Economic Policy Uncertainty and the Yield Curve

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Abstract

This paper analyzes the impact of economic policy uncertainty on the term structure of real and nominal interest rates. We derive a general equilibrium model where the real side of the economy is driven by government policy uncertainty and the central bank sets money supply endogenously following a Taylor rule. We analyze the impact of government and monetary policy uncertainty on nominal yields, short rates, bond risk premia and the term structure of bond yield volatility. Furthermore, we show that our standard affine yield curve model is able to capture both, the shape of the term structure of interest rates as well as the hump-shaped bond yield volatility curve. Finally, the empirical analysis shows that, whereas higher government policy uncertainty leads to a decline in yields, and an increase in bond yield volatility, monetary policy uncertainty does not have a significant contemporaneous effect on movements in the yield or volatility but is however an important predictor for bond risk premia.

JEL classification:

Key Words: Term structure modeling, yield volatility curve, policy uncertainty, bond risk premia

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1 Introduction

Governments and central banks shape the environment and structure of financial markets in an fundamental way. They have a key influence on the decision making process of every individual, firm or institution acting in those financial markets. On the one hand, governments affect firms and investors in many different ways, ranging from levying taxes, providing subsidies, enforcing laws and regulating competition or defining environment policies. On the other hand central banks manage the county’s money supply, oversee the commercial banking system and set nominal short term interest rates, such as for instance the LIBOR which is of fundamental importance for every economy.

In this paper we analyze the impact of policy uncertainty on the term structure of interest rates, its corresponding volatility curve and bond risk premia. To this end, we develop a general equilibrium model for the term structure of interest rates where the real side of the economy is subject to government policy uncertainty and the nominal side of the economy is affected by monetary policy uncertainty shocks. Our model setup allows us to derive an approximate analytical solution for the general equilibrium. A key feature of our model is that the central bank is setting the money supply following a Taylor rule. With such an active policy making, real shocks affect the nominal side as well, which will be important to reproduce the salient features of the nominal term structure and its volatility curve.

As our analysis shows, government and monetary policy uncertainty are affecting nominal yields, the term structure of volatility, and bond risk premia in a fundamentally different way. Our model predicts that it is mainly the uncertainty about the impact of the current government policy which is the fundamental driver of contemporaneous movements yields and the volatility curve. We show that, increased government policy uncertainty adversely affects the trend component of real output growth and therefore renders capital investments more risky and will eventually induce investors to favor safe assets such as government bonds. This flight-to-quality behavior will raise government bond prices and therefore drives down its yields. Along the same lines of Bloom (2009), who argues that productivity growth falls because higher uncertainty causes firms to temporarily pause their

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1We follow the approach of Kogan & Uppal (2001) and approximate our model with respect to the risk aversion parameter around the explicit equilibrium computed under the log-utility assumption. Such perturbation methods have been successfully applied in many other studies such as, e.g., Hansen et al. (2008).
investment, in our model economy, higher fiscal policy uncertainty will not only negatively affect the long run growth path of production but also increase its volatility and therefore worsening the economic growth prospects, which are fundamental to the agents consumption-investment allocation problem.\footnote{Our model is similar in style to the long run risk model of Bansal & Yaron (2004), however with the key distinctive difference that the long run growth component and the market price of output risk are both driven by the same underlying risk factor, namely fiscal policy uncertainty. Furthermore, our setting can also be compared to the literature on real business cycle analysis. For instance, shocks to trend growth exhibit fundamentally different effects on the (real) economy as opposed to transitory fluctuations. The agents or county’s reaction to temporary shocks is to borrow in the short run to smooth out consumption. However, if the shock is more persistent, the long run consumption level has to be adjusted as borrowing for an infinite time horizon is not longer possible.}

Furthermore, government policy uncertainty also plays an important role in capturing the term structure of bond yield volatility, especially the empirically observed hump-shape, i.e., bond yield volatility is tends to be highest at about 2 year maturity. Our framework allows us to generate a volatility amplification mechanism that can help explain the ‘excess bond yield volatility puzzle’, i.e., that empirical bond yields, especially at the long end of the term structure, cannot be reproduced by standard affine models of the term structure of interest rates (see Shiller (1979) and Piazzesi & Schneider (2006)).\footnote{A possible solution to this problem is to introduce heterogeneous agents who have different prior beliefs about some fundamental economic variable, such as for instance inflation as in Xiong & Yan (2010) or to introduce time-varying risk preference as in Buraschi & Jiltsov (2007) which are however analytically less tractable.} Although our model of the term structure belongs to the class of affine term structure models as introduced by Duffie & Kan (1996), it is able to generate the empirical hump shape of yield volatility. The key mechanism in our setup that leads to this result is that fiscal policy uncertainty negatively affects the long run growth path of productivity and thus making future growth of output more uncertain which translates into higher and hump-shaped yield volatility.

To explain how a fiscal policy shock from the real side is able to explain both movements in the nominal yield and its corresponding volatility curve, we have to be more explicit about the channels through which government policy affects bond yields and volatility. In our model economy, the central bank authority controls the money supply growth by conducting its monetary policy following a Taylor rule. Given this setup, money supply growth is endogenously determined by deviations from the expected target inflation level, i.e., the growth rate of the equilibrium price level and from fluctuations of real output, i.e., the equilibrium growth rate of capital accumulation around its long run target rate. The reason why we include an active monetary policy rule is because there
is increasing evidence that policy uncertainty or fear measures lead to direct reactions of the central bank (see for instance David & Veronesi (2014)). In our model economy, higher real uncertainty lowers productivity growth, which feeds into the monetary policy through the interest rate rule. Hence, the monetary authority’s efforts to stabilize growth (and inflation) cases it to react to real uncertainty by lowering the cost of capital. Moreover, since we assume money neutrality, nominal shocks do not have an impact on the real side of the economy. However, the converse is not true since the equilibrium price level growth is driven by the capital accumulation growth which implies that the nominal side is also driven by shocks from the real side, namely fiscal policy shocks. Through these two channels, i.e., inflation and capital accumulation growth targeting, the money supply growth becomes a function of government policy uncertainty. Lastly, our model allows government policy shocks to carry a risk premium and are therefore inducing time variation in bond risk premia.

The literature on classical asset pricing usually abstracts from modeling the government or how asset prices respond to political news completely. More recently, especially since the European debt crisis starting in 2010 and the 2011 Congress debate about raising the fiscal debt ceiling in the US, policy uncertainty has attracted interest from academia. For instance, Pastor & Veronesi (2012) and Pastor & Veronesi (2013) develop a general equilibrium model where the profitability of each firm in the economy is driven by the latent prevailing government policy and discuss the impact of policy risk on stock prices. Other empirical papers have shown that uncertainty about political outcomes has a significant effect on asset returns and corporate decisions.4 There is also a large strand of literature trying to infer political risk from government bond yields such as, e.g., Huang et al. (2013) who study the relationship between political risk and government bond yields. For an excellent overview of this literature, we refer to Bekaert et al. (2012).

In our model, we also include monetary policy shocks into the interest rate rule in order to explore how they affect the term structure, its volatility curve and bond risk premia. How market interest

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4For instance, early studies include Rodrik (1991) or Pindyck & Solimano (1993) who show empirically that uncertainty about political factors can lead to lower investment expenditures, especially when investment is irreversible. More recently, Durnev (2010) and Julio & Yook (2012) document that firms tend to withhold their investment activity prior to national elections. Gulen & Ion (2012) argue, based on the newly developed policy index by Baker et al. (2012), that policy uncertainty reduces firm and industry level investment and that the magnitude of reduction is substantial. Boutchkova et al. (2012) take the analysis further and show that some industries are more sensitive to political uncertainty than others. Some further related articles analyzing the relationship between political uncertainty and asset returns include Belo et al. (2013), Bialkowski et al. (2008) or Bond & Goldstein (2012).
rates respond to Federal Reserve actions is of central importance for bondholders. Therefore, it is not surprising that there is a large literature studying the fundamental link between the impact of monetary policy and the entire term structure of interest rates and to a lesser extent bond yield volatility.\(^5\)

Lastly, not much is known about whether there exists a link between monetary policy and bond risk premia. In an empirical study, Buraschi et al. (2014) show that monetary (path) shocks are indeed an important source of priced and therefore in explaining the risk premia in bond markets. They show in a standard predictability regression of excess bond returns, that monetary policy shocks account for 15% of the variance of bond excess returns. Our model accommodates a time-varying risk premia in bond returns, which implies that policy uncertainty is a priced risk factor, i.e., the equilibrium inflation process exhibits heteroskedastic time-variation.

If one wants to test the predictions of the model empirically, a natural question that arises in this context is: What is an appropriate measure for economic policy uncertainty and more precisely, for government and monetary policy uncertainty respectively? We choose the economic policy uncertainty (EPU) index developed by Baker et al. (2012) which contains both, uncertainty related to fiscal government policy and uncertainty related monetary policy. We also collected data from the authors webpage www.policyuncertainty.com in order to construct a government (GPU) as well as a monetary policy uncertainty (MPU) index. From the empirical side, we provide some evidence that government policy uncertainty indeed is the main driver in contemporaneous movements in the term structure of interest rates and its volatility curve.

To give a preview of our results, we qualitatively investigate the relationship between government treasury (TB) yields and the EPU, GPU as well as the MPU index in Figure 1 below. The first two spikes of the EPU index are related to the terrorist attacks on the World Trade Center and the 2nd Gulf War. By the end of 2003 and until the outbreak of the financial crisis in 2008, the US economy entered a steady economic growth phase. Both, the EPU and GPU show similar pattern, meaning they declined in this period before they started to peek at the onset of the financial crisis.

\(^5\)For the yield effects see, e.g., Kuttner (2001), Piazzesi (2005), Fleming & Piazzesi (2005), Gürkaynak et al. (2005a), and Wright (2012). For the volatility effects see, e.g., Balduzzi et al. (2001), Piazzesi (2005) or de Goeij & Marquering (2006).
Panel A: Term Structure and Policy Uncertainty  

Panel B: Volatility and Policy Uncertainty

Figure 1: US Treasury bond yields (Panel A) and realized yield volatility (Panel B) with maturity $\tau = 1Y, 2Y, 3Y, 5Y, 7Y$ and $10Y$ and the economic policy uncertainty (EPU) index as constructed by Baker et al. (2012) plus the government (GPU) and monetary policy (MPU) index. Sample period ranges from January 1990 until June 2014. All indexes are scaled to match the scale of the TB yields.

Interestingly, the EPU and GPU index remain at a high level ever since, whereas the MPU index slowly reversed to its pre-crisis level. This might be attributed to the fact that the EPU and GPU index captures political uncertainty, which was especially high during the debt-ceiling crisis of 2011 and lasted until late 2013 where some governmental authorities were even forced to suspend their services temporarily. Furthermore, Figure 1 shows that both, the EPU and GPU index, exhibit a countercyclical pattern with the nominal yields, i.e., when the EPU or GPU index is high, yields tend to go down. This apparent negative relationship is also confirmed by computing a simple sample correlation coefficient which ranges from -0.544 (EPU, 1 year) to -0.32 (GPU, 10 years) for the period January 1990 until June 2014.\(^6\) This suggests that higher economic or (government) policy uncertainty leads to lower TB bond yields. This negative relationship is particularly interesting as current empirical work by Pastor & Veronesi (2013) shows, using also the EPU index of Baker et al. (2012), that political uncertainty raises not only the equity risk premium but also the volatilities and

\(^6\)The time series correlation between the EPU and GPU (MPU) is 0.857 (0.657) and 0.572 between the GPU and MPU index. As can be inferred from Figure 1, the monetary policy uncertainty index appears to have no link with contemporaneous movements in nominal yields. Indeed, the estimated time series correlation is roughly zero along the entire term structure. We collect the all empirical sample correlation of TB yields and the EPU, GPU and MPU indexes in Table 7 as well as realized volatility and EPU, GPU and MPU indexes in Table 8 in Appendix B. Furthermore, the sample correlation between the EPU and the VIX index is 0.44 for the same period.
correlations of stock returns. Thus in our setting, as political risk increases, investors seek safer assets and therefore start to shift from stocks to (government) bonds which is in line with the predictions in Pastor & Veronesi (2013).

Despite the recent attention brought to modeling the impact of policy uncertainty on asset prices, the papers mentioned above either address the empirical link between government bond yields and policy uncertainty or focus on the theoretical impact that a given government policy has on stock returns. Our paper provides a theoretical framework for studying the impact of both government and monetary policy uncertainty on the nominal yield curve and its implications for the term structure of bond yield volatility.

The reminder of the paper is organized as follows. Section 2 presents the model. Section 3 discusses the impact of the latent government policy on the term structure of nominal interest rates and the bond volatility curve. Section 4 summarizes my empirical results and Section 5 concludes. The Appendix contains all proofs, supplementary tables and figures and further technical details.

2 The Baseline Model Economy

Our model economy consists of two sectors: a real sector which is driven by the government policy uncertainty and a monetary sector, which is driven by monetary policy uncertainty, where we assume that the central bank is following a standard Taylor rule to conduct its monetary policy.

2.1 The Real Sector

We consider a production economy where the representative agent produces a single good at a constant return-to-scale production technology. Risk is described by a complete filtered probability space $(\Omega, \mathcal{F}_t = (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ satisfying the usual conditions (right-continuous, increasing and augmented). The filtration $\mathcal{F}_t = (\mathcal{F}_t)_{t \geq 0}$ represents the available information up to time $t$ to the representative agent. The agent produces real output denoted by $Y = (Y_t)_{t \geq 0}$. We assume that productivity $A = (A_t)_{t \geq 0}$ is influenced by a process $g = (g_t)_{t \geq 0}$ which we refer to government policy or real un-
certainty in the sequel. Furthermore, we postulate that fiscal policy uncertainty is the main driver of real risk in the economy as it affects both output and productivity in various ways.

**Assumption 1** (Real Sector). The dynamics of real output $Y_t$, productivity $A_t$ and government policy uncertainty $g_t$ characterize the real sector dynamics as follows

\[
\frac{dY_t}{Y_t} = (\mu_Y + q_A A_t) dt + \sigma_Y \sqrt{g_t} dW_t^Y, \quad Y_0 \in \mathbb{R}_+, \quad (1)
\]

\[
dA_t = (\kappa_A (\theta_A - A_t) + \lambda g_t) dt + \sigma_A \sqrt{g_t} dW_t^A, \quad A_0 \in \mathbb{R}, \quad (2)
\]

\[
dg_t = \kappa_g (\theta_g - g_t) dt + \sigma_g \sqrt{g_t} dW_t^g, \quad g_0 \in (0, \infty) \quad (3)
\]

where $\mu_Y > 0$ is a constant that renders output $(Y_t)$ a nonstationary process consistent with empirical evidence. $\kappa_i > 0, \forall i \in \{A, g\}$ and $q_A, \theta_A, \lambda \in \mathbb{R}$, and $\sigma_j^2 > 0, \forall j \in \{Y, A, g\}$ are positive constants and we assume the following correlation structure\(^7\)

\[
E[dW_t^Y dW_t^A] = \rho_{YA} dt, \quad E[dW_t^Y dW_t^g] = \rho_{YG} dt, \quad E[dW_t^A dW_t^g] = \rho_{AG} dt, \quad \forall t \geq 0 \quad (4)
\]

Given the above setup, government policy uncertainty $g_t$ affects not only trend productivity but also its variation. Productivity $A_t$ represents a mean reverting process, where not only the long run mean is stochastic but also its volatility is time-varying. Note that for $\lambda < 0$, an increase in $g_t$ will have a negative effect on long run productivity and vice versa when $\lambda > 0$. Several interesting sub cases are worthwhile to be discussed. When $\lambda = 0$, we obtain a process with constant long run mean $\theta_A$ and time-varying volatility. If in addition we replace $\sigma_A \sqrt{g_t} dW_t^A$ in Equation (2) by a constant variance process of the form $\sigma_A dW_t^A$ we recover the classical Ornstein-Uhlenbeck process with constant stationary mean $\theta_A$.\(^8\) If we set $\lambda = \kappa_A$ and $\theta_A = 0$, then productivity mean reverts to a stochastic long run level driven by $g_t$. Lastly, setting $\lambda = 0$, replacing $\sigma_A \sqrt{g_t} dW_t^A$ by a time-varying variance process of the form $\sigma_A \sqrt{g_t} dW_t^A$ we obtain the classical Feller square root diffusion process with constant stationary mean $\theta_A$. Furthermore, the government policy uncertainty process $g_t$ describes an unconditionally mean-reverting stationary process who also affects the output growth.

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\(^7\)We define by $\mathcal{F}_t = \sigma \{\omega : W^i_s : s \leq t, \forall 0 \leq s \leq t, \forall i = \{Y, A, g, M^S, m\}\}$ the information available up to time $t$, where $M^S_t$ and $m_t$ represent the money supply and monetary policy uncertainty process which will be formally introduced in Section 2.2. We denote by $E[\cdot] = E[\cdot | \mathcal{F}_t]$ the conditional expectation operator containing all available information up to time $t$.

\(^8\)Note that even in the case when $\lambda = 0$ the processes in 2, it is not equivalent to the classical Feller square root process (see equation (3) for an example.) as its diffusion part is driven by another stochastic process.
rate \( dY_t / Y_t \) through two channels. First, \( g_t \) renders output volatility time-varying which, as we will see in a later section, will in turn lead to a stochastic real market risk. Secondly, it affects the trend output growth rate indirectly as it influences the growth rate of productivity. For example, if \( \lambda < 0 \), higher government policy uncertainty will reduce the long run level of productivity, which will, provided that \( q_A > 0 \), lead to a decline in expected output growth. Next, we analyze in more detail how government policy uncertainty affects key moments of productivity.

**Proposition 1** (Stationary moments of productivity and fiscal policy uncertainty process). Let \( T > 0 \) be fixed and denote by \( C(\cdot, \cdot) \) the unconditional covariance operator, then the first two stationary centered moments of \( A_t \) and \( g_t \) and the covariance are given by

\[
\mathbb{E}[A_t] = \lim_{T \to \infty} \mathbb{E}[A_T] = \theta_A + \frac{\lambda \theta_g}{\kappa_A}, \\
\mathbb{V}[A_t] = \lim_{T \to \infty} \mathbb{V}[A_T] = \theta_A \left( \frac{\sigma_A^2}{2\kappa_A} + \frac{2\kappa_g \lambda \rho_A g \sigma_g + \lambda^2 \sigma_g^2}{2\kappa_A \kappa_g (\kappa_A + \kappa_g)} \right), \\
\mathbb{E}[g_t] = \lim_{T \to \infty} \mathbb{E}[g_T] = \theta_g, \quad \mathbb{V}[g_t] = \lim_{T \to \infty} \mathbb{V}[g_T] = \frac{\theta_g \sigma_g^2}{2\kappa_g}, \\
C[A_t, g_t] = \lim_{T \to \infty} C_t[A_T, g_T] = \frac{\theta_g \sigma_g (2\kappa_g \rho_A g \sigma_A + \lambda \sigma_g)}{2\kappa_g (\kappa_A + \kappa_g)}. 
\]

A first important observation is that the long run level of government policy uncertainty \( \theta_g \) affects all moments. First it raises (lowers) the unconditional expected productivity growth whenever \( \lambda > 0 \) \((\lambda < 0)\). Second, it not only raises the stationary long run level of \( g_t \) proportionally, but also the variance of \( A_t \) and \( g_t \), which implies that increasing \( \theta_g \) will lead to higher fiscal policy uncertainty and simultaneously also higher variance of productivity. Another important parameter is \( \lambda \) which measures the impact of fiscal policy uncertainty on the drift term of productivity. In what follows, we assume that the impact of \( g_t \) is negative, i.e., \( \lambda < 0 \). Then it follows that, assuming \( \rho_A g = 0 \) for simplicity, the effect of \( \lambda \) has three important implications. First it lowers expected growth of productivity proportionally to \( \theta_g / \kappa_A \). Second, it increases volatility of \( A_t \) linearly and lastly it renders \( C[A_t, g_t] \) negative, which will be of central importance in order for the model to capture the stylized facts of bond yield volatility.

8
2.2 Preferences and Monetary Sector

In our model economy, there exists a single good which is produced by an infinitely lived representative agent who can either consume or reinvest it in a constant-returns-to-scale production technology. The agent exhibits time separable constant relative risk aversion preferences that depend on both consumption $C_t$ and real cash balances $M^d_t$ and $P_t$ is the price level in the economy.

**Assumption 2** (Preferences of Representative Agent). Let $U(X_t)$ denote expected utility and $\beta > 0$ the subjective discount factor. The agent's preferences are given by

$$U(X_t) = E_t \int_t^\infty e^{-\rho(u-t)}U(X_u)du$$

where $U(X_t) = \frac{1}{\gamma} (X_t^{\gamma} - 1)$, $X_t = C_t (M^d_t)^\xi$, $0 \leq \xi \leq 1$ (10)

The parameter $\gamma$ is equal to one minus the coefficient of risk aversion. Thus for $\gamma = 0$, the representative agent has separable log-utility of the form $U(C_t, M^d_t) = \log(C_t) + \xi \log(M^d_t)$. The parameter $\xi$ measures the degree of transaction service to the agent. If $\xi = 0$ money does not provide any service, whereas $\xi = 1$ implies that the agent needs to hold exactly one unit of currency for every unit of consumption holdings. Given the specification in Equation (10), higher level of monetary holdings provide a higher level of transaction services but at a decreasing return to scale, i.e., $\frac{\partial^2 X_t}{\partial (M^d_t)^2} < 0$. We can now proceed with the formulation of the agent’s capital budget constraint.

**Assumption 3** (Capital budget constraint). The real after-tax return on capital that can either be allocated to consumption $C_t$ or cash balances $M^d_t$ and/or reinvested $dK_t$ is given by

$$C_t dt + M^d_t dt = K_t \frac{dY_t}{Y_t} - \delta K_t dt - dK_t$$

where $K_t \frac{dY_t}{Y_t}$ is total output, $\delta K_t dt$ is capital depreciation with $\delta \in [0,1]$ and $dK_t$ is time $t$ period investment.

Substituting output growth from Equation (1) and rearranging we obtain that the capital accumulation process satisfies

$$\frac{dK_t}{K_t} = - \left( \frac{C_t}{K_t} + \frac{M^d_t}{K_t} \right) dt + (\mu_y + q_A A_t - \delta) dt + \sigma_y \sqrt{\gamma t} dW_t^y.$$
The capital accumulation process is decreasing in the optimal control variables consumption $C_t$ and money demand $M_t^d$, which is intuitive as higher $C_t$ and/or $M_t^d$ diminish available resources to be invested in the production technology $K_t$. Similar to real output, capital is nonstationary whenever $\mu_y \neq \delta$ and time-varying in productivity $A_t$. Furthermore, Equation (12) implies money-neutrality, i.e., monetary shocks do not have an effect on the real side of the economy.\footnote{Whether or not real output and capital are money-shock-neutral is debated in macroeconomics for a long time. The neo-classical Keynesian literature argues that any increase in the supply of money has to be offset by an equivalent proportional rise in prices and wages. A recent paper using a similar setup I use is Ulrich (2013) who sticks to the neo-classical view that monetary shocks have no impact on real quantities. There are however a number of reasons how inflation may affect the real economy as for instance discussed in Fisher & Modigliani (1978). They argue that because many private contracts are not indexed, inflation has a direct influence on purchasing power. A first quantitative study that allows for dependence of the expected return on capital on inflation is Pennacchi (1991) who uses survey data to identify inflationary expectations. Another channel through which inflation can affect the real side of the economy is through taxation of nominal asset returns. This channel was exploited by Buraschi & Jiltsov (2005) to account for the violation of the expectation hypothesis and the determination of the inflation risk premium. Since policy uncertainty affects the real and the nominal side of the economy (through the endogenous equilibrium price level) I assume money-neutrality throughout the paper but acknowledge that an extension, which is feasible, to let the capital accumulation process be a function of the price level is an interesting theoretical idea which is worthwhile to be considered.}

We now turn to the monetary side of the economy. Following Buraschi & Jiltsov (2005), we assume that there exists a central bank that controls the money supply $M_t^S$ on the basis of a Taylor-type-rule. More precisely, the monetary authority aims at targeting (1) a long term nominal constant money growth rate $\mu_M$, (2) a capital growth rate $\bar{k}$ and (3) an inflation rate equal to $\bar{\pi}$. Transitory deviations from optimal long run money growth are assumed to have stochastic volatility. In this setup, we assume that time-variation in money supply is driven by monetary policy uncertainty denoted by $m_t$. In short, we assume that the money supply process has the following form

**Assumption 4 (Monetary Sector).** The central bank controls money supply growth according to a standard Taylor rule of the following form

$$\frac{dM_t^S}{M_t^S} = \mu_M dt + \eta_1 \left( \frac{dK_t^*}{K_t^*} - \bar{k} dt \right) + \eta_2 \left( \frac{dp_t^*}{p_t^*} - \bar{\pi} dt \right) + \sigma_M \sqrt{m_t} dW_t^M \quad (13)$$

$$dm_t = \kappa_m (\theta_m - m_t) dt + \sigma_m \sqrt{m_t} dW_t^m \quad (14)$$

where $\mu_M \in \mathbb{R}$ and $\sigma_M > 0$ are the unconditional constant mean and volatility of money growth and $\kappa_m$, $\theta_m > 0$, $\sigma_m$ represent the speed of mean reversion, the long run level and volatility of the monetary policy uncertainty $m_t$. The parameters $\eta_1$ and $\eta_2$ determine the weighting of the central bank of the two target growth rates of real output and inflation. We assume that the monetary policy innovation $W_t^m$ is only correlated with the money supply innovation $W_t^M$, i.e., $\mathbb{E}[dW_t^M dW_t^m] = \rho^{Mm} dt$ and
independent of all other sources of risk in the economy.

The magnitude of \( \eta_1 \) and \( \eta_2 \) determine the sensitivity of money supply growth with respect to deviations of endogenous capital and inflation from their long run target levels. For example, a transitory positive shock to equilibrium capital accumulation \( \frac{dK_t^*}{K_t} - \bar{k}dt > 0 \) will force the central bank authority to reduce the money supply provided \( \eta_1 < 0 \). If such a policy rule is in place, the monetary authority reacts to output growth that is over potential or target output growth, by shrinking the money supply which will in turn drive up interest rates and therefore slow down investment. On the other hand, a transitory negative shock to inflation leads to \( \frac{dp_t^*}{p_t} - \bar{\pi}dt < 0 \) and thus when \( \eta_2 < 0 \) the central bank’s response is to increase the money supply. In the case when \( \eta_1 = \eta_2 = 0 \) money supply is exogenous, i.e., the monetary authority does not follow a Taylor rule to control the money stock meaning that the central bank does not react to deviations from the long run capital nor inflation growth rate anymore. Furthermore, given the money supply rule in Equation (13), monetary policy uncertainty renders money supply time-varying in \( m_t \). Having introduced both the real and monetary side of the economy we now characterize the representative agent’s equilibrium.

**Definition 2 (Equilibrium Capital Stock and Money Holdings).** The representative agent’s equilibrium is defined as a vector of optimal consumption and money demand controls \([C_t^*, M_t^d]\) and equilibrium price process \(p_t^*\) with value function \(V(t, K_t, A_t, g_t) = \mathbb{E}_t \left[ \int_t^\infty e^{-\beta(u-t)} U(C_u, M_u^d)du \right]\) such that the dynamic Hamilton-Jacobi-Bellmann programming problem is solved

\[
0 = \frac{\partial V(t, K_t, A_t, g_t)}{\partial t} + \max_{\{C_t, M_t^d\}} \left\{ U(C_t, M_t^d) + AV(t, K_t, A_t, g_t) \right\}
\]  

(15)

and subject to the representative agent’s preferences as in Assumption 2, the intertemporal budget constraint Assumption 3, the real factor structure as in Assumption 1 with Equations (1) and (3), the monetary policy rule in Assumption 4, money market-clearing \(M_t^S = p_t^*M_t^{d*}\) and transversality condition \(\lim_{T \to \infty} \mathbb{E}_t \left[ e^{-\beta TV(t, K_t, A_t, g_t)} \right] = 0\). Let \(A\) denote the infinitesimal generator when applied to the function \(f\), then

\[
Af(X) = \sum_{i=1}^N \frac{\partial f}{\partial x_i} \mu(x^i) + \frac{1}{2} \sum_{i,j=1}^N \frac{\partial^2 f}{\partial x^i \partial x^j} d\langle X^i, X^j \rangle_t
\]  

(16)

where \(X_t\) is an \(N\)-dimensional Itô process of the form \(dX_t = \mu(X)dt + \sigma(X)dW_t\).
In order to solve the problem in Equation (15), one has to first determine the optimal consumption and money demand controls \( \{C_t^*, M_t^{d*}\} \) of the representative agent for a given equilibrium price process \( p_t^* \). Having obtained \( \{C_t^*, M_t^{d*}\} \) one can then determine the equilibrium capital \( K_t^* \) and price process \( p_t^* \) that satisfies simultaneously the money market clearing condition \( M_t^S = p_t^* M_t^{d*} \) as well as the monetary policy rule (Assumption 4). For the problem defined in Equation (15), an explicit solution can only be obtained in the case the representative agent has log-utility. In that case, the resulting optimal consumption and money demand holdings are proportional to capital \( K_t \). However, in the case \( \gamma \neq 0 \), we will show, using perturbation methods (up to first order approximation in \( \gamma \)), that the asymptotic optimal controls \( C_t \) and \( M_t^d \) remain linear in the state variables \( K_t, A_t \) and \( g_t \) and therefore allow for an explicit affine representation of the term structure of real and nominal interest rates.

**Proposition 3** (Perturbed equilibrium of the representative agent’s investment and consumption problem).

1. Denote by \( X_t = (A_t, g_t) \) the state vector. Assume a power series expression for \( \phi(X_t) \) in \( \gamma \) as follows

\[
\phi(X_t) = \phi_0(X_t) + \gamma \phi_1(X_t) + O(\gamma^2),
\]

where \( \phi_0(X_t) = \phi_{00} + \phi_{0A} A_t + \phi_{0g} g_t \) is obtained from the logarithmic utility case.

2. Let \( Q, \phi_{00}, \phi_{0A}, \phi_{0g} \in \mathbb{R} \) be constants, then the representative agent’s value function is

\[
V(t, K_t, X_t) = a(t) H(K_t, X_t) = \frac{e^{-\beta t}}{\beta} H(K_t, X_t) = \frac{e^{-\beta t}}{\beta \gamma} \left( \left( e^{\phi(X_t) K_t^Q} \right)^\gamma - 1 \right)
\]

3. The agent’s first order asymptotic optimal controls are

\[
C_t^* = \frac{\beta K_t}{1 + \xi} \left[ 1 + \gamma (L - \phi_0(A_t, g_t)) \right], \quad M_t^{d*} = \xi C_t.
\]
4. The equilibrium first order asymptotic capital accumulation $K_t^*$ and price process $p_t^*$ satisfy

$$\frac{dK_t^*}{K_t^*} = \mu_{K^*}(A_t, g_t)\, dt + \sigma_Y \sqrt{g_t} dW_t^Y$$

(20)

$$\frac{dp_t^*}{p_t^*} = \left[ \frac{\mu_M - \eta_1 \bar{k} - \eta_2 \bar{\pi}}{1 - \eta_2} + \frac{\eta_1 - 1}{1 - \eta_2} \mu_{K^*}(A_t, g_t) - g_t \frac{\eta_1 - 1}{1 - \eta_2} \sigma_Y^2 \right] dt$$

$$+ \frac{\sigma_M \sqrt{m_t}}{1 - \eta_2} dW_t^M + \frac{(\eta_1 - 1) \sigma_Y \sqrt{g_t}}{1 - \eta_2} dW_t^Y.$$  

(21)

where $\mu_{K^*}(A_t, g_t) := \mu_Y + q_A A_t - \beta - \delta + \gamma \beta (\phi_0(A_t, g_t) - L)$ denotes the equilibrium drift of the capital accumulation process, $L := \log \left( \frac{\beta^{1 + \xi} \xi}{(1 + \xi)^{1 + \tau}} \right)$ is a constant, $\phi_0(A_t, g_t) = \phi_{00} + \phi_{0A} A_t + \phi_{0g} g_t$ is an affine function and $\phi_{00}, \phi_{0A}, \phi_{0g}$ are constants and given in Appendix A.2.

Since nominal shocks have no real effects, the equilibrium capital accumulation process is only driven by the real sector of the economy, i.e., output productivity $A_t$ and the fiscal policy uncertainty process $g_t$. Note that for $\gamma = 0$ the equilibrium capital drift $\mu_{K^*}$ becomes independent of $g_t$. The weighting factor $\eta_2$ on inflation-target deviation enters non-linearly into the equilibrium price process. For $\eta_2 \approx 1$ small innovations in either $A_t, g_t$ or $m_t$ result in large changes in the equilibrium price process. Furthermore, since the price process is endogenous and the central bank authority is controlling money supply growth following a Taylor-rule, the equilibrium price process is driven by both real and monetary shocks. Hence, government policy uncertainty enters the nominal side of the economy through two different channels, namely though its affect on both the equilibrium capital accumulation process and equilibrium inflation. Therefore, by endogenizing money supply growth, i.e., $\eta_1 \neq 0$ and $\eta_2 \neq 0$, there exists an important link between shocks from the real to the nominal sector which will prove to be essential in order to capture key empirical properties of both the yield curve and its corresponding term structure of yield volatility.

3 The Term Structure of Nominal Interest Rates

Having obtained the dynamics of the equilibrium price level, one can now solve for the term structure of both nominal and real bond prices. Let $B(t, \tau)$ be the nominal pure discount bond paying one unit of currency in $t + \tau$ periods. The price of the nominal bond must satisfy the following Euler
equation
\[ B(t, \tau) = e^{-\beta \tau} \mathbb{E}_t \left[ \frac{U_C(C^*_{t+\tau}, M^d_{t+\tau})}{U_C(C^*_t, M^d_t)} \right] = e^{-\beta \tau} \mathbb{E}_t \left[ \frac{\exp\{-\log(K^*_{t+\tau})\}}{\exp\{-\log(K^*_t)\}} \right] \]
which states that in equilibrium the investor should be indifferent between consuming one more unit of currency now or investing one unit of currency in the \( t + \tau \) period nominal discount bond. In other words, Equation (22) allows for a standard inter-temporal substitution argument saying that the marginal cost of reducing consumption today and hence investing into a nominal bond is \( B(t, \tau)U_C(C^*_t) / P_t \) which in equilibrium has to be equal to the marginal utility of future consumption that can be obtained through the bond investment \( U_C(C^*_{t+\tau}) / P_{t+\tau} \). We now derive explicit formulas for the nominal bond price in the presence of policy uncertainty.

**Proposition 4 (Equilibrium Nominal Term Structure of Interest Rates).** Under time-separable CRRA utility as in Equation (10), the nominal discount bond \( B(t, \tau) \) with maturity \( \tau \) is given by
\[ B(t, \tau) = \exp \left\{ -b_0(\tau) - b_A(\tau)A_t - b_g(\tau)g_t - b_m(\tau)m_t \right\} \]
where
\[ b_A(\tau) = C_A \frac{1 - e^{-\kappa_A \tau}}{\kappa_A}, \]
\[ -b'_g(\tau) = Z_{0g}(\tau) + Z_{1g}b_g(\tau) + Z_{2g}b_g^2(\tau), \]
\[ b_m(\tau) = \frac{-Z_{1m} + H_m \text{Cot} \left( \frac{1}{2} \left( -H_m \tau - \text{Tan} \left( \frac{2\sqrt{Z_{0m}Z_{2m}}}{H_m} \right) \right) \right)}{2Z_{2m}}, \quad H_m = 4Z_{0m}Z_{2m} - Z_{1m}^2, \]
\[ b_0(\tau) = \int_0^\tau C_0(u)du \]
and the constant parameters \( Z_{0m}, Z_{1i}, Z_{2i}, i \in \{g, m\} \) and \( Z_{0g}(\tau), C_0(\tau) \) are time-to-maturity functions that only depend on the structural model parameters of the economy and are defined in Appendix A.3.

The nominal term structure of interest rates in Proposition 4 belongs to the general affine class of term structure models of discount bond prices introduced by Duffie & Kan (1996). Using (23) one can derive the time \( t \) yield curve \( Y(t, \tau) \), with maturity \( \tau \) as follows
\[ Y(t, \tau) := -\frac{1}{\tau} \log(B(t, \tau)) = \frac{b_0(\tau)}{\tau} + \frac{b_A(\tau)}{\tau}A_t + \frac{b_g(\tau)}{\tau}g_t + \frac{b_m(\tau)}{\tau}m_t \]
The affine yield model in (28) is driven by three factors. As noted by Longstaff & Schwartz (1992), Litterman & Scheinkman (1991) and Constantinides (1992), such a three factor structure model of the term structure is able to reproduce most if not all empirically relevant shapes of the yield curve, such as upward sloping, inverted or hump shapes. This affine property in the factor loadings implies linearity of the local variance as in Vasicek (1977), Cox et al. (1985) and others. Whereas the target growth rates for output $\bar{k}$ and inflation $\bar{\pi}$ solely affect the intercept of the yield curve but not its slope, their weighting factors $\eta_1$ and $\eta_2$ not only affect both the slope and the intercept of the yield curve but do so in a non-linear way. Similarly, the subjective discount factor $\beta$, the depreciation rate $\delta$ and output $(\mu_Y)$ and money supply growth rate $(\mu_M)$ only influence the level of the term structure. The parameter $\xi$, is not only entering in (27) and therefore affecting the intercept of the term structure but also directly affects the loading factors on productivity $A_t$, $g_t$ and $m_t$ therefore its shape. From (24), provided that $C_A > 0$, it follows that the productivity factor loading $b_A(\tau)$ is positive. Furthermore, the parameter $\lambda$ has a key impact not only on the level of the term structure but also on its slope. First, $\lambda$ affects the level of yields as both $C_A = C_A(\lambda)$ and $C_g = C_g(\lambda)$ who enter $b_0(\tau)$ are functions of $\lambda$. Second, this in turn implies that $\lambda$ also affects the slope of the term structure as both $b_A(\tau)$ and $b_g(\tau)$ depend on it. Lastly, $\lambda$ also determines the long run level of productivity $A_t$. To see this, recall that the trend growth rate of the productivity process $A_t$ is not only dependent on the long run level of productivity $\theta_A$ but also on the long run level government policy $\theta_g$, i.e. $\mathbb{E}[A_t] = \theta_A + \frac{\theta_g}{\kappa_A}$ (see Equation (5) in Proposition 1). Suppose that $\theta_g > 0$ and $\lambda < 0$, and $\theta_A + \frac{\lambda \theta_g}{\kappa_A} < 0$, the term $\frac{b_A(\tau)}{\tau} A_t$ will be negative with high probability and therefore leading to a negative and declining yield curve for any maturity. On the other hand as long as the loading factor $b_A(\tau)$ is positive for any $\tau$ and provided that $\theta_A + \frac{\lambda \theta_g}{\kappa_A} > 0,^{10}$ the yield curve can increase in time to maturity even though the impact of $g b_g(\tau)$ and $m b_m(\tau)$ is negative. In the following section we explore the impact of government and monetary policy uncertainty on the nominal term structure of interest rates in more detail.

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10This condition is needed to ensure that the yield curve is increasing in maturity for reasonable parameter values. Since both $b_g(\tau), b_m(\tau) < 0$ and only $b_A(\tau) > 0, \forall \tau \geq 0$, if $\theta_A + \frac{\lambda \theta_g}{\kappa_A} < 0$ the term structure would be negative and an increasing in $\tau$ which is not consistent with empirical evidence.
3.1 Fitting the model to the data

In order to fit the model to the term structure data we proceed as follows. We estimate a subset of parameters using maximum likelihood estimation (MLE) and calibrate the remaining parameters to the nominal yield and its corresponding volatility curve simultaneously. We estimate the parameters of the policy uncertainty variables $g_t$ and $m_t$ using (exact) maximum likelihood estimation. Table 1 summarizes the results. The estimated parameters between the GPU index and the MPU index differ mainly in the magnitude of the speed of mean reversion parameter $\kappa$. More specifically Table 1 shows that $\kappa_m$ is about half the magnitude of $\kappa_m$. This implies that a fiscal policy shock will have a more permanent effect than a monetary shock has. More concretely, the half-life of a shock in $g_t$ when $\kappa_g = 0.203$ is $-\log(0.5)/\kappa_g = 1.48$ months which implies that it takes about six weeks for a shock to government policy uncertainty to die out by half. Similarly for the monetary policy shock, we have $-\log(0.5)/\kappa_m = 0.72$ months, in other words it takes about three weeks for a monetary policy shock to die out by half. Thus, shocks to monetary policy uncertainty are more transitive which leads the process to mean-revert faster to its long run mean $\theta_m$. Next, the parameters related to GDP growth $dY_t/Y_t$ and money supply growth $dM^S_t/M^S_t$ are set equal to their unconditional first and second sample moments. Furthermore, the correlation parameters $\rho^{Yg}$ and $\rho^{Mm}$ are set to their unconditional sample correlation coefficients. The preference parameters $\beta$, $\rho$ and $\xi$ as well as the structural model parameters $\delta$, $\eta_1$, $\eta_2$, $\bar{k}$ and $\bar{\pi}$ are set to reasonable values also used in other papers of the term structure such as for instance Buraschi & Jiltsov (2005), Ulrich (2013) or Xiong & Yan (2010). Our benchmark case is the log-utility investor, i.e., $\gamma = 0$. Finally, the parameters related to $\text{Table 1}$: The label 'GPU' and 'MPU' refer to the government and monetary policy index, respectively. The row 'Estimates' represents the maximum likelihood estimator and the row 'Stand. Error' shows the asymptotic robust standard errors ('Sandwich estimator') of the parameters which is based on the outer product of the Jacobian of the log-likelihood function. Estimation period is January 1990 to June 2014 (295 data points) using monthly data.

<table>
<thead>
<tr>
<th></th>
<th>GPU</th>
<th>MPU</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_g$</td>
<td>0.203</td>
<td>0.418</td>
</tr>
<tr>
<td>$\theta_g$</td>
<td>0.931</td>
<td>0.935</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>0.326</td>
<td>0.285</td>
</tr>
<tr>
<td>$\kappa_m$</td>
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</tr>
<tr>
<td>$\theta_m$</td>
<td>0.021</td>
<td>0.04</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>0.058</td>
<td>0.018</td>
</tr>
</tbody>
</table>

\[\text{11} \text{Since for the Feller diffusion the transition density is known in closed-form (non-central chi-squared), the transition density does not need to be approximated via quasi maximum likelihood techniques.}\]
the productivity process $A_t$ namely, $\kappa_A, \theta_A, \sigma_A$ and $q_A$, $\lambda$ are calibrated to match key features of the empirical term structure of interest rates and its corresponding volatility curve. The parameters not related to $g_t$ and $m_t$ are summarized in Table 2 below. Given the parameters in Table (1) and

Table 2: Summary of parameter values selected for Monte-Carlo Analysis: The parameter values for $\xi$, $\delta$, $\eta_1$, $\eta_2$ are taken from Buraschi & Jiltsov (2005) who estimate a similar model. All parameter values for the process $A_t$ as given in Equation (2) and $q_A$ are calibrated to match the average yield curve and bond volatility curve over the sample period January 1990 to June 2014. $A_0$ refers to the initial value of $A_t$.

(2) we simulate the economy $M = 1’000$-times on a monthly basis for fifty years using a standard Euler-Maruyama scheme to obtain simulated values for the three Itô diffusions $A_t$, $g_t$ and $m_t$. Having obtained simulated values for the state vector $Z_t = (A_t, g_t, m_t)$ we then compute the nominal yield curve and its corresponding term structure of bond yield volatility with time to maturity of ten years and then average over the number of Monte-Carlo simulation runs.\(^{12}\) From Panel A, we see that the model is able to fit the medium (3 years) to long run maturities very accurately, whereas at the short run the model implied yield curve is underestimating the actual level of yields. However, the overall mean error along the entire term structure is only 2.88%.\(^{13}\) Furthermore, Panel B shows that the fitted model is able to reproduce the hump-shape in the term structure of bond yield volatility which is very difficult to match using affine models of the term structure. Indeed, as Buraschi & Jiltsov (2005) note, even with their more flexible specification of essentially affine price of risk, their model cannot fully match the second moments of yields. Whereas our model captures the short end of the yield volatility curve fairly accurately, it overestimates the actual bond volatility at medium to longer maturities. Nonetheless, the error is with 4.88% relatively small.

\(^{12}\) We set the starting values of the government and monetary policy uncertainty to one, i.e. $m_0 = g_0 = 1$.

\(^{13}\) The error is calculated as $\frac{1}{T} \sum_{r \in T} [\hat{Y}(t, \tau) - Y(t, \tau)]/\bar{m}$ where $\bar{m} = \frac{1}{T} \sum_{r \in T} Y(t, \tau), \hat{Y}(t, \tau)$ is the fitted yield curve and $T = 6$, i.e. the number of maturities.
Figure 2: Comparison of empirical and fitted affine yield curve model. Panel A compares the empirical unconditional nominal yield curve based on monthly zero-coupon bonds with the model implied yield curve. Panel B compares the model-implied bond volatility term-structure to the empirical unconditional realized volatility term structure. Unconditional realized volatility is computed using monthly log-yield changes as described in Equation (36) with time to maturity is one, two, three, five, seven and ten years. Parameter values are given in Table 1 and 2, respectively. Lines in blue indicate Monte-Carlo averages of nominal yields (Panel A) and realized volatility (Panel B).

3.2 Yield curve and policy uncertainty

We now test a series of implications regarding the effect of policy uncertainty on the yield curve based on our fitted model parameters. Our preliminary empirical analysis between nominal bond yields and policy uncertainty as measured by the Baker et al. (2012) - index shows that there is significant negative correlation between economic policy uncertainty and movements in the yield curve. Splitting the index into government and monetary policy uncertainty shows that the GPU index maintains high negative correlation whereas the MPU index seems to have no correlation with nominal yields at any maturity. Using our affine yield model, we can compute the model-implied correlation in a straightforward way. More specifically, we can show that a higher level ($\theta_g$ or $\theta_m$) in either fiscal or monetary policy uncertainty will lead to a downward shift in yields.

Proposition 5. Model-implied correlation and level uncertainty effect on yield curve.

1. Nominal yields are negatively correlated with either fiscal or monetary policy uncertainty, i.e.

$$\rho[Y(t,\tau), g_t] \leq 0, \quad \rho[Y(t,\tau), m_t] \leq 0, \quad \forall \tau \geq 0$$
2. Bond yields are decreasing in both $\theta_g$ and $\theta_m$.

$$\frac{\partial Y(t, \tau)}{\partial \theta_i} = \frac{\kappa_i}{\tau} \int_0^\tau b_i(u) du < 0, \quad i \in \{g, m\}, \quad \forall \tau \geq 0.$$ 

The results in Proposition 5 above are in line with the empirical observations. The reason why the model implied correlation is negative for any maturity can be directly deduced from the covariance between $Y(t, \tau)$ and $g_t$ which is,

$$C[Y(t, \tau), g_t] = \frac{b_A(\tau)}{\tau} \frac{\theta_g \lambda \sigma^2_g}{2\kappa_g (\kappa_A + \kappa_g)} + \frac{b_g(\tau) \theta_g \sigma^2_g}{2\kappa_g}. \quad (29)$$

Given the fitted parameters in Table (1) and (2), the factor loading $b_g(\tau)$ is always negative which implies that the second term in Equation (29) will also be negative. The first term is negative as well because $b(\tau) \geq 0$ and $\lambda < 0$ so that overall the covariance is negative for any $\tau$. In this setting, we obtain a model-implied average (along maturity $\tau$) correlation of -0.3934 which is very close to the empirical sample correlation between nominal yields and the GPU (-0.4037). Likewise, since $b_m(\tau) \leq 0$ we obtain $C[Y(t, \tau), m_t] = \frac{b_m \sigma^2_m}{2\kappa_m} \frac{b_m(\tau)}{\tau} \leq 0, \quad \forall \tau \geq 0$. Comparing this model correlation coefficient of -0.0019 to its empirical counterpart (-0.0013) we see that the model is able to match both sample correlation coefficients. Next, we carry out a comparative static exercise where we analyze the dependence of the nominal yield curve with respect to changes in some selected model parameters. We start the analysis by varying the government policy variable $\theta_g$ and the impact factor $\lambda$. In Figure 3 we plot $Y(t, \tau)$ as a function of time to maturity $\tau$. A first important observation from Panel A is that when the long run level of government policy uncertainty is high and $\lambda < 0$, the nominal bond trades at a higher discount and hence the yield curve shifts downwards as predicted by the results in Proposition 5. The intuition for this result is that government policy uncertainty affects the long run trend of productivity negatively when $\lambda < 0$. Recall that $\mathbb{E}[A_t] = \theta_A + \frac{\lambda g_t}{\kappa_A}$ and since the equilibrium growth rate of capital is driven by productivity $A_t$, a higher $g_t$ combined with $\lambda < 0$ leads the growth rate of capital to decline which in turn leads to an increase in bond prices. Therefore, the representative investor reduces investment into capital and thereby pushes up nominal bond prices which leads to a decrease in yields across all maturities. Furthermore, as $\theta_g$ increases the stationary variance of $A_t$, policy uncertainty renders long run economic prospects more uncertain, which is why the representative agent’s demand for more safe assets increases. Next we analyze the effect of
changing the level of risk aversion $\gamma$, the parameter $q_A$ and the monetary policy reaction variables $\eta_1$ and $\eta_2$. Panel A in Figure 4 shows that increasing the level of risk aversion has a significant effect on the level of the nominal term structure. When the representative investor is more risk averse, in equilibrium, his demand for the bond is higher which leads to a lower nominal yields. Note that from the equilibrium capital growth rate $\mu_{K^*}(A_t, g_t) := \mu_Y + q_A A_t - \beta - \delta + \gamma \beta (\phi_0(A_t, g_t) - L)$ with $\phi_0(A_t, g_t) = \phi_{00} + \phi_{0A} A_t + \phi_{0g} g_t$, when $\gamma \neq 0$ we have that government policy uncertainty is also driving capital accumulation growth. Since $\phi_{0A} > 0$ and $\phi_{0g} = -\frac{(1+\xi)\sigma_Y^2 + 2\lambda \phi_{0A}}{2(\beta + \kappa_A)} < 0$ we have for $\lambda < 0$ that

$$\mu_{K^*}(A_t, g_t) > \mu_{K^*}^{\log}(A_t) = \mu_Y + q_A A_t - \beta - \delta$$

(30)

where $\mu_{K^*}^{\log}(A_t)$ is the capital accumulation growth rate when the representative investor has log-utility and $L < 0$ for the parameters as in Table (2). Furthermore, since $C_A = \frac{(\eta_1 + \eta_2(1+\xi))(q_A + \beta \gamma \phi_{0A})}{1-\eta_2}$ a higher level of risk aversion also lowers $C_A$ which in turn causes the impact of $b_A(\tau)$ to decline and therefore amplifies the downward shift in the term structure even further. In the very extreme case

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14The effect is only accurate up to a first order perturbation in $\gamma$. Due to the highly non-linear dependence of the optimal controls $C_t^*$ and $M_t^{d*}$ on $\gamma$, the effect of changing risk aversion is accurate only in a vicinity around the log-utility case.
Nominal Yield curve $Y^N(t, \tau)$

Figure 4: Nominal yield curve $Y(t, \tau)$ as given in Equation (28) with time to maturity $\tau$ when the level of risk aversion $\gamma$ (Panel A), productivity impact factor $q_A$ (Panel B), and the central bank reaction parameters $\eta_1$ and $\eta_2$ (Panel C and D) vary. If not otherwise stated the parameter values are given in Table 2. Solid lines in red and blue indicate the Monte-Carlo average of the nominal yields when $\gamma$ (Panel A), $q_A$ (Panel B), $\eta_1$ and $\eta_2$ (Panel C and D) vary. Dashed lines correspond to 5th and 95th percentile of the estimated yield curve respectively.

when $\gamma$ becomes very large, then $C_A < 0$ and the yield curve becomes negative. Next, the parameter $q_A$ weights the degree of time-variation in output growth. For the boundary case when $q_A = 0$ the growth rate of GDP becomes entirely constant and the nominal term structure is independent of productivity $A_t$ as $C_A = b_A(\tau) = 0$, $\forall \tau \geq 0$. Thus the higher the level of time-variation in trend output growth, and provided that $\theta_A + \frac{\lambda \theta_A}{\kappa_A} > 0$, a larger impact of $A_t$ increases the level of nominal yields considerably (see Panel B). Finally, the remaining Panels C and D show the sensitivity of the
term structure with respect to changes in the central bank reaction parameters \( \eta_1 \) and \( \eta_2 \). Panel C compares the case when the central bank is pursuing an active \( \eta_1 \neq 0, \eta_2 \neq 0 \) versus a passive \( \eta_1 = \eta_2 = 0 \) money supply policy. In the latter case, the money supply process is exogenous and reduces to

\[
\frac{dM_S^t}{M_S^t} = \mu_M dt + \sigma_M \sqrt{m_t} dW^M_t
\]  

Thus the real and the nominal side are completely decoupled from each other. However, this does not necessarily imply that the term structure shifts downward as it depends on the relative impact of the capital accumulation growth versus the inflation. As \( \eta_1 > 0 \), above target capital accumulation growth, i.e., \( \frac{dK^*_t}{K^*_t} - \bar{k}dt > 0 \) will eventually lead to an increase in money supply and in equilibrium to a higher discount on the nominal bond. By a similar argument, whenever actual inflation is below target inflation, i.e., \( \frac{dp^*_t}{p^*_t} - \bar{\pi}dt < 0 \), the central bank’s response is to increase the money supply as \( \eta_2 < 0 \). Thus, whether on average the term structure shifts upward or downward, when either \( \eta_1 = 0 \) or \( \eta_2 = 0 \), crucially depends on whether the central bank is more responsive to deviations of target capital versus inflation growth, i.e., whether \( \frac{dK^*_t}{K^*_t} - \bar{k}dt \) has a larger impact than \( \frac{dp^*_t}{p^*_t} - \bar{\pi}dt \). From Panel D, this also implies that the real sector has a significant impact on money supply and therefore on the nominal side of the economy. In the next section, we further discuss the impact of government and monetary policy on the equilibrium short rate and the market prices of risk.

### 3.3 Equilibrium nominal short rate and the term premium

We now discuss how the short end of the term structure of interest rates and the bond risk premium (term premium) is affected by fiscal and monetary policy uncertainty. From Equation (23) we can derive the nominal short rate by taking the limit \( \tau \to 0 \) and derive the model-implied term premium.

**Proposition 6** (Equilibrium nominal short rate and bond risk premium). With time-separable CRRA utility, we have the following first order asymptotic results:

1. The nominal short rate \( R_t \) and the nominal price of output risk \( \lambda^{N,Y}_t \) as well as the market
price of monetary risk $\lambda_{t}^{N,M}$ are given by

$$R_{t} = \left(\mu_{Y} - \beta - \delta - \tilde{k}\right)\eta_{1} + \beta + \mu_{M} - \eta_{2}\left(\mu_{Y} + \pi - \delta\right) + \frac{\beta\left(\eta_{1} - \eta_{2}\right)(L - \phi_{00})}{1 - \eta_{2}} - \frac{\sigma_{M}^{2}}{\left(\eta_{2} - 1\right)^{2}}m_{t}$$

$$+ \left(q_{A} + \gamma\beta\phi_{0A}\right)\left(\frac{\eta_{1} - \eta_{2}}{1 - \eta_{2}}\right)A_{t} - \left(\frac{\left(\eta_{1} - \eta_{2}\right)^{2}\sigma_{Y}^{2}}{(\eta_{2} - 1)^{2}} + \frac{\beta\phi_{0A}(\eta_{1} - \eta_{2})}{\eta_{2} - 1}\right)g_{t}$$

$$\lambda_{t}^{N,Y} = \frac{\eta_{1} - \eta_{2}}{1 - \eta_{2}}\sigma_{Y}\sqrt{g_{t}}, \quad \lambda_{t}^{N,M} = \frac{\sigma_{M}}{1 - \eta_{2}}\sqrt{m_{t}}. \quad (32)$$

2. The bond risk premia $RP(t, \tau)$ per unit of time is given by

$$RP(t, \tau) := \frac{1}{dt}E_{t}\left[dB(t, \tau) - R_{t}dt\right] = b_{g}(\tau)\rho^{gY}\sigma_{g}\lambda_{t}^{N,Y}\sqrt{g_{t}} + b_{m}(\tau)\rho^{Mm}\sigma_{m}\lambda_{t}^{N,M}\sqrt{m_{t}} \quad (34)$$

where $b_{g}(\tau)$ and $b_{m}(\tau)$ are time to maturity $\tau = T - t$ functions given in Equation (25) and (26), respectively.

The nominal short rate and the nominal market prices of risks are all influenced by both the real and nominal sector of the economy. This is a direct consequence of A4, i.e., the assumption that the monetary authority sets money supply growth according to a Taylor-rule, which implies that money supply growth is a function of the real side of the economy. In the special case when money supply is entirely decoupled from the real sector, i.e., $\eta_{1} = \eta_{2} = 0$ the nominal short rate reduces to $R_{t} = \mu_{M} + \beta - \sigma_{M}^{2}m_{t}$ and $\lambda_{t}^{N,Y} = 0$ so that the real side does not affect the nominal short rate and output risk is no longer a nominal risk factor. In the general case when $\eta_{1} \neq 0$ and $\eta_{2} \neq 0$, the nominal price of risk decomposes into two state-dependent market prices of risks namely, $(\lambda_{t}^{N,Y})$ and $(\lambda_{t}^{N,M})$, which are driven by government and monetary policy risk, respectively. Furthermore, the sign of those market prices of risk is determined by $\eta_{1}$ and $\eta_{2}$, the parameters controlling the intensity of adjustments to the long run real output growth target $\tilde{k}$ and inflation target $\tilde{\pi}$ and therefore can become negative depending on the values of $\eta_{1}$ and $\eta_{2}$. Additionally, $R_{t}$ depends on the money supply control variables $\eta_{1}$ and $\eta_{2}$ in a non-linear way. This suggests that relatively small fluctuations in either productivity $A_{t}$, monetary or fiscal policy uncertainty may lead to drastic movements in the nominal short rate. Lastly, risk aversion affects $R_{t}$ through three different channels. First, it affects

---

15 The results in Proposition 6 reveal that equilibrium relations such as the expected bond excess premium and interest rate volatility as well as the forward term premium, i.e., violation of the expectation hypothesis, will be driven by government and monetary policy uncertainty whenever $\eta_{1} \neq 0$ and $\eta_{2} \neq 0$. 

23
its level (second term in Equation (32)), more precisely, an increase in $\gamma$ leads to a decline in the short rate as $\frac{\beta(n - m)(1 - \delta_m)}{\eta_2 - 1} > 0$. Secondly, risk aversion affects of both real sector variables $A_t$ as well as $g_t$ and therefore impacts also the slope at the short end of the term structure.

### 3.4 Bond Yield Volatility

In this section, we analyze the impact of government and monetary policy uncertainty on bond yield volatility. Many empirical studies find that long run bond yields exhibit higher volatility than implied by the expectation hypothesis. For instance, using a rational expectation model of the term structure of interest rates, long term bond yields should not be too volatile as averaging smooths out variability in yields. Shiller (1979) shows empirically that long term bond yields exhibits excess volatility relative to their model-implied values. Furthermore, Piazzesi & Schneider (2006) show, using a representative agent-based framework, that the model explains a smaller fraction of observed volatility of the long-end yields than of the short-end yields. Xiong & Yan (2010) argue that excess bond volatility might be due to differences in beliefs about the long run level of inflation. They show that a higher belief dispersion leads to volatility amplification which allows them to account not only for the empirically observed high bond yield volatility but also for the hump-shape of the term structure of bond volatility.\(^{17}\)

We will now explore the key determinants in our model that allows to reproduce the hump-shape in bond volatility. Using the results from Proposition 1, one can now derive the unconditional variance of nominal yields.

**Corollary 1 (Term Structure of Nominal Bond Yield Variance).** Let $Y(t, \tau)$ denote the current time $t$ yield with maturity $\tau$. Then the unconditional term structure of bond yields is given by

$$
\mathbb{V}(Y(t, \tau)) = \frac{b_A^2(\tau)}{\tau^2} \mathbb{V}(A_t) + \frac{b_g^2(\tau)}{\tau^2} \mathbb{V}(g_t) + \frac{b_m^2(\tau)}{\tau^2} \mathbb{V}(m_t) + 2 \frac{b_A(\tau) b_g(\tau)}{\tau^2} \mathbb{C}(A_t, g_t) \quad (35)
$$

where

$$
\begin{align*}
\mathbb{V}(A_t) &= \frac{\theta_g (\kappa_A (\kappa_A + \kappa_g) \sigma_A^2 + \lambda \sigma_g^2)}{2 \kappa_A \kappa_g (\kappa_A + \kappa_g)}, \\
\mathbb{V}(g_t) &= \frac{\theta_g \lambda \sigma_g^2}{2 \kappa_g (\kappa_A + \kappa_g)}, \quad \mathbb{V}(m_t) = \frac{\theta_m \sigma_m^2}{2 \kappa_m}
\end{align*}
$$

\(^{16}\)Note that an increase in risk aversion implies that $\gamma$ becomes more negative, since $\gamma$ is one minus the coefficient of risk aversion.

\(^{17}\)Closely related to this sometimes termed ‘excess volatility puzzle’ phenomenon are also the findings of Gürkaynak et al. (2005b) who document, that bond yields exhibit excess sensitivity to macroeconomic announcements.
Corollary (1) shows that bond yield volatility is affected by fiscal policy uncertainty through three channels. First, it enters as a factor in the unconditional volatility formula, second, it enters through the cross term with the productivity factor $A_t$ whenever the loading parameter $\lambda \neq 0$ and third its parameters $\kappa_g$, $\theta_g$ as well as $\sigma_g$ affect the stationary variance $\mathbb{V}(A_t)$. Furthermore, the parameters of both the productivity process, i.e., $\kappa_A$, $\lambda$ and $\sigma_A$ as well as the fiscal policy uncertainty process $\kappa_g$ and $\sigma_g$ enter the volatility expression government in a non-linear complex way, rendering unambiguous comparative statics difficult. In order to explore the key determinants in generating the hump-shape in bond yield volatility, we plot in Figure 5 the contribution of both the fiscal policy uncertainty and its covariance term with productivity.\textsuperscript{18} As Figure 5 shows, the contribution of

![Contribution to bond volatility](image)

**Figure 5:** Term Structure of Nominal Bond Yields: The plot shows the contribution of government policy factor $b_g^2(\tau)/\tau^2$ and the cross term $b_A(\tau)b_g(\tau)/\tau^2$.  

both the factor loadings $b_g(\tau)$ and the cross term $b_A(\tau)b_g(\tau)$ are hump shaped, which causes the term structure of yield volatility to exhibit a similar pattern. Further, the Figure indicates that the covariance term $C(A_t, g_t)$ is contributing significantly to generating the hump at around 2 year maturity. Lastly, we analyze the sensitivity of the term structure of bond yield volatility with respect

\textsuperscript{18}We did not include the contribution of the productivity factor as it is a monotonically decreasing function in $\tau$ and the factor $m_t$ which has, with the given parameter set, very little impact on the term structure of bond yield volatility.
to the model parameters in more detail. After all, allowing $A_t$ and $g_t$ to be correlated is simply a necessary condition in order to be able to reproduce the hump-shape, but it is not sufficient as its shape crucially depends on the structural model parameters. For this reason, we investigate the behavior of the term structure of bond yield volatility with respect to changes in some selected model parameters. The contribution of $\hat{g}_t$ depends on both the model parameters of the processes $A_t$ and $g_t$ as well as the structural model parameters. Panel A in Figure 6 above shows that the term structure

![Nominal Term Structure of Bond Volatility](image1)

![Nominal Term Structure of Bond Volatility](image2)

![Nominal Term Structure of Bond Volatility](image3)

![Nominal Term Structure of Bond Volatility](image4)

**Figure 6:** Sensitivity of bond volatility term structure with respect to model parameters: Solid lines in red and blue indicate the Monte-Carlo average of the nominal yields when $q_A$ (Panel A), $\kappa_A$ (Panel B), $\lambda$ and $\sigma_A$ (Panel C and D) vary. Dashed lines correspond to 5th and 95th percentile of the estimated yield curve respectively.

of bond yield volatility exhibits a fundamentally different shape whenever $\lambda$ is negative or when $\lambda$
is set to zero in which case government policy uncertainty does not affect the drift of productivity. Not only is the level of the term structure significantly higher than compared to the case when $\lambda = 0$ but also, bond yield volatility becomes hump-shaped in time to maturity. A similar conclusion can be drawn from Panel C where the speed of mean reversion $\kappa_A$ varies. When $\kappa_A$ is low, a positive (negative) shock to $A_t$ raises (lowers) the level of the productivity factor $A_t$ not only in the current but also in the subsequent periods preceding the shock. In other words, a more permanent shock ($\kappa_A \uparrow$) implies that uncertainty shocks affect productivity longer. Further, Panel B shows that when the growth rate of GDP has a larger time-varying component, i.e., $(q_A \uparrow)$, real shocks have a higher impact on the nominal term structure of bond yield volatility. In the extreme case when $q_A = 0$ and thus GDP grows at a constant rate $\mu_Y$, the productivity factor $\frac{b_A^2(\tau)}{\tau^2} \mathcal{V}(A_t)$ disappears completely which causes bond volatility to decline. Finally, increasing production volatility shows that the term structure shifts upward but becomes monotonically decreasing in time to maturity whenever $\sigma_A$ is large enough. This follows because the factor $\frac{b_A^2(\tau)}{\tau^2} \mathcal{V}(A_t)$ increases proportionally in $\sigma_A^2$ and since $\frac{b_A^2(\tau)}{\tau^2}$ is monotonically decreasing, placing higher weight on that factor lowers total volatility for every maturity. Next we analyze the sensitivity of the of bond yield volatility with respect to changes in the government uncertainty parameters and risk aversion in Figure 7 below. Panel A and B show that the speed of mean reversion level $\kappa_g$ and volatility $\sigma_g$ have similar effect on bond volatility. A more persistent fiscal uncertainty shock (low $\kappa_g$) shows that the bond volatility term structure is not only significantly higher but also hump-shaped in time to maturity. Moreover, as can be seen from Panel B, the magnitude of a policy shock appears to have a similar effect on the term structure of bond volatility. Panel C shows that the long run mean of government policy uncertainty $\theta_g$ increases yield dispersion proportionally. This can also be seen directly from Corollary 1 as

$$\frac{\partial \mathcal{V}(Y(t, \tau))}{\partial \theta_g} = \frac{b_A^2(\tau)}{\tau^2} \left( \kappa_g (\kappa_A + \kappa_g) \sigma_A^2 + \lambda^2 \sigma_g^2 \right) + \frac{b_A^2(\tau)}{\tau^2} \mathcal{V}(g_t) \frac{\sigma_g^2}{2 \kappa_g} + 2 \frac{b_A(\tau) b_g(\tau)}{\tau^2} \frac{\theta_g \lambda \sigma_g^2}{2 \kappa_g (\kappa_A + \kappa_g)} > 0$$

since for this set of parameters $b_g(\tau) \leq 0$, $\forall \tau \geq 0$. Interestingly, increasing the level of risk aversion leads to a parallel decline in bond volatility. This is the result of the impact $\gamma$ has on $b_A(\tau)$ and to a lesser extent also on $b_g(\tau)$. Note that $\frac{\partial C_A}{\partial \gamma} < 0$ and therefore, a higher level of risk aversion reduces the contribution of the productivity factor $A_t$ to the bond variance, which in part explains the downward shift in bond volatility. As a last comparative static analysis we analyze the sensitivity
of the bond volatility term structure when changing the central banks reaction parameters $\eta_1$ and $\eta_2$. Whereas Panel A shows that whenever the money supply process is entirely exogenous, the level of bond yield volatility is reduced significantly, Panel B shows that this reduction is due to setting $\eta_1 = 0$, in which case real or fiscal policy shocks do not affect the money supply directly, but only indirectly through affecting the equilibrium price level $p_t^*$. 

**Figure 7:** Sensitivity of bond volatility term structure with respect to model parameters: Solid lines in red and blue indicate the Monte-Carlo average of the nominal yields when $\kappa_g$ (Panel A), $\sigma_g$ (Panel B), $\theta_g$ and $\gamma$ (Panel C and D) vary. Dashed lines correspond to 5th and 95th percentile of the estimated yield curve respectively.
4 Empirical Analysis

In this section we conduct an empirical analysis in order to examine the testable predictions of our model. Our first prediction can be deduced directly from Corollary 5, which implies that when either fiscal or monetary policy uncertainty increases, i.e., the long run level of policy uncertainty increases \( \theta_i, \ i \in \{ g, m \} \), nominal yields fall. In the first part of our empirical analysis, we investigate the joint effect of government and monetary policy uncertainty on the yield curve and its term structure of volatility by using the economic policy uncertainty (EPU) index developed by Baker et al. (2012), which of course contains both uncertainty related to government as well as monetary policy uncertainty and thus is only an imperfect measure of either source of policy uncertainty. Thus, in our model this would be approximately equivalent to setting \( EPU_t \approx g_t + m_t \). The reason why we carry out this investigation is to first understand the basic relationship between contemporaneous yields, bond volatility movements as well as bond risk premia with policy uncertainty and second, to explore the differences in the predictive power of fiscal against monetary policy uncertainty in order to understand which part of economic policy uncertainty is driving our results. Furthermore, our model predicts that bond volatility is higher in the case when the economy is subject to government
policy uncertainty and is hump-shaped in time to maturity.$^{19}$ The term structure of bond yield volatility is higher when including a fiscal policy uncertainty factor because of two reasons: First, the affine yield model reduces from a three factor model to a two factor model. However, this is not enough to explain why the term structure should shift downwards since the covariance between the factors maybe negative. However, setting $\lambda < 0$ and since $b_g(\tau) < 0$, $\forall \tau \geq 0$ it follows from Corollary 1 that the covariance term $\frac{b_A(\tau) - b_g(\tau)}{\tau} C(A_t, g_t)$ is positive. Furthermore, we argue that the hump-shape pattern of the term structure of bond volatility as seen from Figure 5 is mainly driven by fiscal uncertainty. As the analysis in the previous section has shown, this hump-shape structure of bond yield volatility is very sensitive to the impact government policy uncertainty has on the economy’s productivity. In other words, whenever fiscal policy uncertainty has a negative impact on the growth rate of productivity, i.e. $\lambda < 0$, bond volatility is not only higher for any $\tau$ but also hump-shaped. Lastly, Equation (99) implies that bond risk premia in our model will be driven by both fiscal and monetary policy uncertainty. We summarize our four hypotheses below.

**Hypothesis**

- **H1**: Higher (lower) economic policy uncertainty decreases (increases) nominal yields. From Proposition 5, this effect is mainly driven by government policy uncertainty.

- **H2**: Higher (lower) economic policy uncertainty increases (decreases) nominal yield volatility. From Corollary 1, this effect is again mainly driven by government policy uncertainty.

- **H3**: The contribution of economic policy uncertainty to the term-structure of bond yield volatility is hump-shaped. From Corollary 1, this effect is again mainly driven by government policy uncertainty.

- **H4**: From Equation (99), bond risk premia are affected by both monetary and government policy uncertainty.

In what follows, we will empirically test the hypothesis posited above, by regressing nominal yields and yield volatility on the EPU index and our time series of government and monetary policy as well as a set of control variables.

$^{19}$The inclusion of monetary policy uncertainty of course increases bond volatility as well. However, given our fitted parameter values, its impact on the bond volatility curve is negligible.
4.1 Data

We obtain monthly Treasury Bill yields with maturity one, two, three, five, seven and ten years from the Federal Reserve Board ranging from January 1990 until July 2014 which gives us a total of 294 observation. From Datastream, we collect monthly data on a total of eight macro variables which include several controls also used by Ang & Piazzesi (2003), ?, Ludvigson & Ng (2009) or Joslin et al. (2014) in their study of the economic determinants of the term structure of nominal interest rates. Four variables characterize real business cycle activity (RA) in our sample. They include employment (Emp), industrial production (IP), the US (ISM) New Orders Index (NO) and new private housing units started (NPHU). The variables personal income (PI), consumer price index (CPI), producer price index (PPI) and the CRB Spot Commodity Index (CRB) summarize our inflation factors (IF). We then compute monthly log-growth rates over one year for each of the macro control variables. As a further proxy for (policy) uncertainty, we also include monthly observations of the VIX index. Further controls related to economic conditions include the Chicago Fed National Activity Index (CFNAI) and the NBER recession dummy variable (NBER) which we obtain from the FRED database (St. Louis Fed).

We also use treasury bond implied volatility (TIV) based on weighted average of 1 month options on treasury bonds with maturity 2, 5, 10 and 30 years as a proxy for bond market volatility. In addition, we collect three time series which we refer to as 'Financial Variables' (FV). They include a measure of credit spread (CS), i.e., the difference in yields of Aaa-rated and Baa-rated corporate bonds, the monthly log growth rate of the S&P composite dividend yield index (DY) which has been shown to have forecasting power by Fama & French (1989) and the term spread, i.e., 10Y yield less the federal funds rate (TS). As a measure for bond yield volatility we construct monthly realized volatility using

\[ V_t(Y(t, \tau)) = \sqrt{\sum_{d=1}^{D-1} \left( \log \left( \frac{Y(d+1, \tau)}{Y(d, \tau)} \right) \right)^2}, \quad Y(t, \tau), \ d \in \{1, \ldots, D - 1\} \]  

where \(d\) denotes the number of daily observations (about 20 business days per month) and \(\tau=1Y, 2Y, 3Y, 5Y, 7Y\) and \(10Y\) in order to construct a time series of monthly realized bond yield volatility.

\[ V_t(Y(t, \tau)) = \sqrt{\sum_{d=1}^{D-1} \left( \log \left( \frac{Y(d+1, \tau)}{Y(d, \tau)} \right) \right)^2}, \quad Y(t, \tau), \ d \in \{1, \ldots, D - 1\} \]  

\[ V_t(Y(t, \tau)) = \sqrt{\sum_{d=1}^{D-1} \left( \log \left( \frac{Y(d+1, \tau)}{Y(d, \tau)} \right) \right)^2}, \quad Y(t, \tau), \ d \in \{1, \ldots, D - 1\} \]  

The reason why we include the CFNAI and NBER regressors is to test whether either monetary or fiscal policy uncertainty has predictive power after controlling for the state the economy is in. This is because it is reasonable to assume that uncertainty about the government’s future policy choice is in general larger in weaker economic conditions.
Finally, in order to compare the magnitude of the impact of each of the regression controls in the cross-section, we standardize all variables to have mean zero and standard deviation equal to one.

4.2 Construction of government and monetary policy uncertainty index

The economic policy uncertainty (EPU) index developed by Baker et al. (2012) has been recently used by a number of studies. For instance, Pastor & Veronesi (2013) show that government policy uncertainty carries a risk premium, and that stocks are more volatile and more correlated in times of high uncertainty. Brogaard & Detzel (2012) use the same index and find that economic policy uncertainty forecast future market excess returns. Similarly, Gulen & Ion (2012) show that policy-related uncertainty is negatively correlated with firm and industry level investment, in other words, when policy uncertainty increases firm’s tend to reduce their investment. In our setting, we wish to distinguish between fiscal or real policy uncertainty and monetary uncertainty. Of course, as Kelly et al. (2013) argue, it is difficult isolate exogenous variation in political uncertainty as it likely depends on various factors such as overall macro uncertainty. In this regard, the EPU index may not only capture government related uncertainty but can be interpreted as a broader measure of uncertainty about economic fundamentals. We try to isolate the different sources of uncertainty, i.e., fiscal versus monetary policy uncertainty, by decomposing the index into uncertainty related to fiscal and monetary policy. The EPU index is constructed from three main components, namely a news impact part which is based on news paper discussing economic policy uncertainty, a component that summarizes reports by the Congressional Budget Office (CBO) that compile lists of temporary federal tax code provisions and a third component called ‘economic forecaster disagreement’ which draws on the Federal Reserve Bank of Philadelphia’s Survey of Professional Forecasters and summarizes data on consumer price forecast dispersion and predictions for purchases of goods and services by state, local and federal government. We argue that disagreement about temporary federal tax code provisions and forecast variation in local state and federal purchases of goods and services are sources of fiscal policy uncertainty, whereas disagreement about future inflation can be related to monetary uncertainty. The news-based component of the index is likely to consist of both real and monetary policy uncertainty. In order to disentangle the two types of uncertainties, we have also downloaded
additional time-series (Categorical EPU Data) of policy uncertainty which are also available from http://www.policyuncertainty.com/. This data set contains time-series on uncertainty related to government and monetary policy as well as further categorical variables. To construct our measure of fiscal policy uncertainty, we extract from the 'categorical EPU Data' set the time-series 'fiscal policy' as well as 'taxes' and 'government spending'. Additionally, we include from the main EPU index the time series 'FedStateLocal Ex disagreement' and the 'Tax expiration' to obtain our index of 'government policy uncertainty' labeled as 'GPU'. We place half of the weight to the time series 'fiscal policy' uncertainty and the other half is equally distributed among the remaining series.\footnote{We divided the time series 'Tax expiration' by a factor of 10 because its impact would otherwise pull the index up substantially at the end of the time series. Furthermore, as tax laws are only altered infrequently, the 'Tax expiration' remains constant over several months up to 2 years and thus its unscaled inclusion would lead to an underestimation in policy uncertainty variation.} Our measure of 'monetary policy' uncertainty which we abbreviate by 'MPU' consists of the time series 'monetary policy uncertainty' and 'CPI disagreement' of which each obtain weight 1/2.

4.3 Policy Uncertainty and the Yield Curve

We start our empirical analysis by investigating the relationship between the yield curve and the EPU index. To be more precise, our first regression is the following

\[ Y(t, \tau) = \beta_0 + \beta_1 PU^i_t + \epsilon_t, \quad i \in \{ EPU, GPU, MPU \} \]  

(37)

where \( PU^i_t \) denotes either the EPU, the GPU or MPU index at time \( t \) respectively and \( \epsilon_t \) is the regression error term. According to our first hypothesis H1 we expect the coefficient in Equation (37) to be negative for all the three indexes. Furthermore, in order to address potential concerns about robustness of our results, we compute Newey-West standard errors with five lags\footnote{We set the number of lags according to Newey & West (1994), who proposed the following formula: \( L = \left[ 4(T/100)^2/9 \right] \), where \( T \) is the length of the time series.} to account for heteroskedasticity and autocorrelation (HAC) in residuals. Additionally, we run the same regression as in Equation (37) above, just replacing the \( PU^i_t \) index with the VIX index. Since the VIX index is also a measure of overall economic uncertainty and positively correlated (0.45) with the EPU index, we likewise expect the regression coefficient to be negative. Finally, we regress \( Y(t, \tau) \) on both the EPU, GPU, MPU and the VIX index to assess which variable exhibits greater predictive
In Table 3 we summarize the results. The first section labeled 'EPU' shows that higher policy uncertainty EPU reduces yields across all maturities and thus is consistent with our model hypothesis H1. The effect is significant and decreasing in \( \tau \), indicating that the short end of the yield curve is more responsive to shocks in policy uncertainty than the long end is. Furthermore, the adjusted \( R^2 \) ranges from 0.18 at the end of the yield curve to 0.29 (1,2 years maturity). Thus, the single factor EPU accounts for roughly 25% of total variability in the term structure of nominal interest rates. Decomposing the index shows that only government policy uncertainty is a statistically

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</tr>
<tr>
<td></td>
<td>( t_{VIX} )</td>
<td>-1.758</td>
<td>-1.992</td>
<td>-2.189</td>
<td>-2.503</td>
<td>-2.508</td>
</tr>
<tr>
<td></td>
<td>( R^2_{adj} )</td>
<td>0.281</td>
<td>0.276</td>
<td>0.275</td>
<td>0.262</td>
<td>0.243</td>
</tr>
</tbody>
</table>

Table 3: Summary of regression results: Table displays the slope coefficients of the regression (1)-(3) \( Y(t, \tau) \) on \( PU_i \), \( i \in \{EPU, GPU, MPU\} \), (4) \( Y(t, \tau) \) on \( VIX \), (5) \( Y(t, \tau) \) on \( EPU \) and \( VIX \) and (6) \( Y(t, \tau) \) on \( GPU, MPU \) and \( VIX \) for \( \tau=1Y, 2Y, 3Y, 5Y, 7Y \) and \( 10Y \). Values in brackets below represent HAC-robust \( t- \) statistics. \( R^2_{adj} \) refers to adjusted coefficient of determination.

\(^{23}\)For all the regressions tables that follow the intercept estimate \( \hat{\beta}_0 \) and corresponding HAC errors are not displayed.
significant predictor whereas monetary policy uncertainty does not have any explanatory power at any maturity. The section labeled 'VIX' in Table 3 shows that, similar to the EPU index, a rise in the VIX index also leads to a statistically significant (only at the 10% confidence level) decline in nominal yields along the entire yield curve. However, there are two important differences compared to the EPU and GPU index above: First, the impact of the VIX is substantially smaller for any $\tau$ as the impact of the EPU index. Second, its explanatory power as measured by $R^2_{adj}$ is also considerably lower, namely 2% compared to 18-29% and 10-20% above. Third, the section labeled 'EPU & VIX' in Table 3 shows that the only significant predictor is the EPU index, as the regression coefficient for the VIX is statistically insignificant across all maturities, once both regressors are added to the regression equation. Lastly, row 'GPU, MPU & VIX' reveals that government policy uncertainty is the most economic and statistically significant predictor along the entire term structure. Whereas the sign remains the same for the VIX regressor, it is now uniformly positive for the MPU index for any $\tau$. Next, we add the economic condition (EC) controls 'CFNAI', 'NBER' and 'TIV', a convexity control variable labeled 'conv',\textsuperscript{24} financial variables (FV) as well as the real activity (RA) and inflation factors (IF) to the regression equation. Overall, Table 4 shows that EPU regression coefficient remains significant for any maturity and across all regressions. However, compared to Table 3 above, its impact is considerably reduced, especially once the financial variables credit spread, S&P 500 dividend yield and the term spread as well as the inflation factors are added to the regression equation. The reason for the decline in magnitude of the EPU coefficient, once we control for financial factors, is because those regressors exhibit strong positive correlation with the EPU index. Not surprisingly, their estimated impact on contemporaneous yield changes is also negative. Their average is -0.467 (CS), -0.54 (DY) and -0.24 (TS). Furthermore, convexity is an important predictor for instantaneous yield movements, as not only is its estimated coefficient positive and significant but also its magnitude is large. Whereas the $R^2_{adj}$ increases only moderately after controlling for economic conditions and real activity, it surges after adding both the financial and the inflation factors (see row 'EC+FV' and 'EC+FV+IF) which suggests that they are able to explain a large part of the variation in contemporaneous movements of the term structure. Indeed, as the row labeled 'Full Regression' shows, growth in the consumer price index (CPI) has a statistically

\textsuperscript{24}We extract the convexity adjustment factor from a principal component analysis of yields. We use the third principal component factor to proxy convexity adjustment.
Table 4: Summary of regression results: The table displays slope coefficients of the regression, $Y(t, \tau)$ on $EPU_t$ and EC controls (EC), $Y(t, \tau)$ on $EPU_t$ and EC, FV variables (EC+FV), $Y(t, \tau)$ on $EPU_t$ and EC, FV, RA variables (EC+FV+RA), $Y(t, \tau)$ on $EPU_t$, and EC, FV, IF controls (EC+FV+IF) and $Y(t, \tau)$ on $EPU_t$, and EC, FV, RA, IF controls (Full Reg.) for $\tau = 1Y, 2Y, 3Y, 5Y, 7Y$ and $10Y$. All regressions include a convexity adjustment factor. Values in brackets below represent HAC-robust $t$-statistics. $R_{adj}^2$ refers to adjusted coefficient of determination. ***, **, * represent 1%, 5%, and 10% statistical significance, respectively.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>1Y</th>
<th>2Y</th>
<th>3Y</th>
<th>5Y</th>
<th>7Y</th>
<th>10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC</td>
<td>$EPU$</td>
<td>-1.51***</td>
<td>-1.46***</td>
<td>-1.37***</td>
<td>-1.18***</td>
<td>-1.03***</td>
</tr>
<tr>
<td>$t_{EPU}$</td>
<td>(-10.28)</td>
<td>(-9.82)</td>
<td>(-9.33)</td>
<td>(-8.12)</td>
<td>(-7.31)</td>
<td>(-6.32)</td>
</tr>
<tr>
<td>$R_{adj}^2$</td>
<td>0.32</td>
<td>0.31</td>
<td>0.31</td>
<td>0.30</td>
<td>0.28</td>
<td>0.26</td>
</tr>
<tr>
<td>EC+FV</td>
<td>$EPU$</td>
<td>-0.54***</td>
<td>-0.60***</td>
<td>-0.63***</td>
<td>-0.62***</td>
<td>-0.57***</td>
</tr>
<tr>
<td>$t_{EPU}$</td>
<td>(-2.98)</td>
<td>(-3.08)</td>
<td>(-3.17)</td>
<td>(-3.20)</td>
<td>(-3.11)</td>
<td>(-2.84)</td>
</tr>
<tr>
<td>$R_{adj}^2$</td>
<td>0.58</td>
<td>0.52</td>
<td>0.48</td>
<td>0.43</td>
<td>0.40</td>
<td>0.37</td>
</tr>
<tr>
<td>EC+FV+RA</td>
<td>$EPU$</td>
<td>-0.40**</td>
<td>-0.45**</td>
<td>-0.46**</td>
<td>-0.45**</td>
<td>-0.42**</td>
</tr>
<tr>
<td>$t_{EPU}$</td>
<td>(-2.20)</td>
<td>(-2.30)</td>
<td>(-2.38)</td>
<td>(-2.41)</td>
<td>(-2.31)</td>
<td>(-2.07)</td>
</tr>
<tr>
<td>$R_{adj}^2$</td>
<td>0.63</td>
<td>0.58</td>
<td>0.55</td>
<td>0.51</td>
<td>0.48</td>
<td>0.45</td>
</tr>
<tr>
<td>EC+FV+IF</td>
<td>$EPU$</td>
<td>-0.28**</td>
<td>-0.32**</td>
<td>-0.34**</td>
<td>-0.34**</td>
<td>-0.30**</td>
</tr>
<tr>
<td>$t_{EPU}$</td>
<td>(-2.21)</td>
<td>(-2.41)</td>
<td>(-2.57)</td>
<td>(-2.62)</td>
<td>(-2.41)</td>
<td>(-2.06)</td>
</tr>
<tr>
<td>$R_{adj}^2$</td>
<td>0.88</td>
<td>0.87</td>
<td>0.85</td>
<td>0.83</td>
<td>0.82</td>
<td>0.81</td>
</tr>
<tr>
<td>Full Regression</td>
<td>$EPU$</td>
<td>-0.20**</td>
<td>-0.22**</td>
<td>-0.24**</td>
<td>-0.24**</td>
<td>-0.21**</td>
</tr>
<tr>
<td>$t_{EPU}$</td>
<td>(-2.02)</td>
<td>(-2.25)</td>
<td>(-2.42)</td>
<td>(-2.49)</td>
<td>(-2.24)</td>
<td>(-1.85)</td>
</tr>
<tr>
<td>Conv</td>
<td>0.52***</td>
<td>0.53***</td>
<td>0.53***</td>
<td>0.50***</td>
<td>0.48***</td>
<td>0.45***</td>
</tr>
<tr>
<td>$t_{Conv}$</td>
<td>(5.51)</td>
<td>(5.48)</td>
<td>(5.45)</td>
<td>(5.46)</td>
<td>(5.36)</td>
<td>(5.40)</td>
</tr>
<tr>
<td>Div. Yield</td>
<td>-0.55***</td>
<td>-0.58***</td>
<td>-0.58***</td>
<td>-0.54***</td>
<td>-0.53***</td>
<td>-0.50***</td>
</tr>
<tr>
<td>$t_{DY}$</td>
<td>(-5.57)</td>
<td>(-5.61)</td>
<td>(-5.42)</td>
<td>(-5.21)</td>
<td>(-5.27)</td>
<td>(-5.37)</td>
</tr>
<tr>
<td>Pr. Housing</td>
<td>-0.50***</td>
<td>-0.52***</td>
<td>-0.53***</td>
<td>-0.53***</td>
<td>-0.51***</td>
<td>-0.49***</td>
</tr>
<tr>
<td>$t_{NPHU}$</td>
<td>(-5.97)</td>
<td>(-5.86)</td>
<td>(-5.97)</td>
<td>(-6.20)</td>
<td>(-6.13)</td>
<td>(-6.26)</td>
</tr>
<tr>
<td>CPI</td>
<td>2.07***</td>
<td>2.18***</td>
<td>2.18***</td>
<td>2.11***</td>
<td>2.01***</td>
<td>1.90***</td>
</tr>
<tr>
<td>$t_{CPI}$</td>
<td>(12.14)</td>
<td>(12.55)</td>
<td>(12.65)</td>
<td>(13.03)</td>
<td>(12.63)</td>
<td>(12.60)</td>
</tr>
<tr>
<td>$R_{adj}^2$</td>
<td>0.90</td>
<td>0.89</td>
<td>0.88</td>
<td>0.87</td>
<td>0.86</td>
<td>0.85</td>
</tr>
</tbody>
</table>

$*$, **, *** represents statistical significance at $p < 0.1$, $p < 0.05$, and $p < 0.01$, respectively.

**significant, very large and positive impact on the term structure of nominal interest rates. This result suggests that higher inflation increases investors demand a higher compensation for investing in the bond market.**

The last part of this analysis is devoted to the individual impact of government and monetary policy uncertainty. In Figure 9 we plot the estimated regression coefficients of the GPU and MPU index together with their 95% HAC-robust confidence intervals. Figure 9 shows that whereas the government policy uncertainty index remains statistically and economically significant after including
all control variables, the monetary policy uncertainty index becomes insignificant once we add all control variables. Furthermore, whereas including economic condition controls does not affect the predictive power of the GPU index, adding the financial variables substantially reduces the impact of fiscal policy uncertainty yet its effect remains statistically significant for any maturity. Similar to the regressions above using the EPU index, the average $R^2_{adj}$ increases from 26.9% (EC) to 50.7% (EC+FV), where a large part of the explained variation can be attributed to the credit spread and dividend yield factor.

4.4 Unspanned macro and government policy risks

A natural question that arises in this context is what do the EPU index and the government and monetary policy uncertainty indexes capture beyond information already contained in the yield curve? In order to address this question, we decompose movements in the yield curve into its principal components (PC). As it is standard in the literature on bond price analysis (see for instance Litterman & Scheinkman (1991) or Joslin et al. (2014)), the three first principal components commonly referred to as ’level’, ’slope’ and ’curvature’ explain most variation of the term structure of nominal interest
rates. We analyze the unconditional sample correlation of the first three principal components with the EPU, GPU and MPU index, respectively. Table 5 suggests that the EPU index exhibits a large negative correlation with the level factor (PC1). This is not surprising given that the above regression results indicated that a higher index value reduces yields by a substantial amount across all maturities. Furthermore, Table 5 shows that the EPU index is moderately positively correlated with the 'Slope' and 'Curvature' factor. The GPU exhibits a very similar pattern. Lastly, the MPU index factor has virtually no correlation with the 'Level' and 'Slope' factor but has large positive correlation with the curvature factor. Since the first PC explains most of the variation in the term structure of nominal interest rates, we expect that a large portion of information contained in the EPU index (to a lesser extent also for the GPU index) is already encoded in the level factor. To test this hypothesis, we run an OLS regression of the EPU, GPU and MPU onto the three PC's as follows

\[ PU_t^i = \beta_0 + \beta_1 L_t + \beta_2 S_t + \beta_3 C_t + \epsilon_t, \quad i = 1, 2, 3, 4, \quad \text{where } i \in \{EPU_t, GPU_t, MPU_t\} \]

where \( L_t \), \( S_t \) and \( C_t \) denote the level, slope and curvature factor respectively. The corresponding adjusted \( R^2 \)'s are 37.8% (EPU index), 29% (GPU) and 17.4% (MPU). Therefore, this implies that 62.2%, 71% and 82.6% of the variation in the EPU, GPU and MPU index are not explained or are unspanned by the three first PCs, respectively.

### Table 5: Sample Correlation Matrix of EPU index and macro variables with first three principal components.

<table>
<thead>
<tr>
<th></th>
<th>'Level'</th>
<th>'Slope'</th>
<th>'Curvature'</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPU</td>
<td>-0.526</td>
<td>0.104</td>
<td>0.311</td>
</tr>
<tr>
<td>GPU</td>
<td>-0.429</td>
<td>0.126</td>
<td>0.312</td>
</tr>
<tr>
<td>MPU</td>
<td>-0.012</td>
<td>0.095</td>
<td>0.417</td>
</tr>
</tbody>
</table>

4.5 The term structure of bond yield volatility and policy risk

Our theoretical results from Section 3.4 suggest that the inclusion of a time-varying government policy risk factor not only raises the level of the yield curve, but also is a key driver in generating the empirically observed hump shape of the bond volatility term structure. We now test these predictions using both the EPU, and our measures of fiscal and monetary policy uncertainty GPU
and MPU including all the control variables from above. Our measure for observed volatility is realized volatility as given in Equation (36) aggregated on a monthly level from business day data. In addition to our control variables from above, we also include treasury implied volatility (TIV) proxy for fixed-income implied volatility.\textsuperscript{25} From our posited hypothesis H2, we expect the sign of the regression coefficient of the EPU and the GPU index to be positive for all $\tau$.\textsuperscript{26} Furthermore, from H3 we expect that economic or fiscal policy uncertainty to have a hump-shaped effect on the term structure of bond yield volatility and thus we should observe that the estimated regression coefficients peaks at two year maturity as the realized bond volatility curve in Figure 2 does. The results in Table 6 are in line with hypothesis H2, i.e., higher economic policy uncertainty increases bond volatility for any maturity. Concerning hypothesis H3, although the impact of economic policy is highly statistically significant, it gradually declines along the entire term structure and thus is not in line with H2. Whereas the results do not change substantially comparing rows 'EPU' to 'EC', we see that once we include the financial control variables, the impact of the EPU index is reduced for any $\tau$ and the explanatory power increases substantially from an average $\bar{R}^2_{adj} = 0.41$ to $\bar{R}^2_{adj} = 0.58$. This suggests, similar to the results in Table 4, that financial factors are also important predictors of bond variance. Analyzing this effect in more detail we see from row 'Full Regression' that this increase in predictive power is mainly due to the regressors credit spread and dividend yield. The latter effect has also been found an important predictor by Fama & French (1989). Also, similar to the results in Table 4 above, inflation factors, especially PPI, seem to have higher forecasting power than real factors as the average adjusted R-squared increases to 0.76 in the former and only to 0.66 in the latter case. Overall, adding more control variables does reduce the magnitude of the EPU index but does not affect its statistical significance for any maturity.

Lastly, we now analyze the impact of the government and monetary policy uncertainty index separately on the term structure of bond yield volatility. Overall, Figure 10 shows that hypothesis H2 and H3 are supported by the data. Not only does realized volatility rise when fiscal policy uncertainty increases, its effect is also hump-shaped (peak at 2 year maturity) in time to maturity. Similar to the yield regressions from above, once we add the financial control variables from above, the estimated

\textsuperscript{25}Treasury bond implied volatility is based on weighted average of very liquid 1 month options on treasury bonds with maturity 2,5,10 and 30 years. In order to obtain monthly data, we compute the sample mean using daily observations.

\textsuperscript{26}From our theoretical analysis from Section 2 and 3.4, the contemporaneous impact of monetary policy uncertainty is marginal and therefore we do not expect to find any large estimates for the MPU index.
Table 6: Summary of regression results: The table displays the slope coefficients of the regression of $Y(t, \tau)$ on $EPU_t$ (EPU), $Y(t, \tau)$ on $EPU_t$ and EC controls (EC), $Y(t, \tau)$ on $EPU_t$, EC and FV variables (EC+FV), $Y(t, \tau)$ on $EPU_t$, EC, FV, RA (EC+FV+RA), $Y(t, \tau)$ on $EPU_t$, EC, FV, IF (EC+FV+IF) and $Y(t, \tau)$ on $EPU_t$, EC, RA and IF controls (Full Regression) for $\tau = 1Y, 2Y, 3Y, 5Y, 7Y$ and 10Y. All regressions also include a convexity adjustment factor. All regressions also include a convexity adjustment factor. Values in brackets below represent HAC-robust $t-$ statistics. $R^2_{adj}$ refers to adjusted coefficient of determination. ***, **, * represent 1%, 5%, and 10% statistical significance, respectively.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>1</th>
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<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>EPU</td>
<td>0.113***</td>
<td>0.114***</td>
<td>0.096***</td>
<td>0.069***</td>
<td>0.051***</td>
</tr>
<tr>
<td></td>
<td>$R^2_{adj}$</td>
<td>0.391</td>
<td>0.435</td>
<td>0.439</td>
<td>0.425</td>
<td>0.389</td>
</tr>
<tr>
<td>EC</td>
<td>EPU</td>
<td>0.119***</td>
<td>0.118***</td>
<td>0.099***</td>
<td>0.072***</td>
<td>0.053***</td>
</tr>
<tr>
<td></td>
<td>$t_{EPU}$</td>
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<td>(8.731)</td>
<td>(8.600)</td>
<td>(7.458)</td>
</tr>
<tr>
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<td>$R^2_{adj}$</td>
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<td>0.450</td>
<td>0.451</td>
<td>0.434</td>
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<td>EC+FV</td>
<td>EPU</td>
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<td>0.070***</td>
<td>0.060***</td>
<td>0.044***</td>
<td>0.031***</td>
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<tr>
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<td>$t_{EPU}$</td>
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<td>(4.632)</td>
<td>(4.441)</td>
<td>(4.916)</td>
<td>(4.417)</td>
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<td>0.451</td>
<td>0.434</td>
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<tr>
<td>EC+FV+RA</td>
<td>EPU</td>
<td>0.066***</td>
<td>0.066***</td>
<td>0.056***</td>
<td>0.041***</td>
<td>0.030***</td>
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<tr>
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<td>$t_{EPU}$</td>
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<tr>
<td>EC+FV+IF</td>
<td>EPU</td>
<td>0.061***</td>
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<tr>
<td></td>
<td>$t_{EPU}$</td>
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<td>0.790</td>
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<tr>
<td>Full Regression</td>
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<td>0.045***</td>
<td>0.039***</td>
<td>0.034***</td>
<td>0.025***</td>
</tr>
<tr>
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<td>$t_{EPU}$</td>
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<td>(4.454)</td>
<td>(5.565)</td>
<td>(5.200)</td>
</tr>
<tr>
<td></td>
<td>TIV</td>
<td>0.028**</td>
<td>0.028***</td>
<td>0.023***</td>
<td>0.012***</td>
<td>0.006*</td>
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<tr>
<td></td>
<td>$t_{TIV}$</td>
<td>(2.412)</td>
<td>(3.769)</td>
<td>(3.722)</td>
<td>(2.725)</td>
<td>(1.795)</td>
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<td>Credit Spread</td>
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<td>0.057***</td>
<td>0.047***</td>
<td>0.040***</td>
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<td></td>
<td>$t_{CS}$</td>
<td>(2.769)</td>
<td>(4.383)</td>
<td>(5.306)</td>
<td>(5.620)</td>
<td>(5.568)</td>
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<tr>
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<td>Div. Yield</td>
<td>0.007</td>
<td>0.020**</td>
<td>0.023**</td>
<td>0.018**</td>
<td>0.013**</td>
</tr>
<tr>
<td></td>
<td>$t_{DY}$</td>
<td>0.500</td>
<td>1.962</td>
<td>2.368</td>
<td>2.572</td>
<td>2.386</td>
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<tr>
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<td>PPI</td>
<td>0.096***</td>
<td>0.103***</td>
<td>0.086***</td>
<td>0.063***</td>
<td>0.048***</td>
</tr>
<tr>
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<td>$t_{PPI}$</td>
<td>4.861</td>
<td>7.032</td>
<td>6.953</td>
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<td>7.160</td>
</tr>
<tr>
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<td>$R^2_{adj}$</td>
<td>0.714</td>
<td>0.823</td>
<td>0.816</td>
<td>0.822</td>
<td>0.819</td>
</tr>
</tbody>
</table>

* * *  $p < 0.01$,  **  $p < 0.05$,  *  $p < 0.1$

impact of the GPU is substantially reduced yet remains statistically significant. Overall, whereas the impact of the GPU index is statistically significant and positive for any maturity, an increase in monetary policy uncertainty leads to a decline in nominal yields.
4.6 Bond Risk Premia

In this section we explore the predictive power of the EPU index as well as our government and monetary policy indexes on future bond excess returns. From the model-implied bond risk premia in Equation (99), both types of policy uncertainties should explain movements in the term premium. To fix notation, we let \( t \in \{1,\ldots,T\} \), and define \( \tau_i \) as the \( i \)-th year maturity with \( i \in \{1, 2, 3, 5, 7, 10\} \). We denote by
\[
r_{t+1}^{E,\tau_i} := \log \left( B^N(t+1, \tau_i - 1) \right) - \log \left( B^N(t, \tau_i) \right) - Y(t,1)
\]
the continuously compounded (log) excess return of an \( \tau_i \)-year maturity bond in period \( t + 1 \). In other words, \( r_{t+1}^{\tau_i} := \log \left( B^N(t+1, \tau_i - 1)/B^N(t, \tau_i) \right) \) denotes the return from buying a \( \tau_i \) bond at time \( t \) and selling it after a holding period of one month \( (t + 1) \) as a \( \tau_i - 1 \) bond. Furthermore, the log forward rate at a time \( t \) for entering contracts between time \( t + \tau_i - 1 \) and \( t + \tau_i \) is given by
\[
F(t, \tau_i) = \log \left( B^N(t, \tau_i - 1)/B^N(t, \tau_i) \right) \text{ with } i \in \{1, 2, 3, 5, 7, 10\}.
\]
In bond risk premia analysis, it is very common to compare the predictive power of a new predictor variable against two routinely used control variables. The first one is the Campbell & Shiller (1991) slope control, which can be obtained by regressing the change in yield \( Y(t+1, \tau_i - 1) - Y(t, \tau_i) \) onto the current slope of the

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**Figure 10:** Summary of regression results: The graph displays the slope coefficients of the regression of \( \mathbb{V}[Y(t,\tau)] \) on \( EPU_t \) (Simple), \( \mathbb{V}[Y(t,\tau)] \) on \( EPU_t \) and EC controls (EC), \( \mathbb{V}[Y(t,\tau)] \) on \( EPU_t \) EC and FV variables (EC+FV), \( \mathbb{V}[Y(t,\tau)] \) on \( EPU_t \), EC, RA and IF controls (Macro Factors) for \( \tau = 1Y, 2Y, 3Y, 5Y, 7Y \) and \( 10Y \). All regressions also include the TIV control variable. Vertical lines represent HAC-robust 5% confidence intervals.
yield curve which we denote by $S_{t}^{\tau_{i}} = \frac{Y(t,\tau_{i}) - Y(t,1)}{\tau_{i} - 1}$. The second control variable is the Cochrane & Piazzesi (2005) factor, which is constructed based on a tent-shaped linear combination of forward rates. As a further set of control variables, we extract the first three principal components from our yield data set. Finally, we also include the economic condition and financial variables and the set of macroeconomic factors as above. We start our analysis by first investigating the predictive power of the EPU index, before decomposing it into its fiscal and monetary policy uncertainty parts. In Figure 11 we plot the estimated impact of the EPU index together with the Campbell & Shiller (1991) and Cochrane & Piazzesi (2005) factors in Panel A and in Panel B the slope coefficients of the EPU index including all other controls. Panel A shows that an increase in the EPU index

**Figure 11:** Summary of regression results: Panel A displays the slope coefficients of the regression $r_{t+1}^{E,\tau_{i}}$ on the EPU index (Simple), $r_{t+1}^{E,\tau_{i}}$ on the EPU index and Cochrane & Piazzesi (2005) factor (CP), $r_{t+1}^{E,\tau_{i}}$ on the EPU index and the Campbell & Shiller (1991) slope factor (CS) and $r_{t+1}^{E,\tau_{i}}$ on the EPU index and the Cochrane & Piazzesi (2005) factor, the Campbell & Shiller (1991) slope factor and the first three principal components (PCA). Panel B displays the slope coefficients of the regression $r_{t+1}^{E,\tau_{i}}$ on the EPU index, CP, CS, PCA and EC controls (Econ. Cond), $r_{t+1}^{E,\tau_{i}}$ on the EPU, CP, CS, PCA, EC and FV controls (Fin. Var), $r_{t+1}^{E,\tau_{i}}$ on the EPU, CP, CS, PCA, EC, FV and Real activity controls (Real Factors) and $r_{t+1}^{E,\tau_{i}}$ on the EPU, CP, CS, PCA, EC, FV, RA and inflation factor controls (Inf. factors). Vertical lines represent HAC-robust 5% confidence intervals.

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28 A detailed description of the construction of this factor is given in Cochrane & Piazzesi (2005). In order to avoid collinearity problems, we only include the current one year yield $Y(t,1)$ and the two- and five year forward rates and we do not restrict the regression coefficients to sum up to one.
increases the bond risk premia for any maturity. Furthermore, its impact is statistically significant, at least for longer maturities as the 95% confidence intervals do not include zero. The predictive power decreases as we add the CP -, the CS factor and the first three principal components. The average $R^2_{adj}$ increases gradually from 20% (Simple) to 34% (CP) and jumps up to 79% once we add the slope factor and remains about at the same level when the PCA factors are added to the regression equation (80%). Panel B shows that once we control for economic condition, financial variables and macro factors, the estimated impact of economic policy uncertainty is reduced and becomes insignificant for longer maturities. Lastly, we analyze the forecasting power of the GPU and MPU index individually. Overall, whereas Panel A shows that the GPU index does not exhibit any significant predictive power once all control variables are included, Panel B shows that monetary policy uncertainty remains an important predictor at any maturity. The coefficient has the expected positive sign and is significant and increasing in maturity. The estimated slope coefficients of the MPU index are also economically significant, especially for longer horizons. For instance, for any standard deviation increase in the MPU index, the expected 10Y bond excess return is predicted to

Figure 12: Summary of regression results: Panel A displays the slope coefficients of the regression $r_{t+1}^{E,τ}$ on the GPU index, CP, CS, PCA and EC controls (Econ. Cond), $r_{t+1}^{E,τ}$ on the GPU, CP, CS, PCA, EC and FV controls (Fin. Var), $r_{t+1}^{E,τ}$ on the GPU, CP, CS, PCA, EC, FV and Real activity controls (Real Factors) and $r_{t+1}^{E,τ}$ on the GPU, CP, CS, PCA, EC, FV, RA and inflation factor controls (Inf. factors). Vertical lines represent HAC-robust 5% confidence intervals. Panel B shows the same regressions but instead replacing the GPU with the MPU index.
increase by 24 Basispoints. The average adjusted $R^2_{adj}$ after having included all regression controls is 89%.

5 Conclusion

In this paper, we present a tractable dynamic equilibrium model of the term structure of interest rates with fiscal (real) and monetary policy risks. Consistent with the data, our model is able to reproduce the flight-to-quality behavior, i.e., the observation that investors tend to increase their bond holdings in times of higher economic or government policy uncertainty and thereby lowering treasury bond yields. We calibrate our model to data and provide a detailed analysis of the dependence of the nominal yield curve and its corresponding term structure of nominal bond yield volatility on the structural model parameters. Even though our model belongs to the class of affine term structure models it is still able to reproduce not only higher level of bond volatility but also captures, the empirically observed hump-shape of the term structure of bond volatility. In order to achieve this result, two key feature in our model are essential. First, that the long run growth path of productivity is time-varying and negatively dependent on government policy uncertainty ($\lambda < 0$) and secondly, that real policy risks affect the nominal side of the economy whenever the central bank responds actively to deviations of output and inflation growth from their respective target rates.

Even though our simple empirical analysis has only illustrative character, it sheds some light on the relationship between movements of the yield and bond yield curve in response to shocks in government and monetary policy uncertainty. However, the empirical results seem to confirm our hypothesis that, first, increased policy uncertainty leads to an increased demand in bonds and therefore reducing their yields and, second, that higher economic policy uncertainty not only raises bond volatility but is also a key driver of generating the empirically observed hump-shaped structure of the bond yield volatility curve. Finally, decomposing the EPU index into its components, confirms our hypothesis that it is mainly uncertainty related to government policy that is responsible first, for generating the contemporaneous negative relationship between policy uncertainty and nominal yields and second for generating the empirically observed high and often hump-shaped term structure of bond yield volatility. On the other hand, the explanatory power of the MPU index for contemporaneous yield
and bond volatility movements is mixed. However, monetary policy uncertainty is a key predictor for bond risk premia for any maturity.
A Proofs

A.1 Proof of Proposition 1

Proof. We start by deriving an \( n \)-th order recursive moment formula for the expectation of the government policy uncertainty process \( g_t \) with dynamics

\[
dg_t = \kappa_g (\theta_g - g_t) \, dt + \sigma_g \sqrt{g_t} dW_t^g
\]  

To compute its moments, let \( f(g) = g^n \) where \( n \in \mathbb{N} \), then an application of Itô’s lemma and using Equation (38) gives

\[
dg_t^n = ng_t^{n-1}dg_t + \frac{1}{2} n(n-1)g_t^{n-2}\sigma_g^2 g_t \, dt
\]  

\[
= \left( -n\kappa_g g_t^n + g_t^{n-1}(n\kappa_g \theta + \frac{n}{2}(n-1)\sigma_g^2) \right) \, dt + ng_t^{n-1}\sigma_g \sqrt{g_t} dW_t^g
\]  

Integrating from \( t \) to \( T \), taking expectations on both sides, using Fubini’s theorem and the law of iterated expectation, differentiating with respect to \( T \) gives

\[
\psi'_t(T) = \Upsilon_0(T) + \Upsilon_1(T) \psi_t(T)
\]  

\[
\Upsilon_0(T) = \mathbb{E}_t \left[ g_{t+\tau}^n \right] \left( n\kappa_g \theta_g + \frac{n}{2}(n-1)\sigma_g^2 \right)
\]  

\[
\Upsilon_1(T) = -n\kappa_g
\]  

Conditional on \( \kappa_g > 0 \), the solution to Equation (40) is

\[
\psi_t(t + \tau) = e^{\psi_1 \tau} g_t^n + \int_{t}^{t+\tau} \Upsilon_0(t + \tau - u) e^{\psi_1 (t+\tau - u)} du
\]  

Then the first moment satisfies

\[
\psi'_g(t, T) := \frac{\mathbb{E}_t[g_T]}{dT} = \kappa_g (\theta_g - \mathbb{E}_t[g_T])
\]  

\[
\psi_g(t, t) := g_t > 0
\]  

where Equation (45) represents the initial condition. The solution this fist order linear differential equation in (44) can be represented as

\[
\psi_g(t, T) := \mathbb{E}_t[g_T] = \theta_g + (g_t - \theta_g) e^{-\kappa_g \tau}
\]
and similarly, using Equation (46) one obtains that the second moment is given by

$$\psi_{g^2}(t, T) := \mathbb{E}_t[g_T^2] = g_t^2 e^{-\kappa_g \tau} + \frac{(e^{-\kappa_g \tau} - e^{-2\kappa_g \tau})}{2\kappa_g} (2\theta g \kappa_g + \sigma_g^2) g_t + \frac{\theta_g (2\theta g \kappa_g + \sigma_g^2) (1 - e^{-\kappa_g \tau})^2}{2\kappa_g}$$

$$\psi_{g^2}(t, t) := g_t^2 > 0$$

from which one can immediately deduce that the variance of $g_t$ is

$$\mathbb{V}_t[g_T^2] = \mathbb{E}[g_T^2] - \mathbb{E}[g_T]^2 = \frac{\left(2g_t (e^{-\kappa_g \tau} - e^{-2\kappa_g \tau}) + \theta (1 - e^{-\kappa_g \tau})^2\right) \sigma_g^2}{2\kappa_g}$$

and its unconditional variance is

$$\mathbb{V}[g_t] = \frac{\theta \sigma_g^2}{2\kappa_g}$$

Along the same line of argumentation, integrating Equation (2), applying Fubini's theorem and the law of iterated expectations we obtain that the conditional expected value of $A_t$ satisfies

$$\psi'_A(t, T) := \frac{\mathbb{E}_t[A_T]}{dT} = \kappa_A (\theta_A - \psi_A(t, T)) + \lambda \psi_g(t, T)$$

$$\psi_A(t, t) = A_t \in \mathbb{R}$$

where Equation (49) represents the initial condition for the process $A_t$. Using the expression for $\psi_g(t, T)$ in Equation (46), the solution can be obtained using standard methods. Passing to the limit, i.e., $T \to \infty$, gives the stationary expectation of $A_t$ in Proposition 1. Next, in order to derive the second moment of the productivity process $A_t$, we have to first derive the an expression for the product expectation $\psi_{Ag}(t, T) := \mathbb{E}_t[A_{t+\tau}g_{t+\tau}]$. An application of Itô’s formula to $A_t g_t$ results in the following dynamics

$$d(A_t g_t) = [(\kappa_A \theta_A + \rho^A g \sigma_A g) g_t + \kappa_A \theta_A A_t - (\kappa_A + \kappa_g) A_t g_t + \lambda g_t^2] \, dt + \sigma_A g_t^{3/2} \, dW_t^A + \sigma g_t^{3/2} \, dW_t^g$$

Then, as above, integrating from $t$ to $T = t + \tau$, applying Fubini’s theorem and taking time $t$ conditional expectation shows that $\psi_{Ag}(t, T)$ satisfies

$$\psi'_{Ag}(t, T) := \frac{\mathbb{E}_t[A_{T} g_T]}{dT} = (\kappa_A \theta_A + \rho^A g \sigma_A g) \psi_g(t, T) + \kappa_A \theta_A \psi_A(t, T) - (\kappa_A + \kappa_g) \psi_{Ag}(t, T) + \lambda \psi_{g^2}(t, T)$$

$$\psi_{Ag}(t, t) = A_t g_t$$

where Equation (50) represents the initial condition. Solving this first order differential equation with time-dependent functions $\psi_g(t, T), \psi_{g^2}(t, T)$ and $\psi_A(t, T)$ gives the conditional covariance ex-
expression. Having obtained explicit conditional moments for $A_t$, $g_t$ and $A_t g_t$, the stationary co-
variance expression immediately follows from letting $T \to \infty$, i.e. $\lim_{T \to \infty} \mathbb{E}_t[A_{t+T}, g_{t+T}] = \mathbb{E}_t[A_{t+T}, g_{t+T}] = \mathbb{E}_t[A_{t+T}, g_{t+T}] - \mathbb{E}_t[A_{t+T}] \mathbb{E}_t[g_{t+T}]$. Finally, by similar arguments as above and an application of Itô’s formula to $A_t^2$ we obtain that the second moment of $A_t$ satisfies

$$
\psi'_{A^2}(t, T) := \frac{\mathbb{E}_t[A_t^2]}{dT} = 2\kappa_A (\theta_A - \psi_A(t, T)) + 2\lambda \psi_A(t, T) + \sigma_A^2 \psi_A(t, T) \psi_A(t, T)
$$

Solving this ordinary first order differential equation with time-dependent coefficients gives the ex-
pression for the conditional expectation of $A_t$. From $\mathbb{V}_t(A_{t+T}) = \mathbb{E}[A_{t+T}^2] - \mathbb{E}[A_{t+T}]^2$ and letting $T \to \infty$ we immediately obtain the stationary variance expression for $A_t$.

A.2 Proof of Proposition 3

The optimal consumption and investment problem is

$$
\max \mathbb{E}_0 \left[ \int_t^\infty e^{-\beta s} U(C_s, M_s^d) ds \right],
$$

where $U(C_t, M_t^d) = \frac{1}{\gamma} \left( (C_t(M_t^d)^{\gamma} - 1 \right)$ subject to the capital constraint in Equation (11). To simplify the notation, let $X_t = (A_t, g_t)$ such that the value function is given by

$$
V = V(t, K_t, X_t) = \max_{\{C_t, M_t^d\}_{t \leq s < \infty}} \mathbb{E}_t \left[ \int_t^\infty e^{-\beta s} U(C_s, M_s^d) ds \right]
$$

In equilibrium, there exists a value function $V(t, K_t, X_t)$ and control variables $\{C_t, M_t^d\}$ satisfying the HJB equation

$$
- \frac{\partial V(t, K_t, X_t)}{\partial t} = \max_{\{C_t, M_t^d\}} \left\{ U(C_t, M_t^d) + AV(t, K_t, X_t) \right\}
$$
Then by standard time-homogeneity arguments for infinite horizon problems we have that
\[
e^{\beta t}V(K_t, X_t, t) = \max_{\{C_s, M^d_s\}_{t \leq s < \infty}} \mathbb{E}_t \left[ \int_t^\infty e^{-\beta(s-t)} U(C_s, M^d_s) ds \right]
\]
\[
= \max_{\{C_{t+u}, M^d_{t+u}\}_{t \leq u < \infty}} \mathbb{E}_t \left[ \int_t^\infty e^{-\beta u} U(C_{t+u}, M^d_{t+u}) du \right]
\]
\[
= \max_{\{C_u, M^d_u\}_{0 \leq u < \infty}} \mathbb{E}_0 \left[ \int_0^\infty e^{-\beta u} U(C_u, M^d_u) du \right]
\]
\[
\equiv H(K_t, X_t)
\]
(55)

where the third equality follows because the optimal robust control is Markov and \(H(K_t, X_t)\) is independent of time. Therefore we conjecture that the value function has the following form
\[
V(t, K_t, X_t) = a(t)H(K_t, X_t) = e^{-\beta t}H(K_t, X_t) = e^{-\beta t} \left( \left( e^{\phi(X_t)} K_t^{\xi} \right)^{\gamma} - 1 \right)
\]
(56)

where \(\phi : \mathbb{R}^2 \to \mathbb{R}\) is \(C^N\)-differentiable function of the state vector \(X_t\) that needs to be determined in equilibrium. Inserting (56) into the HJB equation in (54), the first order conditions are given by
\[
C_t^* = \frac{K_t \left( K_t^Q e^{\phi(X_t)} \right)^{-\gamma}}{\beta Q} \left( \frac{\beta Q \left( K_t^Q e^{\phi(X_t)} \right)^{\gamma} \left( -\frac{(\gamma-1)\beta^{1-\gamma} Q^{1-\gamma} K_t^{\frac{1-\gamma}{\gamma-1}} e^{\frac{\gamma\phi(X_t)}{1-\gamma}}}{\xi} \right)^{\frac{1-\gamma}{1-\gamma+1}}}{K_t} \right)^{-\xi} \left( \frac{1-\gamma}{\gamma-1} \right)
\]
(57)
\[
M_t^{d*} = \left( (1-\gamma)^{1-\gamma} Q^{1-\gamma} K_t^{1-\gamma} e^{\frac{\gamma\phi(X_t)}{1-\gamma}} \right)^{\frac{1-\gamma}{\gamma-1+1}}.
\]
(58)

In general, the function \(\phi(X_t)\) cannot be obtained in closed form. However, we can obtain an asymptotic expansion to \(g(X_t)\) with respect to the risk aversion parameter \(\gamma\). Assuming a power series expression for \(\phi(X_t)\) in \(\gamma\) as follows
\[
\phi(X_t) = \phi_0(X_t) + \gamma \phi_1(X_t) + O(\gamma^2),
\]

where \(\phi_0(X, t)\) is obtained from the logarithmic utility case, we can then solve the HJB problem in Equation (54) for \(\gamma \neq 0\) in closed form. In order to so, we solve first the HJB problem in the case where utility of the representative agent is logarithmic in the next section.
A.2.1 Log-utility case

Note that for $\gamma \to 0$ the utility reduces to

$$\lim_{\gamma \to 0} U(C_t, M^d_t) = \log(C_t) + \gamma \log \left( M^d_t \right)$$

The optimal consumption and investment problem is

$$\max_{C_t, M^d_t} \mathbb{E}_0 \left[ \int_0^\infty e^{-\rho t} \left[ \log(C_t) + \gamma \log \left( M^d_t \right) \right] dt \right]$$

subject to the capital constraint in Equation (11). In equilibrium, there exists a value function $V^{\log}(t, K_t, X_t) = a(t) H^{\log}(K_t, X_t)$ and control variables $\{C_t, M^d_t\}$ satisfying the HJB equation

$$- \frac{\partial}{\partial t} \left( a(t) H^{\log}(K_t, X_t) \right) = a(t) \left( \max_{\{C_t, M^d_t\}} \left\{ U^{\log}(C_t, M^d_t) + AH^{\log}(K_t, X_t) \right\} \right)$$

We consider the following linear conjecture for the value function $V^{\log}(\cdot)$

$$a(t) H^{\log}(K_t, X_t) = \frac{e^{-\beta t}}{\beta} \left[ Q \log(K_t) + g_0(X_t) \right]$$

where the function $g(X_t)$ is affine in the state variables, i.e.

$$\phi(X_t) = \phi_{00} + \phi_{0A} A_t + \phi_{0g} g_t$$

Applying the generator to Equation (62), using the productivity, the government policy and equilibrium capital accumulation dynamics in Equation (2), (3) and (20) one obtains

$$AV^{\log} = \frac{\partial V^{\log}}{\partial t} + \frac{\partial V^{\log}}{\partial K} \mu_K + \frac{\partial V^{\log}}{\partial A} \mu_A + \frac{\partial V^{\log}}{\partial g} \mu_g + \frac{1}{2} \frac{\partial^2 V^{\log}}{\partial K^2} \sigma_K^2$$

$$= Q \left[ \mu_Y + q_A A_t - \left( \frac{C_t}{K_t} + \frac{M^d_t}{K_t} \right) - \frac{1}{2} \sigma_Y^2 g_t \right]$$

$$+ (\kappa_A (\theta_A - A_t) + \lambda g_t) \phi_{0A} + \kappa_g (\theta_g - g_t) \phi_{0g}$$

The first order optimality conditions for consumption and money holdings are

$$\frac{e^{-\beta t}}{C_t} - \frac{Q e^{-\beta t}}{\beta K_t} = 0 \iff C^*_t = \frac{\beta K_t}{Q}$$

$$\frac{e^{-\beta t} \xi}{M^d_t} - \frac{Q e^{-\beta t}}{\beta K_t} = 0 \iff M^d_{t} = \frac{\beta \xi K_t}{Q}$$
Then substituting the optimal controls $C^*_t$ and $M^d_t$ into Equation (61) and matching coefficients of $\log(K_t)$, $A_t$, $g_t$ and the constant terms, we obtain

$$\log(K_t) : Q = 1 + \xi$$

(68)

and

$$\phi_{00} = \frac{(1 + \xi)(-\theta_g\kappa_g \sigma_Y^2(\beta + \kappa_A) + 2\mu_Y(\beta + \kappa_A)(\beta + \kappa_g) + 2\theta_g\kappa_g \lambda q_A)}{2\beta(\beta + \kappa_A)(\beta + \kappa_g)} - \frac{(1 + \xi)(\beta(\beta + \delta) + \kappa_A(\beta + \delta - \theta_A q A))}{\beta(\beta + \kappa_A)} + L, \quad L := \log \left( \frac{\beta^{1+\xi} \xi^\xi}{(1 + \xi)^{1+\xi}} \right)$$

$$\phi_{0A} = \frac{(1 + \xi)q_A}{(\kappa_A + \beta)}, \quad \phi_{0g} = \frac{(1 + \xi)\left(\frac{2\lambda q_A}{\beta + \kappa_A} - \sigma_Y^2\gamma \right)}{2(\kappa_g + \beta)}.$$

The coefficients are all uniquely determined, state-independent and also independent of $K_t$. Substituting the expressions back into the HJB equations verifies that the guess was correct.

### A.2.2 Perturbed solution

In this section, we derive an asymptotic approximation to the function $\phi(X_t)$ where the expansion taken with respect to the risk aversion parameter $\gamma$. As above, let $V = V(t, K_t, X_t)$ denote the value function as given in Equation (53) where utility is now given by the following non-separable preferences

$$U(C_t, M^d_t) = \frac{1}{\gamma} \left( (C_t(M^d_t)\xi)^\gamma - 1 \right)$$

(69)

From the HJB equation in (54), the optimal consumption $C^*_t$ and money demand $M^d_t$ policy holdings are given by

$$C^*_t = (M^d_t)^{-\xi} \left( \frac{Q(M^d_t)^{-\xi} (K_t^{\gamma Q} e^{\phi(X_t)})^\gamma}{\beta K_t} \right)^{-\frac{1}{\gamma - 1}}$$

(70)

$$M^d_t = \left( \frac{QC_t^{-\gamma} K_t^{\gamma Q-1} e^{\gamma \phi(X_t)}}{\beta \xi} \right)^{-\frac{1}{\gamma - 1}}$$

(71)

Then inserting optimal money demand (71) into the first order condition of consumption (70), using the power series representation of $g(X_t)$ as given in Equation (17) and perturbing the resulting expression
expression around the log-utility case ($\gamma = 0$) (and analogously for optimal money demand), substituting $Q = 1 + \xi$ from Equation (68), the perturbed optimal consumption and money holdings are given by

$$C^*_t, P_t = \frac{\beta K_t}{1 + \xi} \left[ 1 + \gamma \left( \log \left( \frac{\beta^{1+\xi}\xi}{(1+\xi)^{1+\xi}} \right) - \phi_0(X_t) \right) \right] + O(\gamma^2)$$

$$M^*_t, P_t = \frac{\beta \xi K_t}{1 + \xi} \left[ 1 + \gamma \left( \log \left( \frac{\beta^{1+\xi}\xi}{(1+\xi)^{1+\xi}} \right) - \phi_0(X_t) \right) \right] + O(\gamma^2)$$

There are a number of important conclusions that can be drawn from the optimal perturbed solutions in Equations (72) and (73). First, both equations only depend on $\phi_0(X_t)$ and do not depend on $\phi_1(X_t)$ which implies that solving the consumption-investment problem with log-utility is sufficient to fully characterize the optimal perturbed consumption and money holdings up to first order. Secondly, $C^*_t, P_t$ and $M^*_t, P_t$ are affine functions not only of capital $K_t$ but also of the state vector $X_t$. This property of the solution will not only render the equilibrium path process of $K_t$ affine, but also implies that optimal inflation $dp^*_t/p^*_t$ remain affine in the state variables. Next, substituting $C^*_t, P_t$ and $M^*_t, P_t$ into Equation (12) immediately gives the equilibrium capital process $K^*_t$ in Equation (20). To show the equilibrium price dynamics in (21), we apply Itô’s lemma to the money market clearing condition $M^*_t = p^*_t M^*_t$ and obtain

$$dM^*_t = M^*_t dp^*_t + p^*_t dM^*_t + C_t \left( dp^*_t, dM^*_t \right)$$

Then using the optimal controls $C^*_t$ and $M^*_t$ and inserting the money market clearing condition from Equation (74) yields

$$\frac{dp^*_t}{p^*_t} = \frac{dM^*_t}{M^*_t} - \frac{dK^*_t}{K^*_t} - C_t \left( \frac{dp^*_t}{p^*_t}, \frac{dK^*_t}{K^*_t} \right)$$

Inserting the money supply rule of Equation (13) and the equilibrium capital accumulation process into (75) gives the equilibrium price process as in Equation (21). To verify that the guess for the value function $V(\cdot)$ was correct, we substitute the equilibrium values back into the HJB problem in Equation (54).

Assuming that at $t = 0$, markets cleared, i.e. $p^*_0 M^*_0 = M^*_0$. 

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A.3 Proof of Proposition 4

To simplify the notation, let \( \kappa_t^* = \log(K_t^*) + \beta t \). The using the equilibrium capital accumulation process implies that \( \kappa_t^* \) satisfies

\[
dk_t^* = \left( \mu_y + q_A \lambda_t - \delta - \frac{1}{2} \sigma_y^2 g_t + \beta \gamma (g_0(X_t) - L) \right) dt + \sigma_y \sqrt{g_t} dW_t^Y
\]  

(76)

The Euler condition in Equation (22) can then be expressed as

\[
B(t, \tau) = e^{-\beta \tau} \mathbb{E}_t \left[ \frac{UC(C_{t+\tau}^*, M_{t+\tau}^*)}{UC(C_t^*, M_t^*)} \frac{p_t^*}{p_{t+\tau}^*} \right] = e^{-\beta \tau} \mathbb{E}_t \left[ \frac{K_t^*}{K_{t+\tau}^*} \frac{p_t^*}{p_{t+\tau}^*} \right] = e^{-\beta \tau} \mathbb{E}_t \left[ \exp\left\{-\log (K_{t+\tau}^*) \right\} \frac{p_t^*}{p_{t+\tau}^*} \right]
\]

(77)

To solve the problem in Equation (77) we follow Ulrich (2013) and Buraschi & Jiltsov (2005) and define

\[
f = f(\kappa_t^*, p_t^*, A_t, g_t, m_t, \tau) = \mathbb{E}_t \left[ \frac{e^{-\kappa_{t+\tau}^*}}{p_{t+\tau}^*} \right]
\]

(78)

Then, conjecturing a log-linear guess for \( f(\cdot) \) of the form

\[
f(\kappa_t^*, p_t^*, A_t, g_t, m_t, \tau) = \frac{e^{-\kappa_t^*}}{p_t^*} \exp\left\{-b_0(\tau) - b_A(\tau)A_t - b_g(\tau)g_t - b_m(\tau)m_t \right\}
\]

(79)

If our log-linear guess in Equation (79) solves the stochastic problem in (78) then it is also the solution to the following PDE

\[
-\frac{\partial f(\cdot, \tau)}{\partial \tau} = Af(\cdot, \tau), \quad f(\cdot, 0) = \frac{e^{-\kappa_t^*}}{p_t^*}
\]

(80)

The left-hand side of Equation (80) is given by

\[
\frac{\partial f(\cdot, \tau)}{\partial \tau} = \left[ -b'_0(\tau) - b'_A(\tau)A_t - b'_g(\tau)g_t - b'_m(\tau)m_t \right] f
\]

(81)

Setting \( \Theta = \{ \kappa_t^*, p_t^*, A_t, g_t, m_t \} \), an application of Itô’s lemma to the right-hand side of (80) gives

\[
Af(\kappa_t^*, p_t^*, A_t, g_t, m_t, \tau) = \sum_{i \in \Theta} \frac{\partial f}{\partial \Theta^i} \mu_{\Theta^i} dt + \frac{1}{2} \sum_{i \in \Theta} \frac{\partial^2 f}{\partial \Theta^i^2} d(\Theta^i, \Theta^i)_t + \sum_{i,j \in \Theta, i \neq j} \frac{\partial^2 f}{\partial \Theta^i \partial \Theta^j} d(\Theta^i, \Theta^j)_t
\]

(82)
Writing out the expression above, recalling that $\rho^{gm} = \rho^{Ag} = \rho^{MA} = \rho^{Mg} = 0$ and substituting the dynamics of $\kappa_t^*, p_t^*, A_t, g_t$ and $m_t$ respectively we get

$$Af = \frac{\partial f}{\partial \kappa^*} \left( \mu_y + qAAD_t - \delta - \frac{1}{2} \sigma_Y^2 g_t \right) + \frac{\partial f}{\partial p_t^*} p_t^* \left[ \frac{\mu_M - \eta_1 k - \eta_2 \pi}{1 - \eta_2} + \frac{\eta_1 - 1}{1 - \eta_2} \mu_{K^*} (A_t, g_t) - g_t \frac{(\eta_1 - 1) \sigma_Y^2}{1 - \eta_2} \right] + \frac{\partial f}{\partial A} \left[ \kappa_A (\theta_A - A_t) + \lambda g_t \right] + \frac{\partial f}{\partial g} \kappa_g (g_t - g_t) + \frac{\partial f}{\partial m} \kappa_m (g_t - m_t)$$

$$= \frac{1}{2} \left[ \frac{\partial^2 f}{\partial \kappa^2} \sigma_Y^2 g_t + \frac{\partial^2 f}{\partial p_t^2} p_t^* \left[ g_t \left( \frac{\eta_1 - 1}{1 - \eta_2} \right)^2 \sigma_Y^2 + m_t \frac{\sigma_M^2}{(1 - \eta_2)^2} \right] + \frac{\partial^2 f}{\partial A^2} \sigma_A^2 + \frac{\partial^2 f}{\partial g^2} \sigma_Y^2 g_t + \frac{\partial^2 f}{\partial m^2} \sigma_m^2 m_t \right]$$

$$+ \frac{\partial^2 f}{\partial \kappa^* \partial p_t^*} p_t^* \sigma_Y^2 g_t \frac{1 - \eta_2}{1 - \eta_2} + \frac{\partial^2 f}{\partial \kappa^* \partial A} \sigma_A \sigma_Y p^{AY} g_t + \frac{\partial^2 f}{\partial \kappa^* \partial g} \sigma_g \sigma_Y p^{gY} g_t + \frac{\partial^2 f}{\partial p^* \partial A} \left[ \eta_1 - 1 \right] \sigma_A \sigma_Y g_t p^{AY} + \frac{\partial^2 f}{\partial p^* \partial g} \left[ \eta_1 - 1 \right] \sigma_g \sigma_Y g_t p^{gY}$$

Computing the derivatives and separating variables one obtains the following system of first order asymptotic (Riccati) ODE’s

$$A_t : -C_A + \kappa_A b_A(\tau) + b_A'(\tau) = 0, \text{ subject to } b_A(0) = 0,$$

$$g_t : Z_{0g}(\tau) + b_g(\tau) Z_{1g} + Z_{2g} b_g^2(\tau) + b_g'(\tau) = 0, \text{ subject to } b_g(0) = 0,$$  \hspace{1cm} (83)

$$m_t : Z_{0m} + b_m(\tau) Z_{1m} + Z_{2m} b_m^2(\tau) + b_m'(\tau) = 0, \text{ subject to } b_m(0) = 0,$$  \hspace{1cm} (84)

Constants : $b_0'(\tau) = C_0(\tau), \text{ subject to } b_0(0) = 0,$  \hspace{1cm} (85)
where

\[ C_A = (q_A + \gamma \beta \phi_{0A}) \left( \frac{\eta_1 - \eta_2}{1 - \eta_2} \right), \]

\[ Z_{0g}(\tau) = (q_A^2 + 2\gamma \beta \phi_{0A}) \left( \frac{\eta_1 - \eta_2}{1 - \eta_2} \right)^2 \left( 1 - e^{-\kappa_A \tau} \right)^2 \frac{\sigma^2}{\gamma^2} + \left( \frac{\eta_1 - \eta_2}{\eta_2 - 1} \right)^2 + \frac{\gamma \beta \phi_{0g} (\eta_1 - \eta_2)}{\eta_2 - 1} \]

\[ - b_A(\tau) \left( \frac{(\eta_2 - 1) \lambda + (\eta_1 - \eta_2) \rho \delta A \sigma_A \delta Y}{\eta_2 - 1} \right), \]

\[ Z_{1g} = \kappa_g + b_A(\tau) \rho \lambda g A \sigma_A + \frac{(\eta_2 - \eta_1) \rho \delta g Y \sigma_g \delta Y}{\eta_2 - 1}, \]

\[ Z_{2g} = \sigma^2 g / 2, \quad H_g(\tau) = \sqrt{4Z_{0g}(\tau)Z_{2g} - Z_{1g}^2}, \quad \] (86)

\[ Z_{0m} = \frac{\sigma^2 m}{(\eta_2 - 1)^2}, \quad Z_{1m} = \kappa_m + \frac{\rho \mu m \sigma_m \sigma M}{1 - \eta_2}, \quad Z_{2m} = \frac{\sigma^2 m}{2}, \quad H_m = \sqrt{4Z_{0m}Z_{2m} - Z_{1m}^2} \quad \] (87)

\[ C_0(\tau) = \left( \frac{\rho \mu Y - \beta - \delta - \bar{k}}{1 - \eta_2} \right) \eta_1 + \beta + \mu M - \eta_2 (\rho \mu Y + \bar{k} - \delta) \]

\[ - \sum_{i \in \{A, g, m\}} b_i(\tau) \theta_i \kappa_i - \frac{\gamma \beta (\eta_1 - \eta_2)(L - \phi_{00})}{1 - \eta_2} \] (88)

yields the desired expression. For the existence of a solution to the bond pricing PDE that excludes arbitrage opportunities, requires that the Riccati equations in (83) and (84) above, satisfy the following periodicity condition

\[ 4Z_{0g}(\tau)Z_{2g} - Z_{1g}^2 < 0, \quad 4Z_{0m}Z_{2m} - Z_{1m}^2 < 0, \quad \forall \tau \geq 0, \quad \] (89)

This condition essentially rules out singularities of the solution of the Riccati equation above, i.e., for \( \tau \geq 0 \), the function \( b_i(\tau) \) \( i \in \{g, m\} \) is continuous in \( \tau \).

### A.4 Nominal short rate under CRRA-utility

**Proof.** In this section we derive the first order asymptotic nominal short rate and the market prices of real and nominal risks when the investor has CRRA utility. The nominal short rate is defined as the following limit

\[ R_t := \lim_{\tau \to 0} Y(t, \tau) = \lim_{\tau \to 0} \frac{1}{\tau} \left( \log(B(t, \tau)) \right) \] (90)
To show Equation (32) we make repeated use of Bernoulli’s rule. For instance, to compute \( \lim_{\tau \to 0} \frac{b_0(\tau)}{\tau} \) we set \( f(\tau) := \int_0^\tau C_0(u)du \) and \( g(\tau) = \tau \). Then, using Leibniz’ integral rule we obtain

\[
\lim_{\tau \to 0} \frac{f(\tau)}{g(\tau)} = \lim_{\tau \to 0} \frac{f'(\tau)}{g'(\tau)} = \lim_{\tau \to 0} C_0(\tau) = \frac{(\mu_Y - \beta - \delta - \bar{k})\eta_1 + \beta + \mu_M - \eta_2(\mu_Y + \bar{\pi} - \delta)}{1 - \eta_2} + \frac{\beta(\eta_1 - \eta_2)(L - \phi_{00})}{\eta_2 - 1}
\]

and similarly we have

\[
\begin{align*}
\lim_{\tau \to 0} \frac{b_A(\tau)}{\tau} &= C_A, \\
\lim_{\tau \to 0} \frac{b_g(\tau)}{\tau} &= -Z_{0g}(0) = -\left( \frac{\eta_1 - \eta_2}{(\eta_2 - 1)^2} + \frac{\beta \phi_{0g}(\eta_1 - \eta_2)}{\eta_2 - 1} \right), \\
\lim_{\tau \to 0} \frac{b_m(\tau)}{\tau} &= -Z_{0m} = -\frac{\sigma_M^2}{(\eta_2 - 1)^2}
\end{align*}
\]

which gives the expression for the short rate in Equation (32). Next, in order to derive the bond excess return, we apply Itô’s rule to the closed-form bond price formula in Equation (23) and obtain the following dynamics

\[
\frac{dB(t, \tau)}{B(t, \tau)} = \frac{\partial B(t, \tau)}{\partial t} dt + \sum_{i \in \Theta} \frac{\partial B(t, \tau)}{\partial \Theta_i} \mu_{\Theta_i} dt + \frac{1}{2} \sum_{i \in \Theta} \frac{\partial^2 B(t, \tau)}{\partial \Theta_i^2} d\langle \Theta_i, \Theta_i \rangle_t
\]

where \( \Theta = \{A, g, m\} \). Taking time \( t \) conditional expectation on both sides and using Equation (83) and (84) from above, we obtain that the expected infinitesimal bond risk premia is given by

\[
RP(t, \tau) := \frac{1}{dt} \mathbb{E}_t \left[ \frac{dB(t, \tau)}{B(t, \tau)} - R_t dt \right] = b_g(\tau) \frac{\eta_2 - \eta_1}{1 - \eta_2} \sigma_Y \rho \sigma_M \sigma_m \eta_{t} + b_m(\tau) \frac{\sigma_M \sigma_m \rho_{Mm}}{1 - \eta_2} \eta_{t}
\]

which is after defining the nominal market price of risks the expression in Equation (34). The results for the log-utility agent can be easily derived by simply setting \( \gamma = 0 \) in Proposition (6). However, for verifying our results and to explain why we defined the market prices of nominal output \( \lambda^{N,Y}_t \) and fiscal policy risk \( \lambda^{N,g}_t \) as above, we derive both the nominal short rate, the market prices of risk and the term premium from a different angle using the stochastic discount factor approach. Recall that the real short rate can be obtained directly from the real stochastic discount factor which we denote by \( \zeta_t^{R} \). Its dynamics take the general form

\[
d\zeta_t^{R} = \mu(\zeta^{R}, t) dt + \sigma(\zeta^{R}, t) dW_t
\]
where $\mu(\cdot, \cdot)$ and $\sigma(\cdot, \cdot)$ are $\mathcal{F}_t$-measurable bounded scalar drift and vector diffusion processes and $W_t = (W_t^Y, W_t^M)$. In our setup, $\zeta^R = e^{-\rho t} U_C(X_t^R)$ from which, after an application of Itô’s lemma we find

$$
\frac{d\zeta^R}{\zeta^R} = - (\mu_Y + q_A A_t - \delta - \sigma^2_Y g_t) dt - \sigma_Y \sqrt{g_t} dW^Y_t \quad (95)
$$

from which we can read off the real short rate as $r_t = -\mu(\zeta^R, t)$ and the market price of the real productivity innovation risk by $\lambda^{R,i} = \sigma_Y \sqrt{g_t}$. Following Veronesi & Jared (2000) or Piazzesi & Schneider (2006), the dynamics of the nominal stochastic discount factor $\zeta^N_t$ are given by

$$
\frac{d\zeta^N}{\zeta^N} = - \left\{ \frac{\mu_M + \beta(1 - \eta_1) - \eta_1 \bar{k} - \eta_2 \bar{\pi} - (\eta_1 - \eta_2)(\delta - \mu_Y)}{\eta_2 - 1} A_t 
- \frac{(\eta_1 - \eta_2)^2 \sigma_Y^2}{(\eta_2 - 1)^2} g_t - \frac{\sigma_M^2}{(\eta_2 - 1)^2 m_t} \right\} dt 
- \frac{(\eta_1 - \eta_2) \sigma_Y}{\eta_2 - 1} \sqrt{g_t} dW^Y_t 
- \frac{\sigma_M}{\eta_2 - 1} \sqrt{m_t} dW^M_t \quad (96)
$$

from which we deduce that $R_t = -\mu(\zeta^N, t)$. Similarly we have that the nominal market prices of risk is given by $\lambda^{N,i} = -\sigma(\zeta^N, t)$. Next, in order to determine the term premium on nominal bond yields under log-utility, we make use of the closed-form solution of the nominal bond price as given in Proposition 4, and the nominal short rate together with the associated market prices $\lambda^{N,Y}$ and $\lambda^{N,M}$ as in Proposition 6. An application of Itô’s lemma to Equation (23) shows that the risk-neutral dynamics of the bond price equals to

$$
\frac{dB^N(t, \tau)}{B^N(t, \tau)} = R_t dt - b_A(\tau) \sigma_A \sqrt{g_t} d\tilde{W}^A_t 
- b_g(\tau) \sigma_g \sqrt{g_t} d\tilde{W}^g_t 
- b_m(\tau) \sigma_m \sqrt{m_t} d\tilde{W}^m_t \quad (97)
$$

where $\tilde{W}_t^i, i \in \{A, g, m\}$ are $\mathcal{F}_t$-measurable Brownian innovation processes under the risk neutral measure $\mathbb{Q}$. According to Proposition 4, only real fiscal and nominal monetary policy risk are priced. We can decompose the Brownian innovations $\tilde{W}_t^A, \tilde{W}_t^g$ and $\tilde{W}_t^m$ into two orthogonal complements as follows

$$
\begin{align*}
\tilde{dW}_t^A &= \rho^{AY} \tilde{dW}_t^Y + \sqrt{1 - \rho^{AY}^2} \tilde{dW}_t^A, \quad \tilde{W}_t^Y \perp \tilde{W}_t^A \\
\tilde{dW}_t^g &= \rho^{Yg} \tilde{dW}_t^Y + \sqrt{1 - \rho^{Yg}^2} \tilde{dW}_t^g, \quad \tilde{W}_t^Y \perp \tilde{W}_t^g \\
\tilde{dW}_t^m &= \rho^{Mm} \tilde{dW}_t^M + \sqrt{1 - \rho^{Mm}^2} \tilde{dW}_t^m, \quad \tilde{W}_t^M \perp \tilde{W}_t^Y
\end{align*}
$$

(98)
Hence, substituting the decomposed Brownian motions into Equation (97), the bond price dynamics are then given by

\[
\frac{dB^N(t, \tau)}{B^N(t, \tau)} = R_t dt - b_A(\tau)\sigma_A \sqrt{g_t} \left( \rho^{AY} d\tilde{W}^Y_t + \sqrt{1 - \rho^{AY}^2} d\tilde{W}^A_t \right) - b_g(\tau)\sigma_g \sqrt{g_t} \left( \rho^{YG} d\tilde{W}^Y_t + \sqrt{1 - \rho^{YG}^2} d\tilde{W}^g_t \right) - b_m(\tau)\sigma_m \sqrt{m_t} \left( \rho^{YM} d\tilde{W}^M_t + \sqrt{1 - \rho^{YM}^2} d\tilde{W}^m_t \right)
\]

Next, in order to express the bond under the physical measure \( \mathbb{P} \) we apply Girsanov’s theorem to perform a measure change from \( \mathbb{Q} \) to \( \mathbb{P} \) as follows. We set the change of the drift equal to the market price of risk, in other words we have

\[
d\tilde{W}^Y_t = \lambda^{RY}_t R^Y_t dt + dW^Y_t \text{ and } d\tilde{W}^M_t = \lambda^{NM}_t N^M_t dt + dW^M_t
\]

Then, given the correlation structure of the factors we obtain that the equilibrium term premium under the physical measure \( \mathbb{P} \) is given by

\[
\frac{1}{dt} \mathbb{E}_t \left[ \frac{dB^N(t, \tau)}{B^N(t, \tau)} - R_t dt \right] = b_g(\tau)\sigma_g \sqrt{g_t} \rho^{YM} \lambda^{RY}_t + b_m(\tau)\sigma_m \sqrt{m_t} \rho^{YM} \lambda^{NM}_t
\]  

(A.5 Proof of Proposition 5)

The unconditional correlation coefficient of the nominal yield curve \( Y(t, \tau) \) and \( g_t \) is given by

\[
\rho \left[ Y(t, \tau), g_t \right] = \frac{C \left[ Y(t, \tau), g_t \right]}{\sqrt{\mathbb{V}[Y(t, \tau)] \mathbb{V}[g_t]}}
\]

Using the affine expression in Equation (28) and the results from Proposition 1, with \( \rho^{Ag} = 0 \), the unconditional covariance is

\[
C[Y(t, \tau), g_t] = \frac{b_A(\tau)}{\tau} C[A_t, g_t] + \frac{b_g(\tau)}{\tau} \mathbb{V}[g_t] = \frac{b_A(\tau)}{\tau} \frac{\theta_g \lambda \sigma^2}{2 \kappa_g (\kappa_A + \kappa_g)} + \frac{b_g(\tau)}{\tau} \frac{\theta_g \sigma^2}{2 \kappa_g},
\]

where the unconditional variance of term structure is \( \mathbb{V}[Y(t, \tau)] = \frac{\nu_t(\tau)}{\tau^2} \mathbb{V}[A_t] + \frac{\nu_t(\tau)}{\tau^2} \mathbb{V}[g_t] + \frac{\nu_t(\tau)}{\tau^2} \mathbb{V}[m_t] + 2 \frac{b_A(\tau)}{\tau} \frac{b_g(\tau)}{\tau} C[A_t, g_t] \). Along the same line of argumentation, we have that

\[
C[Y(t, \tau), m_t] = \frac{\theta_m \sigma^2 \sigma^2}{2 \kappa_m} \frac{b_m(\tau)}{\tau}
\]  

(A.5 Proof of Proposition 5)

\[32\text{With } \lambda^{RA}_t = 0 \text{ it follows directly that } d\tilde{W}^Y_t = dW^A_t \text{ which is } \mathcal{F}_t-\text{measurable under } \mathbb{P}.\]
and since $b_m(\tau) \leq 0$, $\tau \geq 0$ the result follows immediately. To show the second part of the proposition, recall that both $\theta_m$ and $\theta_g$ enter only into the time-dependent level coefficient $C_0(\tau)$ given in Equation (88). From the affine yield expression in Equation (28) we have $C_0(\tau) = \sum_{i \in \{g, m\}} \kappa_i \theta_i \int_0^\tau b_i(u) du + \tilde{C}_0(\tau)$, where $\tilde{C}_0(\tau)$ collects the remaining terms not dependent on either $\theta_g$ or $\theta_m$, the result follows immediately after taking the partial derivative with respect to $\theta_i$, $i \in \{g, m\}$.

**B Supplementary Tables**

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<th>3Y</th>
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Table 7: Sample correlation matrix of EPU, GPU and index with nominal yields with $\tau = 1Y, 2Y, 3Y, 5Y, 7Y$ and 10Y using data from January 1990 until June 2014.

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<td>0.9728</td>
</tr>
<tr>
<td>7Y</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.9877</td>
</tr>
<tr>
<td>10Y</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 8: Sample correlation matrix of EPU, GPU and index with realized volatility as in Equation (36) with $\tau = 1Y, 2Y, 3Y, 5Y, 7Y$ and 10Y using data from January 1990 until June 2014.
C Supplementary Graphs

If needed, add supplementary graphs here.
References


