_{x,t} \sim N(0, \sigma_x)$, for $x = v, z, g$. Parameters for the persistence and volatility of the preference, productivity, and government shocks are all drawn from Christiano, Rotto, and Rostagno (2010). The firm-specific quality shock $\varepsilon_t(i)$ is independent over time and across firms and log-normally distributed: $\log(\varepsilon_t(i)) \sim N(-\frac{1}{2}\sigma_\varepsilon^2, \sigma_\varepsilon)$. Christiano, Rotto, and Rostagno (2010) and Nuño and Thomas (2013) consider a time-varying variance of the firm-specific shock to capture the effect of an increase in risk on the capital requirement of the banks. In this context, the volatility $\sigma_{\varepsilon,t}$ plays the role of a risk shock on firms. In our framework, we view σ_ε as a parameter of the model and we will consider the effect of a change in the volatility on the optimal strategy of the banks.

For the finance part of the model, there are two key parameters, namely the cost of fire sales (ϕ) and the liquidation cost (θ). The cost of fire sales is difficult to calibrate because it depends a lot on the urgency of the sales. Several papers have investigated the fire sales process, such as Coval and Stafford (2007), Shleifer and Vishny (2011), or Cont and Wagalath (2012) and, recently, Duarte and Eisenbach (2013). We assume a median value of 6.5% and consider a range between 5% and 8%. Measures of the liquidation cost are also very scarce. Acharaya, Sundaram, and John (2005) assume a value of 25%, whereas Garlapi, Shu, and Yan (2005) use a value of 30%. We consider a median value of 30% and a range between 20% and 40%.

The other parameters of the model are determined to match some key observable variables. First, the risk-free rate, $R_{F,t}$, is usually measured using the interest rate on Federal Fed Funds or the 3-month Treasury-bill rate. It has experienced a dramatic fall over the last decades. Using the last 10 years, the average real interest rate is equal to 1.5% and 1.3% per year, when measured using the Federal Fed Funds or T-bills. We calibrate the discount factor as $\beta = 0.99625$, which corresponds to an annualized discount rate of 1.5%. Second, the properties of the return on securities (μ_S and σ_S) are based on the S&P500 over a relatively long period of time (the last 10 years, from 2004 to 2013). Once deflated by the consumer price index, the average return (including dividend payments) is 5% per year (7.5% in nominal terms minus 2.4% of average annual inflation). The average annual volatility of the S&P500 over the same period is 16.7%. We expect the equilibrium to imply $\mu_S = 1.25\%$ per quarter and $\sigma_S = 8\%$ per quarter.

Last, we calibrate the value of securities issued by firms and held by the merchant bank and the value of deposits by households to the deposit bank. For this purpose, we rely on the Financial National Accounts of the United States (Federal Reserve System, 2015). The value of equity of nonfinancial business held by the domestic financial sector is equal to USD $17,002$ billion as of end of 2014. The value of deposits (including checkable, time and savings deposits, and money market fund shares) held by households is equal to USD $10,231$ billion. Therefore, deposits represent approximately 60% of business equity. These numbers suggest that the ratio of securities held by the financial system to the deposits (S_t/Dep_t) is approximately 1.66 in 2014. Over the last 5 years, this ratio is equal to 1.46 on average.

Regarding the assets and liabilities of financial institutions, we have already used data from security brokers and dealers to measure the leverage of merchant banks and data from commercial banks to measure the leverage of deposit banks. In fact, it should be mentioned that most large banks are involved in both investment banking and commercial banking activities. In addition, several large banking institutions are now organized according to a bank holding company. This recent trend is partly due to the reorganization of several large banks through mergers and acquisitions during the subprime crisis (Avraham, Selvaggi, and Vickery, 2012). However, even if this evolution makes the differentiation between the different types of banks difficult to quantify, the two types of activities (collecting deposits vs. financing investment projects) remain. Another difficulty is that our simplified balance sheets do not include several important components of the activities of banks (in particular, loans and bonds). For this reason, our estimates of the leverage of the merchant and deposit banks are not directly comparable to actual measures.

3.2 Impulse Response Analysis

To evaluate the dynamic properties of the model, we investigate the response of the model to shocks. To keep the model as simple as possible and focus on the financial mechanisms, we have introduced only four shocks, all reflecting macroeconomic relations, i.e., the productivity, preference, and government shocks, and the firm-specific quality shock. To illustrate the main mechanisms at play in the model, we comment the impulse responses to a shock on the total factor productivity, so that our results can be compared with previous research, such as Christiano, Motto, and Rostagno (2010), Dib (2010), Angeloni and Faia (2013), and Nuño and Thomas (2013).

In Figures 1 and 2, we display the impulse responses of the main macro variables of the model to a positive, one standard deviation, productivity shock. On impact, the increase in productivity induces an increase in the marginal productivity of capital ($R_{K,t}$) and a decrease in hours worked (H_t). This in turn implies an increase in investment (through Tobin's q increase) and in consumption, so that total output increases.

On the finance side, after the initial positive impact of the productivity shock, the Tobin's q decreases, which results in a sharp decline in the return on assets ($\tilde{R}_{S,t}$) (through Equation (7)) and therefore on the interbank lending rate ($\tilde{R}_{D,t}$). In parallel, after an initial negative impact, the productivity shock implies an increase in the marginal utility of consumption (λ_t), which results in a sharp decline in the deposit rate ($R_{Dep,t}$) (through Equation (5)).

Both firm's securities and household's deposits increase on impact. As a consequence, the equity of the merchant bank increases so as to keep its leverage stable (at $1 + a_t^*/m_t^*$). In contrast, as after the initial impact deposits increase more than interbank loans, the equity of the deposit bank decreases and its leverage increases. The procyclicality of leverage was already emphasized by Adrian and Shin (2010, 2014). The probability of default of the deposit bank increases because of the higher leverage and so does the insurance premium paid by deposit banks to guarantee deposits.

[Insert Figures 1 and 2 here]

An important feature we want to investigate is how our model compares with a model without banks. For this purpose, we construct a model with exactly the same features as our main model but without banks. In this model, there are no financial frictions and the mechanisms by which banks finance their assets are not introduced. Essentially, it implies that the households directly buy the long-term claim on the firm's assets and hold risk-free government bonds. Therefore, the accumulation of capital is determined in the household optimization problem. We calibrate this model with the very same parameters as the main model. Figure 3 allows us to compare the response of the main variables of the model to a total factor productivity shock. The productivity shock implies a much larger increase in investment and consumption in the simple model compared to the complete model. In the absence of banks, the amount of securities issued by the firm is higher than in the presence of banks. Yet the return on capital is barely affected. These effects correspond to the frictions implied by the agency problems raised by financial sector. Similar patters were already obtained by Christiano, Motto, and Rostagno (2010).

[Insert Figure 3 here]

3.3 Equilibrium and Comparative Statics

In this section, we briefly present the equilibrium obtained under our benchmark case. Then, we discuss how the equilibrium is changed when we vary the value of the key finance parameters. These parameters are the following: the volatility of the securities return (σ_S), the cost of fire sales (ϕ), and the liquidation cost (θ). As explained in Section 2, the other parameters describing bank's behavior, i.e., the decisions to hold cash (a_t for the merchant bank and b_t for the deposit bank) and the haircut (m_t), are determined optimally by the banks. Therefore, they are obtained as the solution of the equilibrium model.

Benchmark case. We first notice that the haircut optimally determined by the deposit bank is equal to $m_t^* = 30\%$. This value is in the range of haircut values for equity (European Parliament, 2013). Now, if we consider the merchant bank, the optimal fraction of equity used as collateral is $a_t^* = 79\%$, meaning that the bank only keeps 21% of its equity in cash. Given the value of the haircut, it implies that the optimal leverage of the merchant bank is $1 + a_t^*/m_t^* = 3.6$. This value is relatively high, as the merchant bank does not receive deposits. It is mainly due to the relative importance of the funding gap (the ratio of securities over deposits is 1.66). As the leverage is high and the free cash limited, the probability of default of the merchant bank is 2.7%, which is high compared to actual numbers.[13] As a consequence, the equilibrium interest rate on interbank loans is $R_{D,t} = 3.1\%$, meaning that the risk premium is 1.6%. The threshold of return on securities under which the merchant bank has to sell some of its assets with short notice (fire sales) is $\overline{R}_{S,t} = -4\%$ and the threshold under which it defaults is $\underline{R}_{S,t} = -22.4\%$. Given the expected return $\mu_{S,t} = 5\%$ and the leverage used by the bank, we find an expected return on the merchant bank equity equal to 7.8%.

Regarding the situation of the deposit bank at the equilibrium, we find that it does not hold cash ($b_t^* = 1$). The reason for this result is twofold. First, holding cash is costly for the bank because the risk-free rate is lower than the return on interbank loans, so that the deposit bank prefers to hold loans. Second, one may expect that the bank should hold free cash to avoid a default in case of default from the merchant bank (exactly as it is optimal for the merchant bank to hold cash in case of a depreciation of the securities of the firm). However, this second effect does not compensate the first effect, because a default of the merchant bank almost systematically triggers a default of the deposit bank, due to the large implied loss. Therefore, the deposit bank prefers to hold less cash and make higher profit in normal time and have a higher probability of default when the merchant bank defaults. Given the haircut optimally selected by the deposit bank, its leverage is also relatively high (3.5%). Yet, due to the high probability of default (2.7%), the insurance premium on deposits is also rather high ($IP_{Dep,t} = 0.15\%$). This premium exactly compensates households for the expected loss on deposits due to the possible default of the deposit bank. In other words, the insurance premium accounts for the expected loss, and the government will have to cover the unexpected loss in case of a crisis. The expected loss is equal to 0.18, which corresponds to 0.11% of the value of

[13] We should keep in mind that we do not consider any regulation for the moment and that this number does not take the possibility of recapitalization or bailout into account.

the assets or 0.15% of the value of the deposits (hence the value of the risk premium). The expected return on deposit bank equity is equal to 5.1%, which is consistent with the return on loans of 3.1% and the return on deposits of 1.65%.

Balance sheet of the merchant bank at equilibrium

	Assets	Liabilities	
($R_F = 1.5\%$)	Cash: 13	Interbank loan: 154	($R_D = 3.1\%$)
($R_S = 5\%$)	Securities: 200	Equity: 59	($R_{N_M} = 7.8\%$)

Balance sheet of the deposit bank at equilibrium

	Assets	Liabilities	
($R_F = 1.5\%$)	Cash: 0	Deposits: 120	($R_{Dep} = 1.65\%$)
($R_D = 3.1\%$)	Interbank loan: 154	Equity: 34	($R_{N_D} = 5.1\%$)

Change in the volatility of securities (σ_S). We now consider the effect of changing the key parameters of the model. The main effect of an increase in volatility, from 12% to 20% (the benchmark value is 16%), is an increase in the haircut set by the deposit bank (from 20% to 50%) (see Figure 4). As a result, the merchant bank borrows less on the interbank market and therefore its leverage decreases (from 4.7 to 2.9). In other words, the deposit bank forces the merchant bank to be less risky by deleveraging. This mechanism is consistent with the leverage constraint model (Geanokoplos and Fostel, 2008, Adrian and Shin, 2010, 2014), which results in a procyclical leverage dynamics in the crisis. A consequence is that the return on interbank loans also decreases (from 3.3% to 2.4%).

As the deposit bank has less access to equity financing in a riskier environment, its leverage increases (from 3.5 to 8.9) when the volatility of securities increases from 12% to 20%. The increase in the deposit bank leverage in the crisis is consistent with the theoretical implications of Xiong (2001) and the empirical evidence reported in the introduction. Due to the higher leverage, the expected loss of the deposit bank increases.

This also implies an increase in the return on deposits paid by the deposit bank (from 1.5% to 2%) due to the higher insurance premium. This mechanism is consistent with the equity risk-capital constraint model (He and Krishnamurthy, 2012 and 2013). Such a contrast in the dynamics of the leverage of merchant and deposit banks has been described by He, Khang, and Krishnamurthy (2010).

Change in the cost of fire sales (ϕ). When the cost of fire sales is low, the merchant bank optimally does not hold cash. However, the deposit bank imposes a high haircut ($m_t^* = 50\%$) to avoid the merchant bank to take too much risk (Figure 5). As a consequence, the leverage and probability of default of the merchant bank are low, so that the return on interbank loan is low. In contrast, the deposit bank issues limited equity and mostly relies on leverage. Therefore, it has to pay a 0.1% insurance premium on its deposits. When the cost of fire sales increases from 4% to 8% (the benchmark value is 6%), the merchant bank holds more free cash and the deposit bank decreases the haircut, so that the leverage of the merchant bank increases to 3.6. The probability of default of the merchant bank increases to 3% but the probability of default of the deposit bank remains low (at 0.4%), so that the insurance premium on deposits is negligible.

Change in the liquidation cost (θ). When the cost of liquidation of the merchant bank assets increases, from 20% to 40% (the benchmark value is 30%), the main effect on the equilibrium is the increase in the return on interbank loans (from 2.3% to 3.2%) (Figure 6). The reason is that, in case of a default of the merchant bank, the deposit bank will recover a smaller fraction of its assets. To mitigate this risk, the deposit bank issues more equity and therefore decreases its leverage (from 5.4 to 4.4). Nevertheless, as the probability of defaults of the deposit bank increases (from 1.6% to 1.9%), the insurance premium on deposits increases from 0.1% to 0.5%.

[Insert Figures 4 to 6 here]

4 Systemic Risk

In this section, we focus on two important aspects of the model. First, we describe how to measure systemic risk in this context. Second, we compare this measure to the widely-discussed Systemic Risk (SRISK) measure proposed by Acharya et al. (2012) and Brownlees and Engle (2012).

4.1 Measuring Systemic Risk

We start with the measure of systemic risk implied by our model. In Section 2.7, we have computed the expected loss on deposits implied by the possible default of the deposit banks to determine the insurance premium. The measure shown in Equation (8) is an "unstressed" measure of risk as it is not obtained in a period of stress for the financial system. We recall this measure here:

$$\begin{aligned} IP_{Dep,t} \quad = \quad & -\Bigg\{ \left[(1-\theta)(1+\mu_{S,t}^{(DBD)}) + \frac{m_t^*(1-a_t^*)}{a_t^*}\frac{1+R_{F,t}}{1+m_t^*} - \frac{R_{D,t}}{1+m_t^*} \right] \frac{S_t}{Dep_t} \\ & + (1-b_t^*)(1+R_{F,t}) - (1+R_{Dep,t}) \Bigg\} \Phi_S(\underline{R}_{SD,t}), \end{aligned}$$

where $\mu_{S,t}^{(DBD)} = E_t[\tilde{R}_{S,t} \mid \text{Deposit bank default }] = E_t[\tilde{R}_{S,t} \mid \tilde{R}_{S,t} \leq \underline{R}_{SD,t}]$.

Following the approach proposed by Acharya et al. (2012) and Brownlees and Engle (2012), we define systemic risk as the propensity of a financial firm to be undercapitalized when the financial system as a whole is undercapitalized. This definition corresponds to a "stressed" version of the expected loss above, i.e., in case of a financial crisis. In our model, the crisis naturally corresponds to a shock on the expected return on investment, $\mu_{S,t}$.

To measure systemic risk in this model, we proceed as follows: Parameters ϕ and θ are fixed. The economy is at the equilibrium at the end of date $t-1$. The equilibrium returns ($R_{F,t}$, $R_{D,t}$, and $R_{Dep,t}$) are determined for the next period. At the beginning of period t, banks optimally determine their balance sheet (a_t^*, b_t^*, and m_t^*). The various quantities (S_t, $C_{M,t}$, D_t, $C_{D,t}$, and Dep_t) and thresholds ($\overline{R}_{S,t}$, $\underline{R}_{S,t}$, and $\underline{R}_{SD,t}$) are also determined. Then, a shock hits the economy, corresponding to a shift in the distribution of the firm's return on investment across firms, $\tilde{R}_{S,t}$. Therefore, the probability for $\tilde{R}_{S,t}$ to belong to a given interval corresponds to the new distribution $N(\mu_S^{(Crisis)}, \sigma_S)$.[14]

Definition 1 *The "stressed" expected loss on deposits, $SEL_t = IP_{Dep,t}^{(Crisis)} Dep_t$, is defined as:*

$$
\begin{aligned}
SEL_t \quad = \quad & -\Bigg\{ \Bigg[(1-\theta)(1+m_t^*)(1+\mu_{S,t}^{(DBD|Crisis)}) + \frac{m_t^*(1-a_t^*)}{a_t^*}(1+R_{F,t}) - R_{D,t} \Bigg] D_t \\
& +(1+R_{F,t})C_{D,t} - (1+R_{Dep,t})Dep_t \Bigg\} \Pi_t^{(DBD|Crisis)}. \qquad (11)
\end{aligned}
$$

where the expected return on securities ($\mu_{S,t}^{(DBD|Crisis)} = E_t[\tilde{R}_{S,t} \mid \tilde{R}_{S,t} \leq \underline{R}_{SD,t}; Crisis\ ]$) and the probability of default of the deposit bank ($\Pi_t^{(DBD|Crisis)} = \Pr[\tilde{R}_{S,t} \leq \underline{R}_{SD,t} \mid Crisis\ ]$) are computed using the distribution of $\log(1+\tilde{R}_{S,t})$, i.e., $N(\mu_S^{(Crisis)}, \sigma_S)$ in the crisis. See Appendix 1 for details on how to compute these expressions.

Several elements contribute to the stressed expected loss. First, the SEL is the difference between the expected market value of the assets under stress and the book value of deposits. It should be noticed that the book value of deposits corresponds to an unstressed expected value because the value that has to be repaid is not affected by the crisis. Second, the expected value of the assets held by the deposit bank under stress has two components: the expected value of the interbank loan and the value of cash. The former is subject to a loss in the crisis, whereas the latter is not affected.

As the term in squared brackets in Equation (11) indicates, the expected return of the interbank loan under stress depends on the assets left by the defaulting merchant bank: (1) the market value of the securities held by the merchant bank incurs a loss due to the crisis ($\mu_{S,t}^{(DBD|Crisis)}$) and is further reduced by the liquidation cost; (2) the value of the cash held by the merchant bank depends on the amount of free cash put aside by the bank ($1 - a_t^*$) and on the fraction of collateral (m_t^*) that has to be posted for borrowing money from the deposit bank.[15]

[14] We also consider below an increase in the volatility of the shock, σ_S.

[15] We also notice that the value of the assets seized by the deposit bank in case of default of the merchant bank is decreased by $R_{D,t}$, because interests on loans are supposed to be paid before the default. This interest rate also incorporates the unstressed expected loss by the deposit bank, as it includes the risk premium due to the possible default of the merchant bank.

A simple way to defined the stressed expected loss is:

$$SEL_t = E_t[Dep_{t+1}^{(guar)} - Assets_{D,t+1}^{(DBD|Crisis)}]\ \Pi_t^{(DBD|Crisis)},$$

where $Assets_{D,t+1}^{(DBD|Crisis)}$ is the marked-to-market value of the assets of the deposit bank if it defaults in the crisis, $Dep_{t+1}^{(guar)}$ is the book value of the deposits guaranteed by the deposit insurance, and $\Pi_t^{(DBD|Crisis)}$ is the probability of default of the deposit bank in the crisis.

It is interesting to compare our measure of SEL with the SRISK measure proposed by Acharya et al. (2012) and Brownlees and Engle (2012). The aggregate SRISK is the positive value of the capital shortfall of the banking industry in a crisis, defined as:[16]

$$SRISK_t = E_t[\vartheta Assets_{D,t+1} - Equity_{D,t+1} \mid \text{Crisis }], \tag{12}$$

where the crisis is defined as a large loss in the stock market, and ϑ represents a prudential ratio of equity to assets. Equation (12) can be rewritten as:

$$SRISK_t = E_t[Dep_{t+1}^{(guar)} - (1-\vartheta) Assets_{D,t+1}^{(Crisis)}], \tag{13}$$

where we have used $Assets_{D,t+1} = Dep_{t+1}^{(guar)} + Equity_{D,t+1}$, as the deposit bank has no debt other than deposits. In Equation (13), $Assets_{D,t+1}^{(Crisis)}$ is marked-to-market value of the deposit bank in the crisis and $Dep_{t+1}^{(guar)}$ is the book value of the deposits.

Although the two specifications are closely related, there are two main differences between these measures. First, one difficulty with the definition of the SRISK is the choice of the threshold ϑ. The level of ϑ can be viewed as a decision made by the regulator that is imposed to all banks. But in principle the fraction of assets ($\vartheta Assets_{D,t+1}$) under which a given bank is undercapitalized should be bank specific. In such a case, the determination of ϑ is a much more complicated task. Our measure SEL does not rely on any threshold

[16] More precisely, Acharya et al. (2012) define the SRISK of bank i as $E_t[\vartheta Assets_{i,t+1} - Equity_{i,t+1} \mid Equity_{B,t+1} \leq \vartheta Assets_{B,t+1}]$, where $Equity_{B,t+1}$ and $Assets_{B,t+1}$ denote the aggregate equity and assets of the banking industry, respectively. The condition $Equity_{B,t+1} \leq \vartheta Assets_{B,t+1}$ means a situation when the banking industry as a whole is undercapitalized. Brownlees and Engle (2012) then use this expression by assuming that the condition $Equity_{B,t+1} \leq \vartheta Assets_{B,t+1}$ corresponds to a severe stock market crash (40% in the next 6 months).

because we specifically focus on the cost of a default, i.e., on the lack of assets to repay the deposits. Second, the SRISK considers the cost of a bank undercapitalization. In a crisis, a bank may have difficulties to find funds on the interbank market. The SEL explicitly considers situations where the bank is not able to repay its debt and therefore defaults. The difference is thus that *Assets* in SRISK is the market-to-market value conditional on a crisis, whereas *Assets* in SEL is the market-to-market value conditional on a default in the crisis.

4.2 Stressed Expected Loss

As mentioned above, the (unstressed) expected loss on deposits due to the possible default of the deposit bank is low in our calibration. It is equal to 0.18, i.e., 0.12% of the value of the ex-ante assets (or 0.5% of the ex-ante equity). To investigate the impact of a crisis on our measure of systemic risk, SEL_t, defined in Equation (11), we consider a range of values of return on securities that will define the magnitude of the crisis: the expected return in the crisis, $\mu_S^{(Crisis)}$, ranges between 0 and -40%, and therefore covers a wide range of crisis magnitudes.

In the benchmark case, the probability of default of the deposit bank increases from 6.5% (when the crisis is defined as just a mere 0% return) to 86.7% (when the crisis is defined as a -40% market crash). The resulting stressed expected loss therefore increases from 1.5% of the ex-ante assets (for $\mu_S^{(Crisis)} = 0$) to 17.8%, or 80.9% of the ex-ante equity (for $\mu_S^{(Crisis)} = -40\%$). These numbers are admittedly large, but it should be noticed that they correspond to an environment with no regulation. This issue will be addressed in the next section.

In Figures 7 to 9, we report the probability of default of the deposit bank and the stressed expected loss when we vary the key parameters. These figures allow us to more clearly identify the main drivers of systemic risk in the economy. First, an increase in the volatility of securities return implies a significant increase in the expected loss in case of a crisis (Figure 7). When σ_S is increased, the deposit bank increases its haircut to reduce the risk borne by the merchant bank. As a consequence, the probability of default of the merchant and deposit banks is reduced. However, the larger loss in case of a crisis is not fully compensated by the lower probability of default. Ultimately, the stressed expected loss is increased.

Second, when the cost of fire sales increases, the merchant bank increases its free cash to avoid fire sales (a_t^* decreases). This reduces the extent of the loss in case of a crash. The expected loss after a -40% market crash decreases from 27.5% to 11% of the ex-ante assets when ϕ increases from 4% to 8% (Figure 8). As the figure also shows, increasing again the cost of fire sales would not significantly decrease the SEL.

Third, the role played by the liquidation cost, θ, is different from the one played by the cost of fire sales (Figure 9). When the liquidation cost increases, the decisions of the merchant bank are almost unchanged, except through the increase in the return on interbank loans. However, as a default occurs, the loss is proportional to the liquidation cost, so that overall the increase in the liquidation cost implies an increase in the expected loss. The expected loss after a -40% market crash increases from 17.5% to 23% of the ex-ante assets when θ decreases from 20% to 40%. As before, decreasing the liquidation cost does not significantly decrease the SEL.

[Insert Figures 7 to 9 here]

4.3 Regulation

In the previous section, we have found that, under our parameterization, the stressed expected loss is relatively large when the market fall is large. It can be as high as 20% of the ex-ante assets. In this section, we discuss the possible impact of regulation in reducing systemic risks in the banking system: more specifically, we consider the case of imposing a minimum amount a cash to the deposit bank (liquidity requirement). In our model, this is equivalent to imposing a minimum of equity (capital requirement) or a maximum leverage (leverage constraint). As already discussed, in the model it is always optimal for the deposit bank to hold no cash because the cost of financing through deposits (including the insurance premium) is higher than the risk-free rate. As the bank does not borne the cost of default, there is no market mechanism to incentivize the bank to hold cash.

In Figure 10, we consider the case where the bank is forced to hold a minimum of cash. For this purpose, we decrease the value of b_t from 1 (no cash, corresponding to a leverage of 4.3) to 0.6 (60% of deposits in cash, corresponding to a leverage of 3.1). In doing so, the bank's expected return on equity decreases from 4.4% to 2.8%, showing that decreasing b_t is not optimal from the deposit bank viewpoint. However, as the figure shows, increasing free cash has a considerable effect on both the probability of default and the SEL. In the case of the 40% market crash, the probability of default is reduced from 87% for $b_t = 1$ to 13% for $b_t = 0.6$. In parallel, the SEL is reduced from 17.8% to 3.5% of the value of the assets.

As this exercise illustrates, imposing a minimum amount of cash to the deposit bank seems an effective way to reduce the risk of default of the bank and therefore to mitigate systemic risk.

[Insert Figure 10 here]

5 Conclusion

In this paper, we have described a macro model, in which banks can endogenously default. The amount of cash held by the banks and the extent of leverage are optimally determined. The merchant bank can default when the value of its assets, used as collateral, falls below a certain threshold. Subsequently, the deposit bank can default when the loss on the interbank loan exceeds a certain threshold. The risk premium paid by the merchant bank on its interbank loan and the insurance premium paid by the deposit bank on its deposits are determined at the equilibrium.

This framework allows us to define systemic risk based on the stressed expected loss on deposits, which corresponds to the cost for the taxpayer of a default of the deposit bank. The calibration of the model, based on U.S. estimates, results in a SEL in case of a 40% market crash as high as 18% of the ex-ante assets (or 81% of the ex-ante equity) of the deposit bank. This evaluation is sensitive to some key parameters (such as the volatility of the return on capital, the cost of fire sales, or the liquidation cost). However, changing these parameters is unlikely to decrease significantly the SEL. In contrast, imposing a minimum of free cash to the deposit bank allows for a considerable reduction in the SEL.

6 Appendices

6.1 Appendix 1: Merchant Banks

Case 1: Normal time ($\tilde{R}_{S,t} > \overline{R}_{S,t}$). If the value of securities falls below the amount of collateral requested by the deposit bank, $S_{t+1} < (1+m)D_{t+1}$, the merchant bank has to post some cash as collateral.[17] In this case, the value of cash at $t+1$ is decrease to:

$$\begin{aligned}
C_{M,t+1} &= \underbrace{(1+R_{F,t})C_{M,t}}_{\text{interest received on cash}} - \underbrace{R_{D,t}D_t}_{\text{interest paid on debt}} - \underbrace{\left((1+m_t)D_t - (1+\tilde{R}_{S,t})S_t\right)}_{\text{margin call } (>0)} \\
&= \left[(1-a_t)(1+R_{F,t}) - \frac{a_t}{m_t}R_{D,t} + a_t\frac{1+m_t}{m_t}\tilde{R}_{S,t}\right] N_{M,t}.
\end{aligned}$$

Margin calls are financed using cash only if cash remains positive:

$$C_{M,t+1} > 0 \quad \Rightarrow \quad \tilde{R}_{S,t} > \frac{1}{1+m_t}\left[R_{D,t} - m_t\frac{1-a_t}{a_t}(1+R_{F,t})\right],$$

so that

$$1+\tilde{R}_{S,t} > \frac{m_t}{1+m_t} + \left(\frac{1}{1+m_t}(1+R_{D,t}) - \frac{1-a_t}{a_t}\frac{m_t}{1+m_t}(1+R_{F,t})\right) \equiv 1+\overline{R}_{S,t}.$$

Provided $\tilde{R}_{S,t} > \overline{R}_{S,t}$, there is enough free cash to face the margin call, $a_t\frac{1+m_t}{m_t}\tilde{R}_{S,t}N_{M,t}$. Then, the value of securities increases by the amount of cash used to increase collateral and invested in securities. We have:

$$S_{t+1} = (1+m_t)D_t = (1+\tilde{R}_{S,t})S_t + \underbrace{\left(-a_t\frac{1+m_t}{m_t}\tilde{R}_{S,t}N_{M,t}\right)}_{\text{margin call } (>0)}.$$

In this case, it is easy to verify that the expected equity is given by:

$$\begin{aligned}
E_t[N_{M,t+1}^{(NT)}] &= E_t[N_{M,t+1} \mid \text{Normal time }] \\
&= \left[(1+\mu_{S,t}^{(NT)})a_t\frac{1+m_t}{m_t} + (1+R_{F,t})(1-a_t) - (1+R_{D,t})\frac{a_t}{m_t}\right] N_{M,t},
\end{aligned}$$

[17]We could imagine that the bank prefers to keep collateral in cash. However, in normal time, the expected return of holding securities is higher than the risk-free rate. As the idiosyncratic return on securities is an iid process, it is optimal for the bank to invest its collateral in securities.

where $\mu_{S,t}^{(NT)} = E_t[\tilde{R}_{S,t} \mid \text{Normal time}\ ]$ denotes the expected return on securities in normal time.

Case 2: Fire sales ($\underline{R}_{S,t} < \tilde{R}_{S,t} \leq \overline{R}_{S,t}$). When $\tilde{R}_{S,t} \leq \overline{R}_{S,t}$, we find $C_{M,t+1} \leq 0$, meaning that there is not sufficient cash to cover the margin call. The merchant bank has to sell part of its assets (fire sale) to delever, reduce its debt, and satisfy the margin. In this case, debt should be reduced to maintain the relation:

$$S_{t+1} = (1 + m_t)D_{t+1}.$$

To reduce its debt, the bank uses the available cash and sells part of its securities ($S_{t+1} - (1 + \tilde{R}_{S,t})S_t$). We assume that, in case of fire sale, the merchant bank incurs a cost to sell securities with short notice. The cost is proportional to the value of securities sold, i.e., $-\phi(S_{t+1} - (1 + \tilde{R}_{S,t})S_t) > 0$. If the bank sells $(S_{t+1} - (1 + \tilde{R}_{S,t})S_t)$ of securities, its interbank debt is reduced by $(1 - \phi)(S_{t+1} - (1 + \tilde{R}_{S,t})S_t) > 0$. Eventually, the interbank debt at $t + 1$ will decrease to:

$$D_{t+1} = D_t + (1 - \phi)[\underbrace{S_{t+1} - (1 + \tilde{R}_{S,t})S_t}_{\text{fire sale } (<0)}] - [\underbrace{(1 + R_{F,t})C_{M,t} - R_{D,t}D_t}_{\text{use of cash } (<0)}].$$

Therefore, in addition to the initial loss due to the firm's low return, the value of securities held by the bank will further decrease due to the fire sale:

$$\begin{aligned}
S_{t+1} &= (1 + m_t)D_{t+1} \\
&= (1 + m_t)\left[(1 + R_{D,t})D_t + (1 - \phi)(S_{t+1} - (1 + \tilde{R}_{S,t})S_t) - (1 + R_{F,t})C_{M,t}\right] \\
&= \Psi_t\left[-(1 + R_{D,t})D_t + (1 - \phi)(1 + \tilde{R}_{S,t})S_t + (1 + R_{F,t})C_{M,t}\right] \\
&= \Psi_t\left[-\frac{a_t}{m_t}(1 + R_{D,t}) + a_t(1 - \phi)\frac{1 + m_t}{m_t}(1 + \tilde{R}_{S,t}) + (1 - a_t)(1 + R_{F,t})\right] N_{M,t}.
\end{aligned}$$

where $\Psi_t = (1+m_t)/((1+m_t)(1-\phi)-1)$ is positive if we assume $m_t > \phi$. We notice that the further reduction in securities is such that:

$$
\begin{aligned}
S_{t+1} - (1+\tilde{R}_{S,t})S_t &= \Psi_t\left[-(1+R_{D,t})D_t + (1+R_{F,t})C_{M,t}\right] - \left[\frac{(1+\tilde{R}_{S,t})S_t}{1-(1+m_t)(1-\phi)}\right] \\
&= \Psi_t\left[(1+R_{F,t})(1-a_t) + (1+\tilde{R}_{S,t})\frac{a_t}{m_t} - (1+R_{D,t})\frac{a_t}{m_t}\right] N_{M,t}.
\end{aligned}
$$

The additional decrease in securities value is used by the merchant bank to repay part of its debt $D_{t+1} - D_t$. At the end of period t, the deposit bank receives the interest and principal of its loan $(1+R_{D,t})D_t$. The new values S_{t+1} and D_{t+1} for next period are fine for both merchant and deposit banks because $S_{t+1} = (1+m_t)D_{t+1}$. It is worth emphasizing that, in the case of fire sales with no default, the merchant bank repays the principal and the interest of its debt to the deposit bank.

The fire sales imply a decrease in the value of the equity of the bank due to the induced cost $\phi(S_{t+1} - (1+\tilde{R}_{S,t})S_t) < 0$. The resulting value of equity is:

$$
\begin{aligned}
N_{M,t+1} &= (1+\tilde{R}_{N_M,t})N_{M,t} + \phi\left(S_{t+1} - (1+\tilde{R}_{S,t})S_t\right) \\
&= \left[(1+\tilde{R}_{S,t})a_t\frac{1+m_t}{m_t} + (1+R_{F,t})(1-a_t) - (1+R_{D,t})\frac{a_t}{m_t}\right] N_{M,t} \\
&+ \phi\Psi_t\left[(1+\tilde{R}_{S,t})\frac{a_t}{m_t} + (1+R_{F,t})(1-a_t) - (1+R_{D,t})\frac{a_t}{m_t}\right] N_{M,t} \\
&= \Psi_t\left[(1+\tilde{R}_{S,t})(1-\phi)a_t + (1+R_{F,t})\frac{m_t}{1+m_t}(1-a_t) - (1+R_{D,t})\frac{a_t}{1+m_t}\right] N_{M,t}.
\end{aligned}
$$

The merchant bank will survive if its deleveraging does not imply negative equity:

$$
N_{M,t+1} > 0 \quad \Rightarrow \quad (1+\tilde{R}_{S,t})(1-\phi)a_t + (1+R_{F,t})\frac{m_t}{1+m_t}(1-a_t) - (1+R_{D,t})\frac{a_t}{1+m_t} > 0,
$$

so that

$$
1+\tilde{R}_{S,t} > \frac{1}{(1-\phi)(1+m_t)}\left[(1+R_{D,t}) - (1+R_{F,t})\frac{m_t(1-a_t)}{a_t}\right] \equiv 1 + \underline{R}_{S,t}.
$$

We denote the expected equity of the merchant bank in case of fire sales by:

$$\begin{aligned} E_t[N_{M,t+1}^{(FS)}] &= E_t[N_{M,t+1} \mid \text{Fire sales }] \\ &= \Psi_t \left[(1+\mu_{S,t}^{(FS)})(1-\phi)a_t + (1+R_{F,t})\frac{m_t(1-a_t)}{1+m_t} - (1+R_{D,t})\frac{a_t}{1+m_t}\right] N_{M,t}, \end{aligned}$$

where $\mu_{S,t}^{(FS)} = E_t[\tilde{R}_{S,t} \mid \text{Fire sales }]$.

We notice that $\tilde{R}_{S,t}$ is the change in the value of firm's assets, not the change in the value of securities held by the merchant bank. In case of fire sales, the change in the value of securities held will also take the capital loss into account. Following Equation (7), the return on securities $\tilde{R}_{S,t}$ is log-normally distributed, such that $\log(1+\tilde{R}_{S,t}) \sim N(m_S, s_S)$, where m_S and s_S^2 are the mean and variance of the log-return on securities.[18]

As returns are log-normally distributed, probabilities and conditional expected returns are computed as follows: the probability to be below a given threshold ϑ, $\Pr[\tilde{R}_{S,t} < \vartheta]$, is defined as the cdf $LN(1+\vartheta; m_S, s_S)$. The expected return conditional on $\tilde{R}_{S,t}$ being below ϑ is defined as:

$$E_t[\tilde{R}_{S,t}|\tilde{R}_{S,t} < \vartheta] = \frac{\exp(m_S + s_S^2/2)\ \Phi((\log(1+\vartheta) - (m_S + s_S^2))/s_S)}{\Pr[\tilde{R}_{S,t} < \vartheta]}.$$

6.2 Appendix 2: Deposit Banks

Case 1: No default on deposit banks. The deposit bank is safe if the value of its assets is above the value of its deposits at $t+1$, or $N_{D,t+1} > 0$, so that:

$$1 + \tilde{R}_{D,t} > (1+R_{Dep,t})\frac{Dep_t}{D_t} - (1+R_{F,t})\frac{(1-b_t)Dep_t}{D_t} \equiv 1 + \underline{R}_{D,t}.$$

The threshold $\underline{R}_{D,t}$ depends on the fraction of the assets of the deposit bank kept in cash (b_t). In fact, the case with no bank default can happen under two situations: (1) the merchant bank does not default, or (2) the merchant bank defaults but the deposit bank has enough cash to pay back the deposits.

[18]We deduce that the expected return and variance of $\tilde{R}_{S,t}$ are $\mu_S = E_t[\tilde{R}_{S,t}] = \exp(m_S + s_S^2/2) - 1$ and $\sigma_S^2 = V_t[\tilde{R}_{S,t}] = \exp(2m_S + s_S^2)(\exp(s_S^2) - 1)$, respectively. Our calibration is based on μ_S and σ_S^2 but our computations are based on m_S and s_S^2 to avoid negative values of equity.

- In case (1) ($\tilde{R}_{S,t} > \underline{R}_{S,t}$), the return on interbank loan is always $\tilde{R}_{D,t} = R_{D,t}$.

- In case (2) ($\tilde{R}_{S,t} \leq \underline{R}_{S,t}$), the merchant bank defaults and the return on the interbank loan is given by what is left to the deposit bank after the default of the merchant bank, i.e., the value of the liquidated assets, $(1+\tilde{R}_{S,t})(1-\theta)S_t$, and the cash after interest payment, $[(1+R_{F,t})C_{M,t} - R_{D,t}D_t]$. Therefore, we have:

$$(1+\tilde{R}_{D,t})D_t = (1+\tilde{R}_{S,t})(1-\theta)S_t + [(1+R_{F,t})C_{M,t} - R_{D,t}D_t],$$

  so that

$$1+\tilde{R}_{D,t} = (1+\tilde{R}_{S,t})(1-\theta)(1+m_t) + \frac{m_t(1-a_t)}{a_t}(1+R_{F,t}) - R_{D,t}.$$

  However, the deposit bank does not default, so that the return on interbank loans still satisfies $\tilde{R}_{D,t} > \underline{R}_{D,t}$. This implies that the return on securities satisfies the inequality:

$$\begin{aligned} 1+\tilde{R}_{S,t} \quad > \quad & \frac{1}{(1+m_t)(1-\theta)} \left[(1+R_{Dep,t})\frac{Dep_t}{D_t} \right. \\ & \left. -(1+R_{F,t})\left(\frac{(1-b_t)Dep_t}{D_t} + \frac{m_t(1-a_t)}{a_t} \right) \right] \equiv 1+\underline{R}_{SD,t}. \end{aligned}$$

In case (1), the value of equity of the deposit bank is:

$$\begin{aligned} E_t[N_{D,t+1}^{(NT)}] \quad &= \quad E_t[N_{D,t+1} \mid \text{Normal time or Fire sales }] = (1+\tilde{R}_{N_D,t}^{(NT)})N_{D,t} \\ &= \quad \left[1+R_{F,t} + (R_{D,t}-R_{F,t})\frac{D_t}{N_{D,t}} - (R_{Dep,t}-R_{F,t})\frac{Dep_t}{N_{D,t}} \right] N_{D,t}. \end{aligned}$$

In case (2), the deposit bank does not default but its return on equity is affected by the default of the merchant bank:

$$\begin{aligned} E_t[N_{D,t+1}^{(MBD)}] \quad &= \quad E_t[N_{D,t+1} \mid \text{Merchant bank default }] = (1+\mu_{N_D,t}^{(MBD)})N_{D,t} \\ &= \quad \left[1+R_{F,t} + (\mu_{D,t}^{(MBD)}-R_{F,t})\frac{D_t}{N_{D,t}} - (R_{Dep,t}-R_{F,t})\frac{Dep_t}{N_{D,t}} \right] N_{D,t}, \end{aligned}$$

where

$$1 + \mu_{D,t}^{(MBD)} = (1 + \mu_{S,t}^{(MBD)})(1 + m_t)(1 - \theta) + \left[(1 + R_{F,t}) \frac{m_t(1 - a_t)}{a_t} - R_{D,t} \right].$$

Case 2: Default of deposit banks. The deposit bank defaults when the merchant bank defaults ($\tilde{R}_{S,t} \leq \underline{R}_{S,t}$) and the return on interbank loan is too low to repay its debt ($\tilde{R}_{D,t} \leq \underline{R}_{D,t}$). In this case, we have:

$$1 + \tilde{R}_{D,t} \leq 1 + \underline{R}_{D,t} = (1 + R_{Dep,t}) \frac{Dep_t}{D_t} - \frac{(1 - b_t) Dep_t}{D_t} (1 + R_{F,t}).$$

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Table 1: Correlation with the leverage ratio of different financial institutions

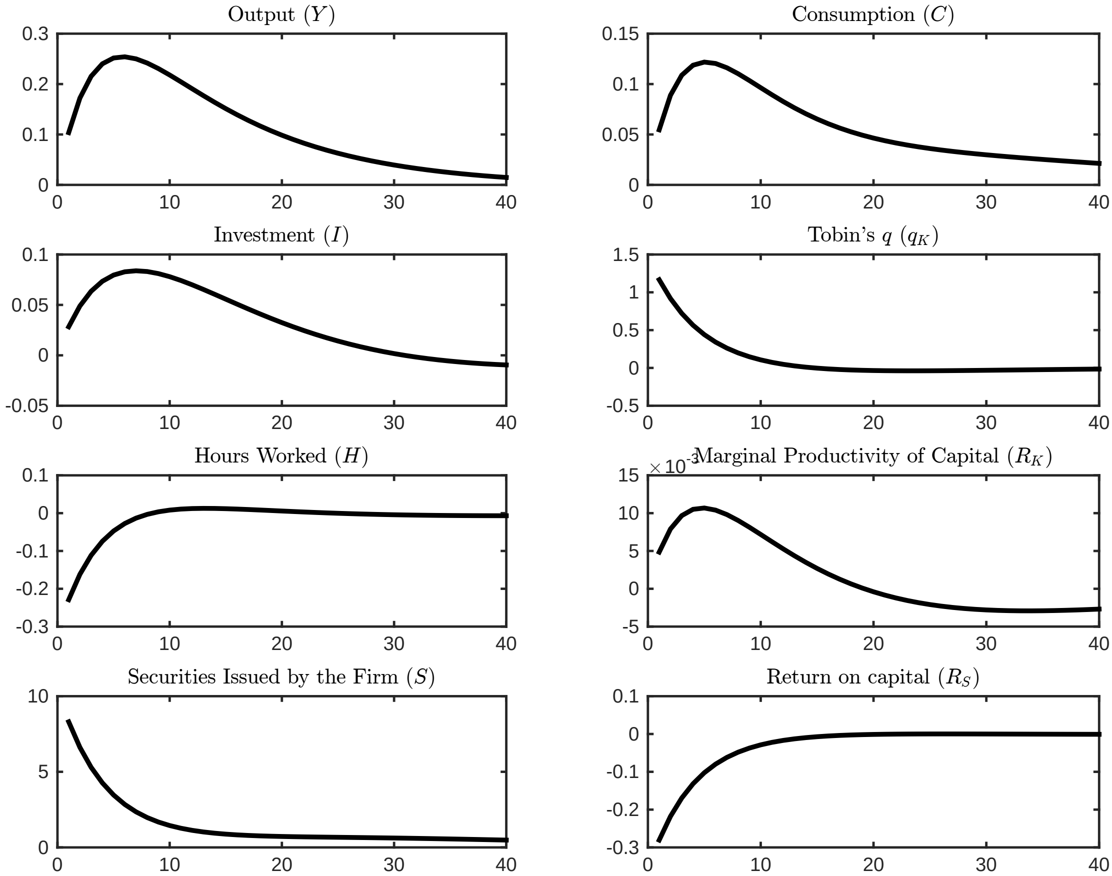
2007-2011	GDP growth	S&P 500 growth	VIX change
Commercial banks	-0.22	-0.39	0.58
Security brokers and dealers	0.61	0.42	-0.45

Table 2: Value of the calibrated parameters

	Symbol	Value	Reference
Households			
Elasticity of subst. of consumption	σ	1	GK
Elasticity of subst. of labor supply	ζ	0.333	GK
Habit formation	γ	0.81	GK
Weight on labor in utility	η	5.584	GK
Firms			
Capital share parameter	α	0.33	GK
Quarterly depreciation rate of capital	δ	2.5%	GK
Investment adjustment cost parameter	κ	26.64	CMR
Fixed cost parameter	Φ	7%	CMR
Authorities			
Tax rate on consumption	τ_c	5%	CMR
Tax rate on labor income	τ_s	24%	CMR
Share of government expenditure	$\bar{g}$	20%	GK/CMR
Shock dynamics (persistence / volatility)			
Preference shock	$\rho_v - \sigma_v$	0.902 – 0.021	CMR
Productivity shock	$\rho_z - \sigma_z$	0.883 – 0.007	CMR
Government shock	$\rho_g - \sigma_g$	0.938 – 0.021	CMR
Finance			
Real discount factor	β	0.99625	(Risk-free rate)
Annualized volatility of securities return	σ_S	[12%; 16%; 20%]	(Volatility of S&P)
Cost of fire sale	ϕ	[4%; 6%; 8%]	
Liquidation cost	θ	[20%; 30%; 40%]	

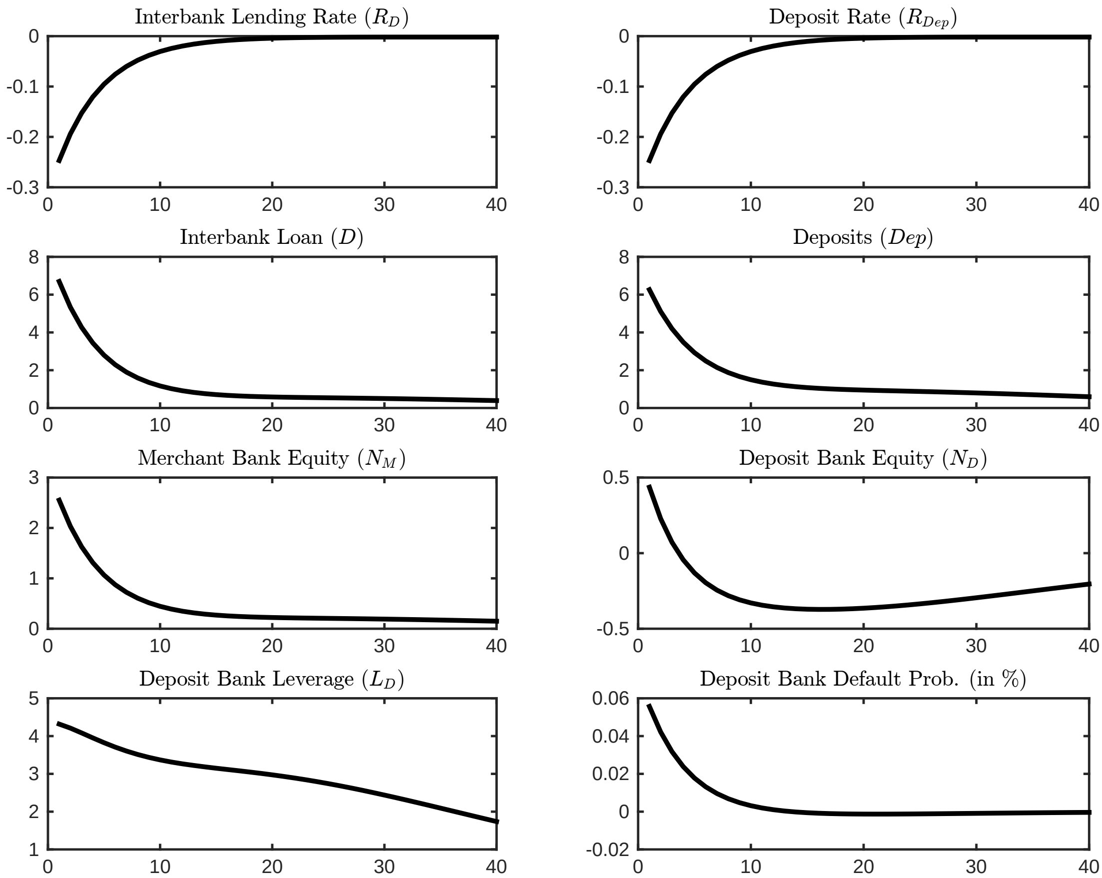
Note: The table reports the value of the calibrated parameters. Most of these values are drawn from Gertler and Kiyotaki (2010) and Christiano, Motto, and Rostagno (2010).

Figure 1: Impulse response to a productivity shock



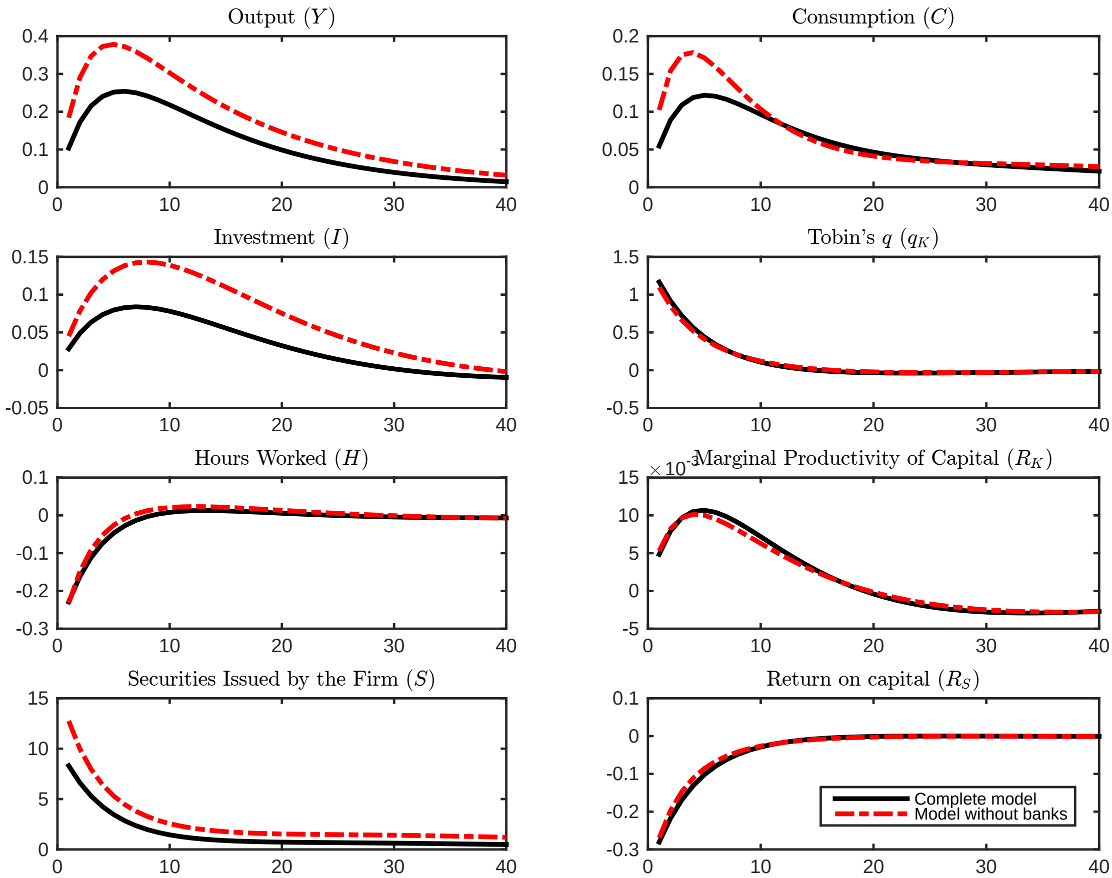
Note: This figure displays the impulse responses of the main variables after a one standard-deviation positive productivity shock.

Figure 2: Impulse response to a productivity shock



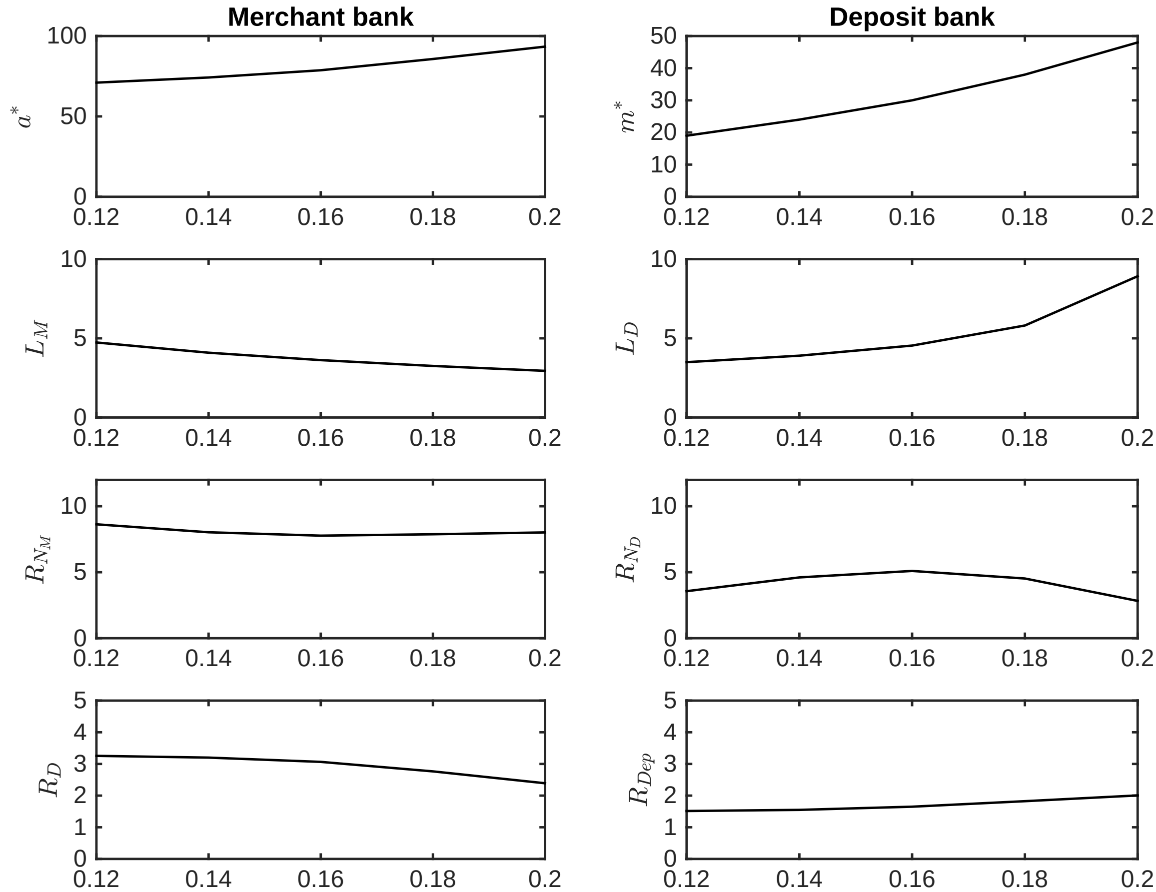
Note: This figure displays the impulse responses of the main variables after a one positive standard-deviation productivity shock.

Figure 3: Impulse response to a productivity shock



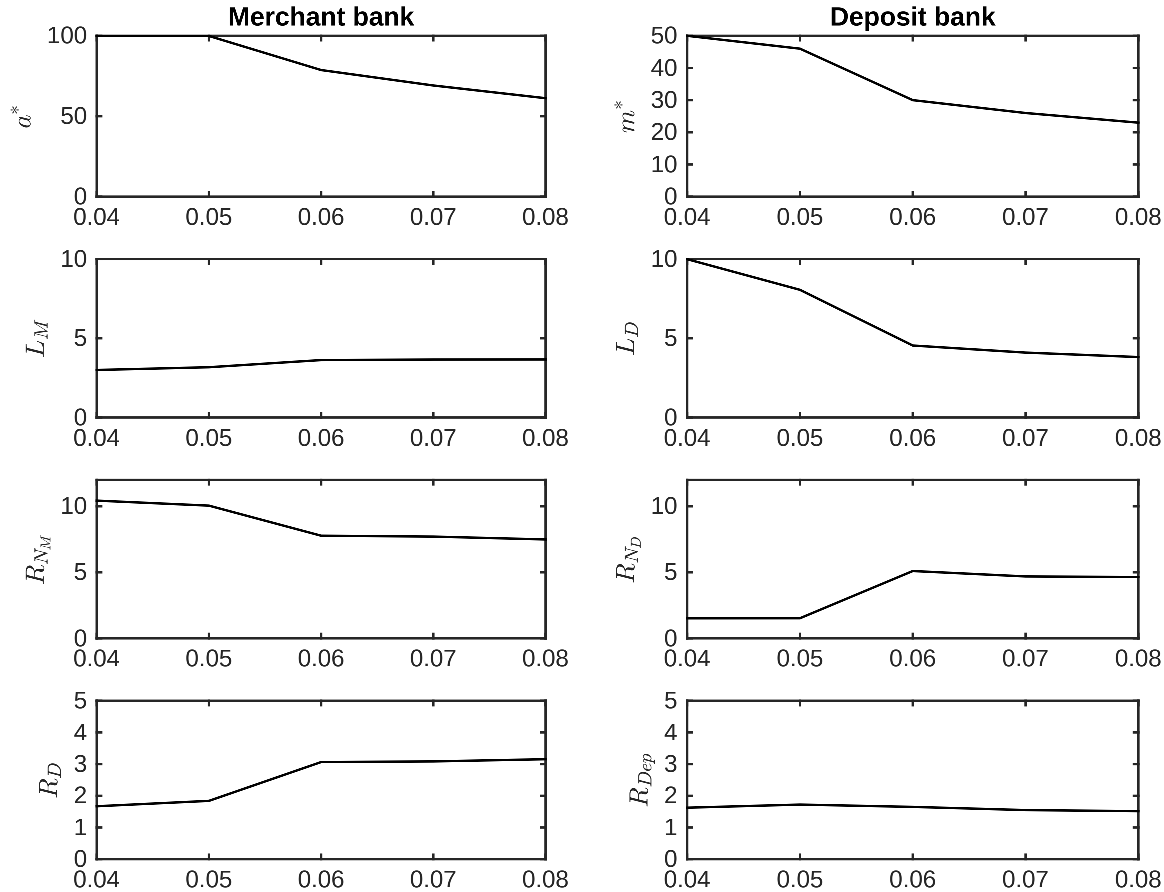
Note: This figure displays the impulse responses of the main variables after a one positive standard-deviation productivity shock. In black, the IRF for the complete model; in dashed red, the IRF for the model without banks.

Figure 4: Equilibrium values when σ_S varies



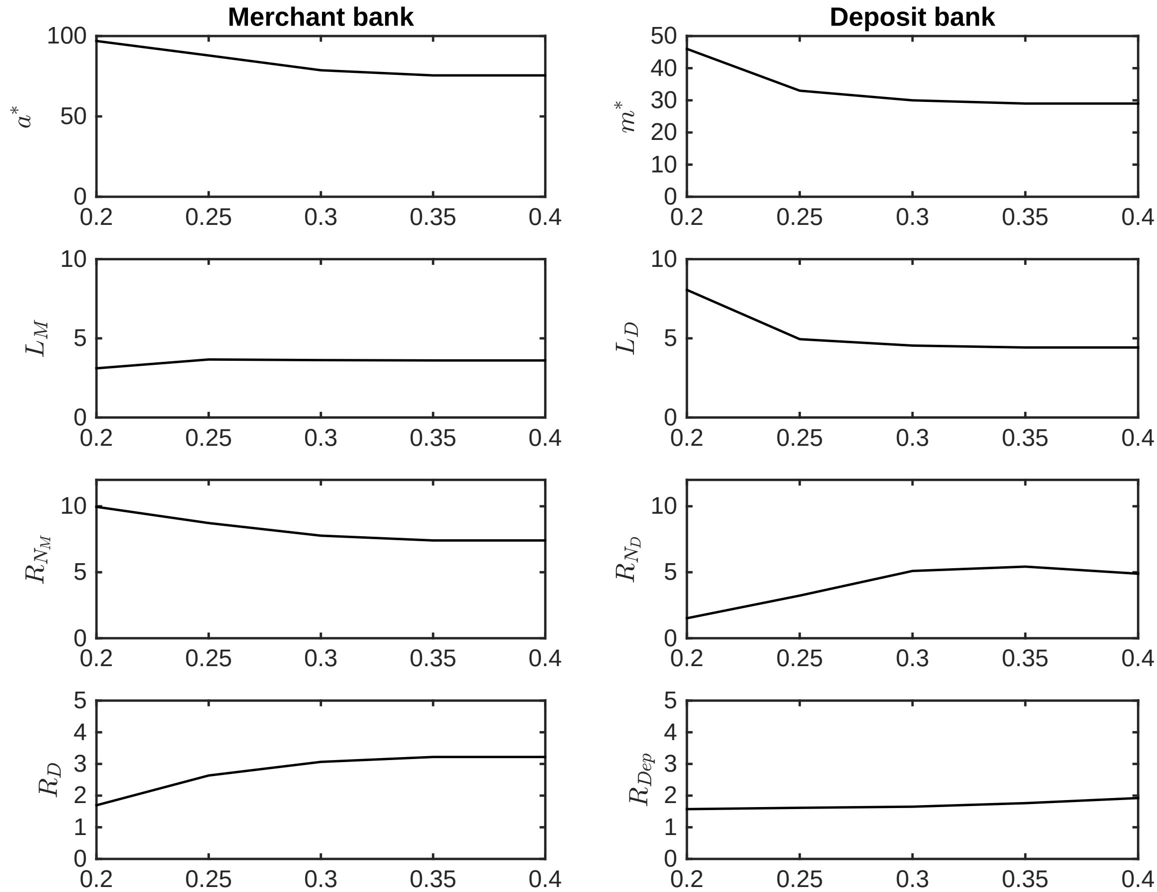
Note: This figure displays the main variables describing the banks when the value of σ_S varies and the other parameters are taken at their benchmark values.

Figure 5: Equilibrium values when ϕ varies



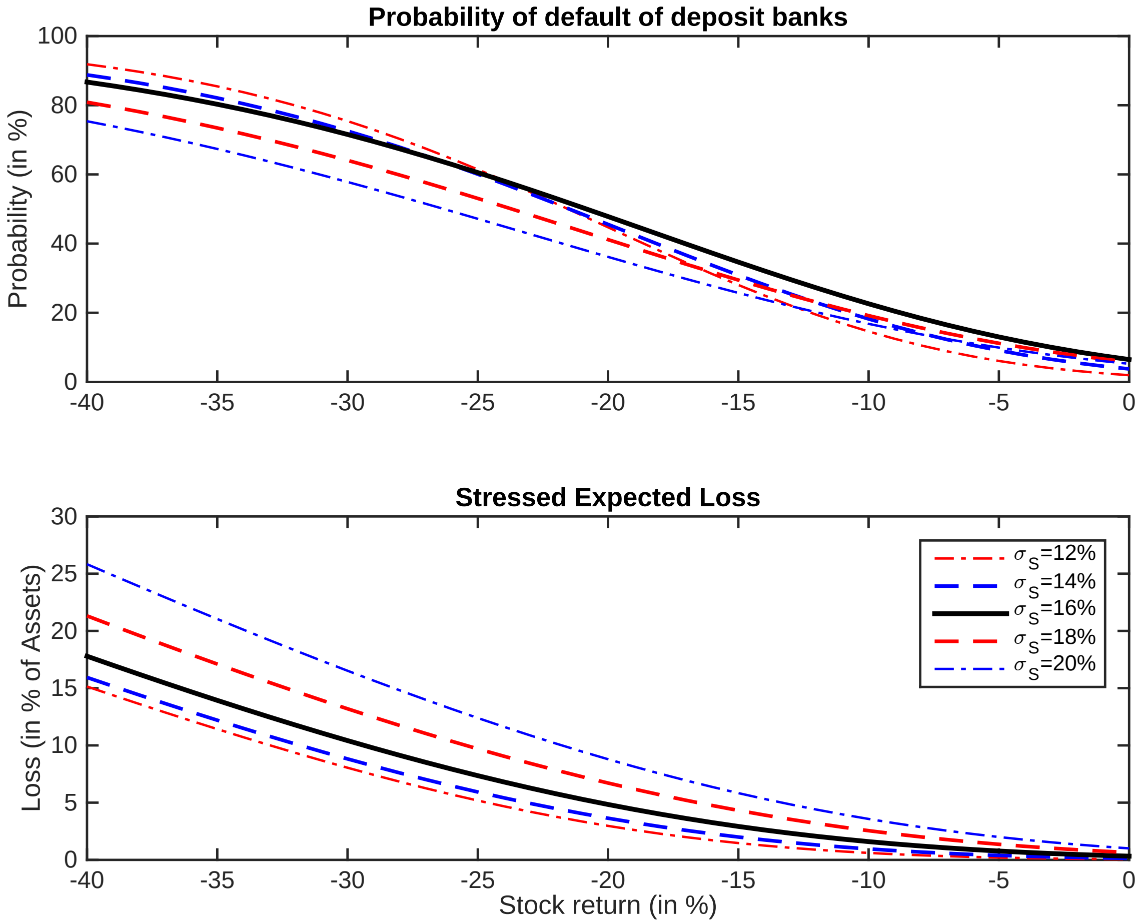
Note: This figure displays the main variables describing the banks when the value of ϕ varies and the other parameters are taken at their benchmark values.

Figure 6: Equilibrium values when θ varies



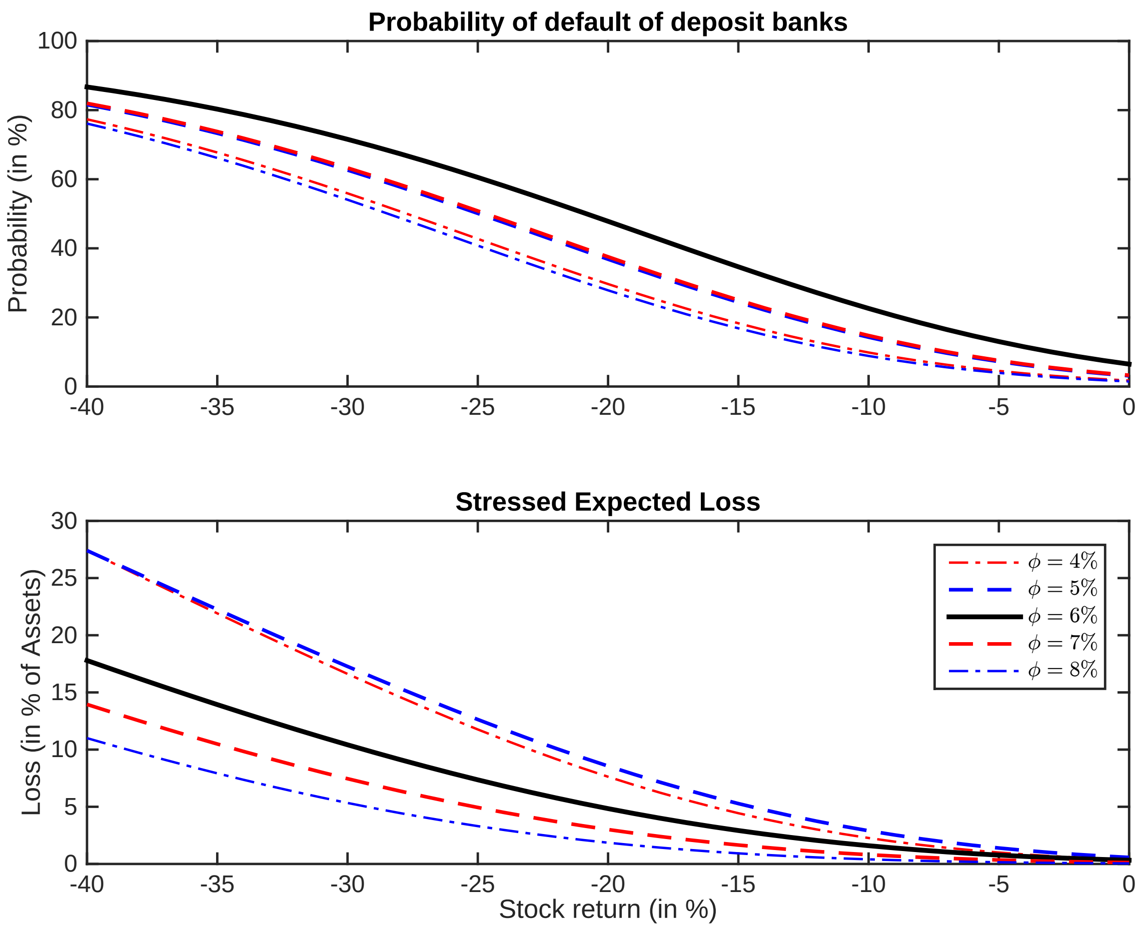
Note: This figure displays the main variables describing the banks when the value of θ varies and the other parameters are taken at their benchmark values.

Figure 7: Probability of Default and Stressed Expected loss when σ_S varies



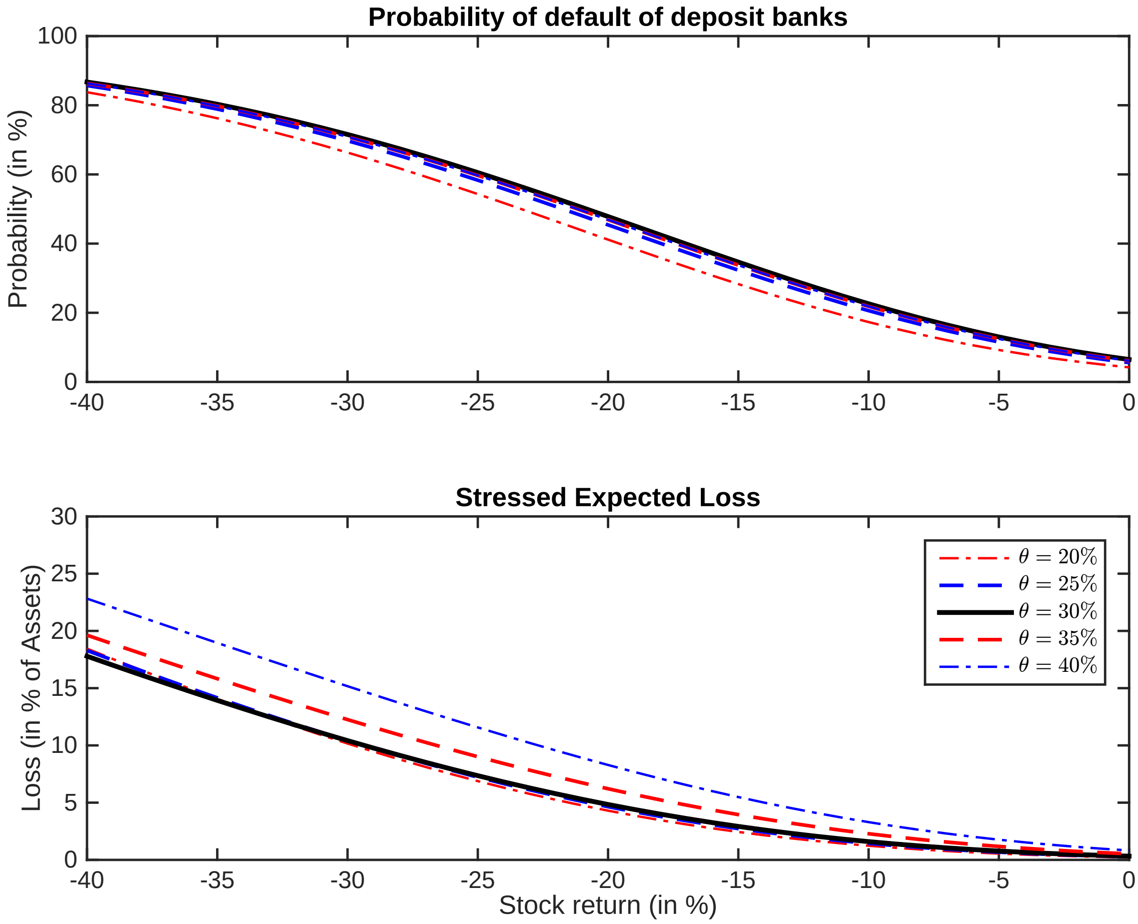
Note: This figure displays the probability of default and the stressed expected loss of the deposit banks when the value of σ_S varies and the other parameters are taken at their benchmark values.

Figure 8: Probability of Default and Stressed Expected loss when ϕ varies



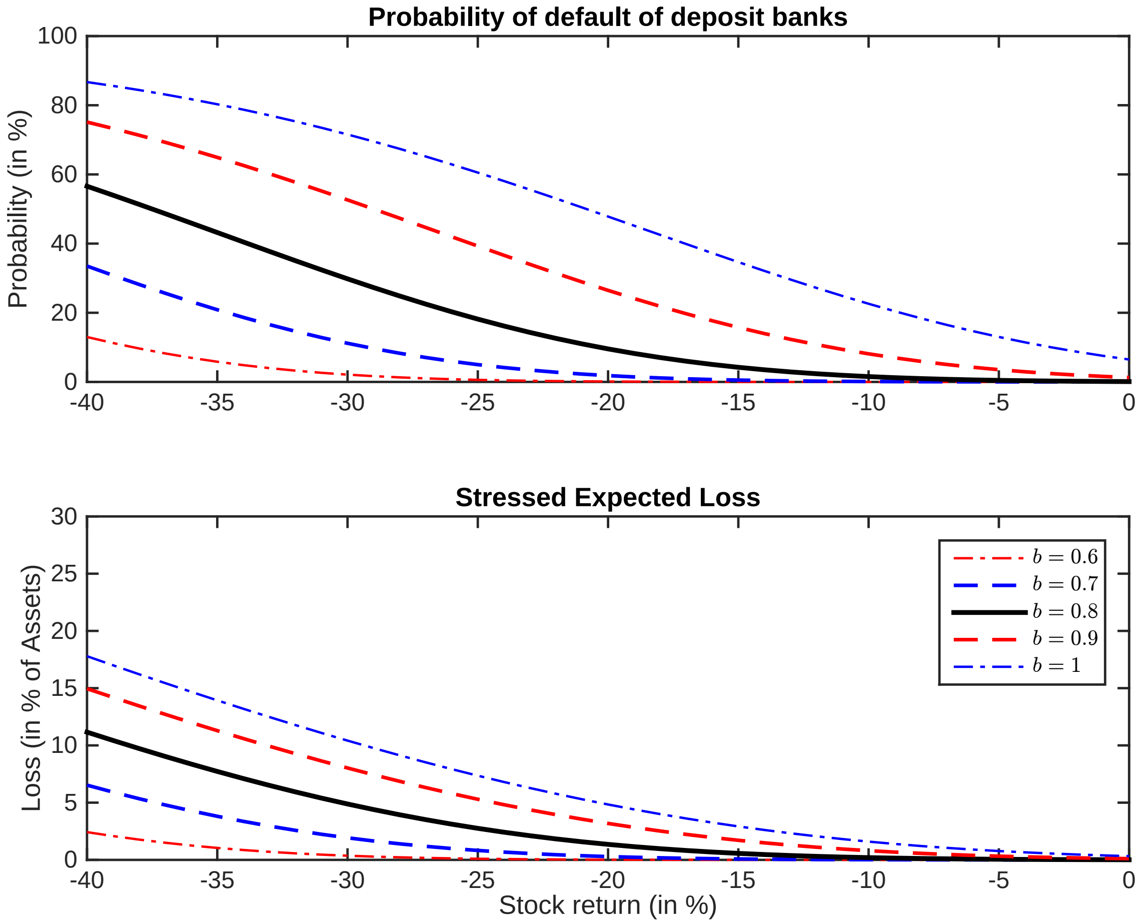
Note: This figure displays the probability of default and the stressed expected loss of the deposit banks when the value of ϕ varies and the other parameters are taken at their benchmark values.

Figure 9: Probability of Default and Stressed Expected loss when θ varies



Note: This figure displays the probability of default and the stressed expected loss of the deposit banks when the value of θ varies and the other parameters are taken at their benchmark values.

Figure 10: Probability of Default and Stressed Expected loss when b varies



Note: This figure displays the probability of default and the stressed expected loss of the deposit banks when the value of b varies and the other parameters are taken at their benchmark values.