# International asset allocation in presence of systematic cojumps

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**Abstract**

The objective of this article is to explain the home bias phenomenon in international asset holdings from an investigation of intraday jumps and cojumps. We hypothesize that global investors will overweigh domestic assets if the benefit from the international diversification is negatively affected by a high level of synchronization and transmission of intraday jumps across markets. Using intraday index-based data for equity traded funds, we provide evidence of significant systematic jump risks in international markets that drive investors to reduce the proportion of foreign assets in their diversified portfolios. Considering the composition of the optimal portfolio in the sense of mean-variance and mean-CVaR approaches, we provide evidence of a negative correlation between the demand of foreign assets and the number of cojumps between domestic and foreign assets.

*Keywords*: systematic jump risk, cojumps, home bias

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## 1. Introduction

This paper contributes to the international home bias puzzle literature by examining whether jump and cojump risks explain a part of this phenomenon. It is well established in the finance literature that price discontinuities or jumps should be taken into account when studying assets price dynamics (Das and Uppal, 2004). The recent development of non-parametric jump identification tests has supported the hypothesis of jumps in financial asset prices. The seminal works in this area include Barndorff-Nielsen and Shephard (2004, 2006) who test for the presence of jumps at the daily level using measures of bipower variation. The same family of within-day jump identification procedures includes the tests developed by Jiang and Oomen (2008), Andersen et al. (2009), Corsi et al. (2010), Podolskij and Ziggel (2010), and Christensen et al. (2011). Andersen et al. (2007) (ABD) and Lee and Mykland (2008) (LM) have developed techniques to identify intraday jumps using high frequency data. All of these jump detection techniques provide empirical evidence in favor of the presence of price discontinuities or jumps.

More recently, researchers are interested in studying cojumps between assets (see Dungey et al., 2009; Lahaye et al., 2010; Dungey and Hvozdyk, 2012; Pukthuanthong and Roll, 2014). For instance, Gilder et al. (2012) examine the frequency of cojumps between individual stocks and the market portfolio and find that there is a tendency for a relatively large number of stocks to be involved in systematic cojumps. Lahaye et al. (2010) study the relationship between asset cojumps and macroeconomic announcements and find that cojumps are partially associated to new macroeconomic announcements. Ait-Sahalia et al (2014) develop a multivariate Hawkes jump-diffusion model to capture jumps propagation over time and across markets and provide strong evidence for jumps to arrive in clusters within the same market and to propagate to other world markets. Bormetti et al. (2013) find that Hawkes one factor model is more suitable to capture the high synchronization of jumps across assets than the multivariate Hawkes model.

The contribution of our paper is to study theoretically and empirically cojumps between international equity indices and their impact on international portfolio allocation. Modern portfolio theory (MPT) suggests that international diversification is an efficient tool to minimize portfolio risks given that international assets are often less correlated and driven by different economic factors. However, one might expect that systematic cojumps will lead to an increase of the correlation between these international assets and thus will affect negatively the benefit from international diversification. On the other side, price jumps may not be systematic and can be specific to a particular market. In fact, jumps could partly be explained by asset-specific events and thus uncorrelated from market movement. This category of jumps is classified as idiosyncratic jumps.

From basic portfolio theory, a risk-averse investor who holds an international portfolio is then exposed to two types of jump risks: systematic jump risk (jumps common to all markets or cojumps) and idiosyncratic jump risk (jumps specific to one market). If his portfolio is well diversified, the idiosyncratic jump risk will be reduced or even eliminated. However, the systematic jump risk will persist and could not be eliminated by diversification. Therefore, an attention has to be paid to systematic jump risk. First, the identification of systematic jumps is important for pricing financial instruments, hedging portfolios, and performing assets allocation. Second, it is also helpful to measure the contribution of the systematic jump risk to the global portfolio risk. This measure represents a good risk indicator for investors when selecting the composition of their portfolios among all available assets.

In this paper, we examine two hypotheses. The first posits that international equity indices have tendency to be involved in cojumps and a part of jump risk is rather systematic. The second posits that the systematic jump risk could explain a part of the lack of international portfolio diversification - home bias - observed in equity financial markets. Introducing the systematic cojumps risk between international equity indices, we would expect an increase of the correlation between these assets and thus a decrease of the diversification benefit. As a result, investors would prefer to invest more in domestic assets.

We restrict our empirical investigation to three international funds SPY, EFA, and EEM, which respectively aim to capture the performance of three equity indices: S&P 500, MSCI EAFE index and MSCI Emerging Markets indices. S&P 500 index is used as a proxy for the US market. MSCI EAFE index is our benchmark for developed markets, excluding the US and Canada whereas the MSCI Emerging Markets is used to capture the performance of emerging equity markets. Our study is based on high frequency intraday returns of the three funds. We apply the technique proposed by Andersen et al. (2007) (ABD) and Lee and Mykland (2008) (LM) to identify all intraday jumps and cojumps of the three funds from January 2008 to October 2013. We capture the time clustering features of intraday jumps and the dynamics of their propagation across markets using a bivariate Hawkes model. Based on the theoretical framework developed by Todorov and Bollerslev (2010), we estimate the sensitivity of three equity indices returns towards the systematic market diffusive and jump risks and show the evidence that jump risk is rather systematic and could not be eliminated by diversification. To study the impact of systematic jumps on international portfolio allocation (second hypothesis), we consider a domestic risk-averse investor who selects the portfolio composition based on one domestic asset and two foreign assets to minimize the portfolio risk, while requiring a minimum expected return. As investors are concerned about negative movements of asset returns, we take the risk of extreme events into account using the Conditional Value at Risk CVaR (Rockafellar and Uryasev (2000)) as a risk measure in our portfolio allocation problem. Contrary to the standard mean-variance approach, which underestimates the risk of large movements of asset returns, the mean-CVaR approach allows us to provide a fairly accurate estimate of the downside risk induced by systematic negative jumps of asset returns. Determining the composition of the optimal portfolio, we analyze how jumps and cojumps affect the demand of the investor for domestic and foreign assets and show the evidence of a strong negative correlation between the demand of foreign assets and the intensity of cojumps between the domestic and foreign assets.

This paper is organized as follows. Section 2 introduces the jump and cojump identification techniques used in our study. Section 3 introduces the portfolio allocation problem in mean-variance and mean-CVaR frameworks. Section 4 describes our data. Section 5 discusses our main empirical findings on cojumps and international asset allocation. Section 6 concludes.

## 2. Jump and cojump detection methodology

### *2.1. Jump test statistics*

This section outlines the theoretical background of jump identification techniques. To explain the basic idea behind these jump detection procedures, we begin for pedagogic reasons with the traditional Barndorff-Nielsen and Shephard (2004, 2006) test, which is initially applied to determine if a day (or a given time window) contains price jumps.

Barndorff-Nielsen and Shephard develop a non-parametric jump test based on two measures of price variation: realized variance (RV) and bipower variation (BV). The first one measures the variation in the prices coming from both continuous and jump components of the total price variation process while the second one is jump robust and only measures variation coming from the continuous part of the process.

We assume that the logarithm of the price of an asset $p_t$ can be generated by the following jump-diffusion model (Merton 1976):

$$dp_t = \mu_t dt + \sigma_t W_t + \kappa_t dJ_t , 0 \le t \le T$$

where $\mu_t dt$ represents the time-varying drift component, $\sigma_t W_t$ represents the time varying volatility component of the asset price, and $W_t$ is a standard Brownian motion. $\kappa_t dJ_t$ represents the time-varying jump component of the process. $\kappa_t$ stands for the size of jumps and $J_t$ is a counting process independent of $W_t$ . We consider M equidistant observations of the logarithm of the price at day t: $(p_{t,i})_{i=1,..,M}$ . The ith intraday return of day t is then defined by:

$$r_{t,i} = p_{t,i} - p_{t,i-1} , \ i = 1,..,M$$

The realized variation of the price at day t is calculated as following:

$$RV_t = \sum_{i=1}^{M} r_{t,i}^2$$

The realized variance converges to the integrated variance (IV) plus a jump component (called also quadratic variation) as the sampling frequency of the price observations increases $(M \to \infty)$ :

$$RV_t \xrightarrow{M \to \infty} \int_{t-1}^{t} \sigma_s^2 ds + \sum_{j=1}^{N_t} \kappa_{t,j}^2$$

where $N_t$ represents the number of within day jump at day t and $\kappa_{t,j}$ denotes the magnitude of jth jump at day t.

Barndorff-Nielsen and Shephard (2004, 2006) propose a jump robust measure of the integrated variation called the bipower variation (BV).

$$BV_t = \frac{\pi}{2}\frac{M}{M-1}\sum_{i=2}^{M}\left|r_{t,i-1}\right|\left|r_{t,i}\right|$$

The bipower variation converges to the integrated variance as the sampling frequency of the price observations increases ($M \rightarrow \infty$) :

$$BV_t \xrightarrow{M\rightarrow\infty} \int_{t-1}^{t}\sigma_s^2 ds$$

Barndorff-Nielsen and Shephard use the difference between realized variance and bipower variation to estimate the sum of jumps within a day. We consider the relative jump measure defined by:

$$RJ_t = \frac{RV_t - BV_t}{RV_t}$$

It measures the contribution of jumps to the total within-day variation of the process. We also introduce a modified form of BNS test statistic proposed by Huang and Tauchen (2005) to identify days with at least one jump. Huang and Tauchen prove that this particular form of the BNS test outperforms the other forms of BNS test applied in the financial literature in terms of size and power. The test statistic of Huang and Tauchen (HT) is defined by:

$$Z_t = \frac{RJ_t}{\sqrt{\left(\frac{\pi^2}{2}+\pi-5\right)\frac{1}{M}\max\left(1,\frac{QV_t}{BV_t^2}\right)}}$$

where:

$$QV_t = M\left(\frac{\pi}{2}\right)^2 \frac{M}{M-3} \sum_{i=4}^{M} |r_{t,i-3}||r_{t,i-2}||r_{t,i-1}||r_{t,i}|$$

The null hypothesis of absence of jumps at day t is statistically rejected with a probability of $1-\alpha$ if:

$$Z_t \succ \Phi_{1-\alpha}^{-1}$$

where $\Phi_{1-\alpha}^{-1}$ represents the inverse of the standard normal cumulative distribution function evaluated at a cumulative probability of $1-\alpha$ .

The previous tests (BNS or HT) are applied to determine if a day (or a given time window) contains price jumps without providing further information about the number, the time and the size of the jumps that occurs during a given time window. Thus, we need to use a test statistic that will enable us to identify all intraday jumps for a given day and to determine the occurrence time and the size of each jump. There are essentially two procedures in the literature used to identify intraday jumps such as Andersen et al. (2007, henceforth ABD) and Lee and Mykland (2008, henceforth LM) tests whereas BNS or HT tests can only check for the discontinuity of asset prices at a daily level. The LM and ABD intraday procedures use the same test statistic but differ on the choice of the critical value. ABD assumes that the test statistic is asymptotically normal whereas LM provide critical value from the limit distribution of the maximum of the test statistic. Moreover, Dumitru and Urga (2011) show that intraday tests of LM and ABD outperform other test procedures mainly if price volatility is not high.

The LM test statistic compares the current asset return with the bipower variation calculated over a moving window with a given number of preceding observations. The statistic $L_{t,i}$ , which tests at time i whether there was a jump from i-1 to i is defined as:

$$L_{t,i} = \frac{|r_{t,i}|}{\hat{\sigma}_{t,i}}$$

where:

$$\left(\hat{\sigma}_{t,i}\right)^2 = \frac{1}{K-2}\sum_{j=i-K+2}^{i-1}\left|r_{t,j-1}\right|\left|r_{t,j}\right|$$

$\hat{\sigma}_{t,i}$ is the realized bipower variation calculated for a window of K observations. It provides a jump robust estimator of the instantaneous volatility. LM emphasize that the window size K should be chosen in a way that the effect of jumps on the volatility estimation disappears. They suggest that it is appropriate to choose the window size *K* between $\sqrt{252 \times \mathrm{M}}$ and 252 × *M*, where *M* is the number of observations in a day. Under the null hypothesis of absence of jumps at anytime in (i-1i], the LM statistic is asymptotically distributed as:

$$\frac{\left|L_{t,i}\right| - C_M}{S_M} \xrightarrow{M \to \infty} \xi$$

where $\xi$ has a cumulative distribution function, $P\left(\xi \le x\right) = \exp(e^{-x})$

We introduce $C_M$ and $S_M$ :

$$C_M = \frac{\sqrt{2\log(M)}}{c} - \frac{\log(\pi) + \log(\log(M))}{2c\sqrt{2\log(M)}}$$

$$S_M = \frac{1}{c\sqrt{2\log(M)}} \text{ and } c = \sqrt{\frac{2}{\pi}}$$

The null hypothesis of absence of jumps at anytime in (i-1,i] is rejected for a given significance level $\alpha$ if:

$$\left|L_{t,i}\right| \succ -\log(-\log(1-\alpha)) * S_M + C_M$$

ABD provide a test statistic which is assumed to be normally distributed in the absence of jumps. A jump is detected with the ABD test on day t in intraday interval i when:

$$\frac{|r_{t,i}|}{\sqrt{\frac{1}{M} BV_t}} \succ \Phi^{-1}_{1-\frac{\beta}{2}}$$

where $\Phi^{-1}_{1-\frac{\beta}{2}}$ represents the inverse of the standard normal cumulative distribution function evaluated at a cumulative probability of $1 - \frac{\beta}{2}$ and $(1-\beta)^M = 1-\alpha$, where $\alpha$ represents the daily significance level of the test.

To identify intraday jumps, we only apply the intraday procedure of LM-ABD in our study. A jump is detected with the LM-ABD test on day t in intraday interval i when:

$$\frac{|r_{t,i}|}{\hat{\sigma}_{t,i}} \succ \theta$$

The critical value $\theta$ is calculated for different significance levels. For a daily significance level of 5% and a sampling frequency of 5 minutes, which corresponds to 77 intraday returns per day in our study, we obtain a critical value of 3.40 using ABD method and 4.40 using LM method. Following Bormetti and al. (2013), we take a critical value $\theta = 4$. We will then study different critical values (3, 4 and 5) to verify the robustness of our results.

*2.2. Cojump identification procedure*

Financial assets can jump simultaneously and we call this price jump a cojump. To identify a cojump in the prices of a pair of assets, we use a two-step procedure: we first identify the intraday jumps of each asset using the LM-ABD method described in the previous section. We then apply the following co-exceedance rule to detect a cojump:

$$1_{\left\{\frac{|r_{m,t,i}|}{\hat{\sigma}_{m,t,i}} \succ \theta\right\}} \times 1_{\left\{\frac{|r_{n,t,i}|}{\hat{\sigma}_{n,t,i}} \succ \theta\right\}} = \begin{cases} 1 : cojump \\ 0 : no \;\; cojump \end{cases}$$

A cojump in the prices of the pair of assets (m,n) is detected on day t in intraday interval i when both assets jump at the same intraday time interval.

In our study, we distinguish between an idiosyncratic jump of a single stock that occurs independently of the market movement and a systematic jump that happens at the market level. As we limit our empirical study to three international equity indices that cover three different markets (US, developed countries excluding US and emerging countries), a jump is considered to be internationally systematic if the three indices jump simultaneously. If only one or two indices are involved in an intraday jump, this jump is not classified as an international systematic jump. It is only considered as systematic within its corresponding market.

## 3. **Portfolio allocation problem**

We consider a risk-averse investor, who selects his portfolio composition based on n assets: one domestic risky asset and n-1 foreign risky assets. We suppose that all assets are expressed in the investor's domestic money. The investor allocates funds across n assets in a way to minimize the risk, while requiring a minimum expected return.

We first consider the standard mean-variance (MV) approach initially formulated by Markowitz (1952). The Markowitz approach defines the risk as the variance of the portfolio return. The MV portfolio optimization problem is formulated as follows :

$$\min_{w} \left(w'\Sigma w\right) \qquad \text{(P1)}$$

subject to :

$w_k \geq 0$ for k = 1,…, n

$$e'w = 1 \qquad (1)$$

$$\mu'w = \overline{\mu} \qquad (2)$$

where:

$w = (w_1, w_2, ..., w_n)'$ is the vector of portfolio weights, $\mu = (\mu_1, \mu_2, ..., \mu_n)'$ is the mean vector of returns and $\Sigma = \text{cov}(r_i, r_k)_{1 \leq i,k \leq n}$ is the variance-covariance matrix of returns. $e = (1,1,..., 1)'$ denotes the vector of ones.

The first constraint (1) is the budget constraint. The second one (2) constrains portfolio's expected return to be equal to a given value $\overline{\mu}$ . The optimization problem (P1) can be solved using quadratic programming techniques.

The MV approach is based on the first two moments of the return distribution. It is well established that if asset returns are normal, the portfolio optimization problem could be reduced to the mean-variance framework. However, in the case of non-normal returns, the optimization problem depends on the preference of the investor. If he only cares about the mean and the variance of the portfolio, the MV framework could be used to obtain the optimal portfolio weights. However, in most cases, the whole distribution of the return should be considered in the optimization problem. Moreover, the variance, as a symmetric risk measure, fails to differentiate between the upside and downside risks. The variance often leads to an overestimation of the risk for positively skewed distribution and an underestimation of the risk for negatively skewed distribution. The variance also fails to estimate the risk of extreme events (fat tail distribution): large losses as well as large gains. As investors are more concerned about negative movements of asset returns, it is important to pay attention to the downside risk when selecting portfolio assets.

The issue of the portfolio allocation under the non-normality of asset returns has been widely studied and several alternatives to the standard MV framework have been proposed (see Guidolin and Timmermann (2003), Jondeau and Rockinger (2006)). Both studies have extended the MV framework to cover higher moments of asset returns by approximating the expected utility using Taylor series expansions. More recently, a particular attention has been given to the downside risk in portfolio allocation and several percentile risk measures have been proposed as an alternative to the variance such as Value at Risk VaR (see Basak and Shapiro (2001), Gaivoronski and Pflug (1999)) and conditional Value at Risk CVaR, which is also known as mean excess loss, mean shortfall, or tail VaR (see Rockafellar and Uryasev (1999), Krokhmal, Palmquist, and Uryasev (2002)).

The VaR is an estimate of the upper percentile of loss distribution. It is calculated for specified confidence level over a certain period of time. The VaR is widely used by financial practitioners to manage and control risks; however, its use in portfolio optimization remains very limited. Indeed, the VaR has some undesirable properties (Artzner and al., 1997, 1999), which affect its efficiency as a risk measure such as the lack of sub-additivity implying that VaR of a portfolio with two instruments may be greater than the sum of the individual VaRs of these two instruments. Also, the VaR is usually calculated using scenarios. In this case, VaR is non-convex and non-smooth and the optimization becomes very unstable and leads to multiple local extrema.

Contrary to the VaR, the CVaR has more attractive financial and mathematical properties. It is sub-additive and convex (Rockafellar and Uryasev, 2000). It is also considered as a coherent risk measure (Pflug, 2000) in the sense of Artzner et al. (1997, 1999). The CVaR of a portfolio represents the conditional expectation of losses that exceeds the VaR. This definition ensures that VaR is never higher than the CVaR. The CVaR and VaR optimization problems often lead to similar optimal portfolio as both measures are linked by definition.

The following describes the CVaR optimization approach initially developed by Rockafellar and Uryasev (2000).We first define the loss function of a portfolio composed of N assets. It is defined for a given vector of weights w by:

$$f(w,r) = -\sum_{n=1}^{N} w_n r_n = -w'r$$

where r is the random vector of asset returns.
The probability of $f(w,r)$ not exceeding a threshold $\alpha$ is given by:

$$\Psi(w,\alpha) = \int_{f(w,r)\le\alpha} p(r)dr$$

where $p(r)$ is the density function of the vector of returns. $\Psi$ is a function of $\alpha$ for a fixed vector of weights w and represents the cumulative distribution function for the loss associated with the vector of weights w.

We denote $\alpha_\beta(w)$ and $\phi_\beta(w)$ as the values of the VaR and the CVaR of the loss function associated of w and a confidence level $\beta$.

$$\alpha_\beta(w) = \min(\alpha \in R : \Psi(w,\alpha) \ge \beta)$$

$$\phi_\beta(w) = (1-\beta)^{-1} \int_{f(w,r) \geq \alpha_\beta(w)} f(w,r)\, p(r) dr$$

Following Rockafellar and Uryasev (2000), we provide the expression $\phi_\beta(w)$ using the function $F_\beta$ defined as follows:

$$F_\beta(w,\alpha) = \alpha + (1-\beta)^{-1} \int [f(w,r) - \alpha]^+ p(r) dr$$

where $[x]^+ = \max(x;0)$

Rockafellar and Uryasev (2000) demonstrate that $F_\beta(w,\alpha)$ is convex and continuously differentiable as a function of $\alpha$. It is also related to the CVaR of the loss function by the following formula:

$$\phi_\beta(w) = \min_{\alpha \in R}\left(F_\beta(w,\alpha)\right)$$

Rockafellar and Uryasev (2000) also prove that minimizing $\phi_\beta(w)$ over all $w \in R^N$ is equivalent to minimizing $F_\beta(w,\alpha)$ over all $(w,\alpha) \in R^N \times R$.

$$\min_{w \in R^N} \phi_\beta(w) = \min_{(w,\alpha) \in R^N \times R} F_\beta(w,\alpha)$$

To simplify the expression of $F_\beta$, we need to approximate the integral in the definition of $F_\beta$. One possible solution is to generate a random collection of the vector of returns $\left(r^{(1)}, r^{(2)}, ..., r^{(q)}\right)$ and approximate $F_\beta$ as the following:

$$\tilde{F}_\beta(w,\alpha) = \alpha + \frac{1}{q(1-\beta)} \sum_{i=1}^{q} \left[f(w, r^{(i)}) - \alpha\right]^+$$

Replacing the loss function by its expression gives:

$$\tilde{F}_{\beta}(w,\alpha) = \alpha + \frac{1}{q(1-\beta)} \sum_{i=1}^{q} \left[ -w'r^{(i)} - \alpha \right]^{+}$$

By introducing the auxiliary variable $u_i$, the minimizing of $\tilde{F}_{\beta}$ is equivalent to the linear equation:

$$\alpha + \frac{1}{q(1-\beta)} \sum_{i=1}^{q} u_i$$

subject to: $u_i \geq 0$ , $u_i + w'r^{(i)} + \alpha \geq 0$ for i = 1,…,q

If we add the budget and the expected target return constraints, the mean-CVaR optimization problem is given by:

$$\min_{\alpha,\omega} \alpha + \frac{1}{q(1-\beta)} \sum_{i=1}^{q} u_i \quad \text{(P2)}$$

subject to : $e'w = 1$ , $\mu'w = \overline{\mu}$ , $w_k \geq 0$ for k = 1,…, n

$u_i \geq 0$, $u_i + w'r^{(i)} + \alpha \geq 0$ for i = 1,…, q

The mean-CVaR optimization (P2) problem can be solved using linear programming techniques. We note that if asset returns are normally distributed and with $\beta \geq 0.5$, value of the Mean-Variance and mean-CVaR approaches are equivalent and give the same optimal portfolio weights (Rockafellar and Uryasev, 2000). In this paper, we apply both approaches to determine the optimal portfolio composition and study how the departure from the normality caused by the presence of jumps affects the optimal portfolio composition.

## 4. Data description

We use intraday data of three international exchange traded funds in our empirical investigation: SPDR S&P 500 (SPY), iShares MSCI EAFE (EFA) and iShares MSCI Emerging Markets (EEM). SPDR S&P 500 ETF aims to replicate the performance of S&P 500 index by holding a portfolio of the common stocks that are included in index, with the weight of each stock in the portfolio substantially corresponding to the weight of such stock in the index. The S&P 500 index is a US stock market index containing the stocks of 500 large-Cap corporations, and thus a proxy for the whole US stock market.

The iShares MSCI EAFE ETF aims to replicate the performance of the MSCI EAFE index. The MSCI EAFE Index is a stock market index that captures the stock market performance of developed markets outside of the U.S. & Canada and thus a proxy for the international equity. It includes Europe, Australia and Far East equity markets.

The iShares Emerging Markets ETF aims to replicate the performance of the MSCI Emerging Markets index. The iShares Emerging Markets Index is a stock market index that captures the stock market performance of emerging markets. It covers over 800 securities across 21 markets and represents approximately 11% of world market cap.

Our empirical analysis is based on intraday prices of the three funds from January 2008 to October 2013. Prices are sampled every five minutes from 9:30 to 15:55, to smooth the impact of market microstructure noise.

## 5. Empirical findings

### *5.1. Intraday jump identification*

This section summarizes the results from applying LM-ABD intraday jump detection test. A particular attention is given to the intraday volatility pattern (Rognlie, 2010 and Dumitru and Urga, 2011), which can lead to spurious jump detection. To ameliorate the robustness of our jump detection procedure, we correct the intraday volatility pattern using a jump robust corrector proposed by Bollerslev and al. (2008). Appendix A provides a detailed description of the volatility pattern corrector used in our study.

We estimate the realized bipower variation using a window of 155 intraday returns, which corresponds to two days of intraday returns sampled at a frequency of five minutes. Jumps are detected with a critical value $\theta = 4$ (Bormetti and al., 2013), which means that the intraday jump return size is at least four times greater than the estimate of the local volatility. We also apply critical values of 3 and 5 to study the robustness of our results. In this section, we only provide empirical results of LM-ABD jump detection procedure for $\theta = 4$ . Detailed results for $\theta = 3$ and 5 are available upon request.

Table 1 provides the number of total, positive and negative intraday jumps detected over the period of study. We respectively identify 1119, 1114 and 1024 intraday jumps for SPY, EFA and EEM funds, which corresponds respectively to 0.989%, 0.986% and 0.900%

of the total number of intraday returns over the period of study. Thus, the number of detected intraday jumps is slightly higher in developed markets (US and EFA) than in emerging markets. The difference between the number of intraday jumps identified within each region of the world is in line with the previous results in the literature indicating that the degree of comovement and integration is higher in developed markets than emerging ones. A positive (negative) jump is a jump with positive (negative) return. The results show the number of negative jumps is more than 56% of total number of detected jumps for each fund, which is slightly greater than positive ones. Stock markets tend to be more linked together when prices are decreasing. Table 1 also shows basic statistics about the distribution of intraday jump returns. The mean of intraday jump returns of SPY (-4.3e-04), which is in absolute value two times higher than EFA and EEM (-2.5e-04 and -2.1e-04, respectively) indicates that the negative movements of intraday prices are more severe for the US market during the period of our study. The intraday jump return volatility is higher for emerging markets (0.0058) than for developed ones (around 0.0047). At a daily level, Table 2 shows the percentage number of days with at least one intraday jump is around 40% of the total number of days of the period of study (1468 days) for three ETFs (Table 2).

Table 3 shows some statistics of detected cojumps. We find that SPY and EFA funds cojump 586 times over the period of study. This represents 52% of the total number of detected jumps. The SPY and EEM funds cojump 510 times over the same period. The number of cojumps of EEM with SPY is slightly greater to the cojumps with EFA (458), which indicates that emerging equity markets are more linked to the U.S market than to other developed markets covered by the EFA fund. The three funds are involved in 366 cojumps during the period of study. As for jumps, the number of negative cojumps is higher than positive cojumps for three funds. Table 4 presents the probability to have at least one cojump between SPY and EFA is 0.27 at a daily level. This probability is lower for SPY and EEM (0.23) or EFA and EEM (0.22). The probability to have the three funds involved in a one simultaneous jump or more is 0.18. Figures 2 and 3 show the variation of the daily intensity of jumps (JI) and cojumps (CJI) during the period of study. These time-varying jump (cojump) intensities are calculated each day using a rolling six-month window of observations as follows:

$$JI = \frac{\sum_i 1_{\left\{Jump_i^n\right\}}}{N_{days}} \text{ and } CJI = \frac{\sum_i 1_{\left\{Jump_i^m \cap Jump_i^n\right\}}}{N_{days}}$$

where $N_{days}$ is the number of days of the observation period (120 days in our case).

$1_{\left\{Jump_i^n\right\}}$ is an indicator function of jump occurrence for the asset n at the intraday interval i. $1_{\left\{Jump_i^m \cap Jump_i^n\right\}}$ is an indicator function of cojump occurrence for the asset m and n at the intraday interval i.

We notice that the daily jump and cojump intensities have increased significantly during the financial crisis of 2008 – 2009 for the three funds. There is a pattern that the US market was the first to reach the peak of the jump intensity during the crisis followed by developed markets and then emerging markets. The striking evidence is during a peak in January 2010, a drop in June 2012, and a jump in December 2012. The US market seems to be followed more closely by the developed markets. The emerging markets are the most lagged from the other two. The results support the evidence found by Ait-Sahalia, Cacho-Diaz, and Laeven (2014). Figure 3 shows the cojump intensity between the US and developed market is highest, followed by the US and emerging markets, and the developed market and emerging markets. The intensity three markets jump simultaneously is lowest. Overall, the lead/lag of jumps is similar to pattern of lead/lag in financial crisis. That is, the unusual increase of the intraday jumps (both positive and negative) seems to be initially triggered in the US markets and then propagated to other markets in the world.

### *5.2. Time and space clustering of intraday jumps*

We previously show that intraday jumps seem to be initially triggered in the US market and then propagated to other markets in the world. The pattern of intraday jumps also suggests that jumps tend to appear in clusters within the same region (Figure 4). Thus, It seems that international intraday jumps have tendency to propagate both in time (in the same market) and in space (across markets).This dependency between the occurrences of jumps that we observe in the data cannot be reproduced by the standard Poisson Process, which is on the contrary based on the hypothesis of independence of the increments, meaning that the numbers of jumps on disjoint time intervals should be independent. So we need to use an alternative model to reproduce the time and space propagation of asset jumps. We employ the Hawkes process (Hawkes, 1971), which is a self-excited point process whose intensity depends on the path followed by the point process. This process has been applied in different sciences such as seismology and neurology and more recently in finance (in modeling the trading activity, for example). To our knowledge, there are only two papers, which have employed the Hawkes processes to model the dynamics of asset jumps in financial markets. Ait-Sahalia, Cacho-Diaz, and Laeven (2014) are interested in capturing the patterns of contagion in the international equity markets using a multivariate Hawkes jump-diffusion model, but restrain their empirical analysis to the daily data of major international equity indices. The Hawkes processes were also applied in Bormetti et al. (2013) to reproduce the time clustering of jumps. However, they find that the extension of the Hawkes processes to a multi-asset framework is inconsistent with data and instead propose to use Hawkes factor models to capture the cross-sectional dependencies of jumps. They conduct their empirical study based on the high frequency data of 20 high cap Italian stocks.

The following describes the Hawkes processes that we use in the current paper to model the dynamics of intraday jumps of the three international equity funds. We first begin with the univariate Hawkes process, which can be used to capture the time clustering of intraday jumps for one asset. The intensity process is defined by:

$$d\lambda_t = \beta(\lambda_\infty - \lambda_t)dt + \alpha dN_t$$

where $N_t$ is the number of jumps occurring in the time interval [0, t]. A jump occurrence at a given time will increase the intensity, which increases the probability of another jump (self excitation). The intensity increases by $\alpha$ whenever a jump occurs, and then decays back towards a level $\lambda_\infty$ at a speed $\beta$. These parameters can be estimated using the method of maximum of likelihood. Given the jump arrival times $t_1,\ t_2,\ldots,t_n$, the likelihood function is written as:

$$L(t_1,t_2,...,t_n) = -\lambda_\infty t_n + \sum_{i=1}^{n}\frac{\alpha}{\beta}\left(e^{-\beta(t_n-t_i)}-1\right)+\sum_{i=1}^{n}\log\left(\lambda_\infty+\alpha A(i)\right),$$

where $A(i)=\sum_{t_j \prec t_i} e^{-\beta(t_i-t_j)}$ for $i \geq 2$ and $A(1)=0$. The details of the maximum likelihood estimation are provided in Ogata (1978) and Ozaki (1979).

Now we consider the multidimensional framework of the Hawkes process defined by:

$$d\lambda_{i,t} = \beta_i(\lambda_{i,\infty} - \lambda_{i,t})dt + \sum_{j=1}^{n}\alpha_{i,j}dN_{j,t} \qquad \text{i and j = 1…n}$$

Under this model, a jump in market j increases the jump intensity within the same market by $\alpha_{j,j}$ (self excitation) and cross markets i by $\alpha_{i,j}$ (cross excitation). Then the jump intensity of market i reverts exponentially to its average level $\lambda_{i,\infty}$ at a speed $\beta_i$. In the empirical analysis, we find that the numerical resolution of the multidimensional Hawkes model in the case of three markets is problematic as the number of parameters is too large to estimate. So we restrain the calibration procedure to the bivariate model given by:

$$\begin{cases} d\lambda_{1,t} = \beta_1 \left(\lambda_{1,\infty} - \lambda_{1,t}\right)dt + \alpha_{1,1} dN_{1,t} + \alpha_{1,2} dN_{2,t} \\ d\lambda_{2,t} = \beta_2 \left(\lambda_{2,\infty} - \lambda_{2,t}\right)dt + \alpha_{2,1} dN_{1,t} + \alpha_{2,2} dN_{2,t} \end{cases}$$

In this case we only have 8 parameters to estimate: $\Theta = \left(\lambda_{1,\infty}, \lambda_{2,\infty}, \beta_1, \beta_2, \alpha_{1,1}, \alpha_{1,2}, \alpha_{2,1}, \alpha_{2,2}\right)$.

Table 8 Panels A, B and C show the results of the maximum likelihood estimation of the bivariate Hawkes model for SPY/EFA, SPY/EEM and EFA/EEM, respectively. We first remark that the model parameters are all statistically significant implying that the bivariate Hawkes model fits the data of intraday jump occurrences of three funds. The values of the parameters measuring the degree of self-excitation $\alpha_{1,1}$ and $\alpha_{2,2}$ are large. This result provides clear evidence that the US market, developed markets (excluding the US) and emerging markets are strongly self-excited meaning that the occurrence of a jump at a given time increases the probability of other jumps within the same market. The comparison between three funds suggests that the self-excitation activity is higher in the US market than other markets. The values of the parameters $\alpha_{1,2}$ and $\alpha_{2,1}$, measuring the degree of transmission of jumps between markets are relatively small compared to the self-excitation parameters. The degree of transmission of jumps between markets seems to be asymmetric with a stronger transmission from the US market to developed markets ex-US (2.40 e-03) and emerging markets (2.23 e-03). The degree of the reverse transmission of jumps from developed markets (1.79 e-03) or emerging markets (2.16 e-03) to the US market are statistically different from zero but remains relatively small. The degree of transmission of jumps from the developed markets ex-US to emerging markets (2.58 e-03) is higher than that from emerging markets to developed markets ex-US (2.16 e-03).

*5.3. Systematic diffusive and jump risks*

This section studies the contribution of intraday jumps of asset prices to the total systematic risk in equity markets. We have shown in previous sections that international equity markets are characterized by a high level of synchronization between jumps as well as a high degree of transmission of these jumps across markets. Such interconnection between jumps will increase the systematic jump risk in the international equity markets.

In the following, we briefly review the theoretical framework that we use to disentangle and estimate the sensitivity towards systematic diffusive and jump risks in the context of factor models. We first assume that the intraday log-price processes for the aggregate market index, denoted by $p_{0,t}$, and the ith asset, denoted by $p_{i,t}$, follow general continuous-time processes:

$$p_{0,t} = \alpha_{0,t}dt + \sigma_{0,t}d\omega_{0,t} + dJ_{0,t}$$
$$p_{i,t} = \alpha_{i,t}dt + \beta_i^c\sigma_{0,t}d\omega_{0,t} + \beta_i^d dJ_{0,t} + \sigma_{i,t}d\omega_{i,t} + dJ_{i,t}, \text{ i=1...n}$$

where $\omega_{0,t}$ and $\omega_{i,t}$ denote independent standard Brownian motions; $J_{0,t}$ and $J_{i,t}$ denote pure jump processes for systematic market wide jumps and jumps specific to asset i, respectively. The two betas, $\beta_i^c$ and $\beta_i^d$, measure respectively asset i's sensitivities to continuous and discontinuous movements of the market. Aggregating the individual asset processes over multiple days [0; T] readily implies the linear two-factor relation for the T-day return on asset i:

$$r_i = \alpha_i + \beta_i^c r_0^c + \beta_i^d r_0^d + \varepsilon_i$$

where $\alpha_i$ is its drift term, and $r_0^c$ and $r_0^d$ are the continuous and the discontinuous parts of the market return, respectively. The idiosyncratic term $\varepsilon_i$ term is defined similarly from the temporally aggregated asset specific components.

Clearly, when $\beta_i^c = \beta_i^d$, this framework collapses back to the classic, standard one-factor. Over the time interval [0;T], suppose that asset prices are observed at discrete time grids $\frac{\tau}{m}$ where m is the number of observations per one time unit, and $\tau$ = 1,…., mT, and $\Delta p_{i,\tau} = p_{i,\frac{\tau}{m}} - p_{i,\frac{\tau-1}{m}}$ is the intraday return of asset i over the $\left[\frac{\tau-1}{m};\frac{\tau}{m}\right]$ intraday time interval. Following Todorov and Bollerslev (2010), we may estimate $\beta_i^c$ and $\beta_i^d$ using the observed discrete intraday returns of the market and the asset i, $\{\Delta p_{0,\tau}, \Delta p_{i,\tau}\}$ over the period [0; T]:

$$\beta_i^c = \frac{\sum_{\tau=1}^{mT}\left[\left(\Delta p_{i,\tau} + \Delta p_{0,\tau}\right)^2 1_{\left\{\left|\Delta p_{i,\tau}+\Delta p_{0,\tau}\right| \le \theta_{i+0,\tau}\right\}} - \left(\Delta p_{i,\tau} - \Delta p_{0,\tau}\right)^2 1_{\left\{\left|\Delta p_{i,\tau}-\Delta p_{0,\tau}\right| \le \theta_{i-0,\tau}\right\}}\right]}{\sum_{\tau=1}^{mT}\left(2\Delta p_{0,\tau}\right)^2 1_{\left\{\left|2\Delta p_{0,\tau}\right| \le \theta_{2\times 0,\tau}\right\}}}$$

and

$$\beta_i^d = sign\left\{\sum_{\tau=1}^{mT} sign\left\{\Delta p_{i,\tau}\Delta p_{0,\tau}\right\}\left(\Delta p_{i,\tau}\Delta p_{0,\tau}\right)^2\right\} \times \left(\frac{\left|\sum_{\tau=1}^{mT} sign\left\{\Delta p_{i,\tau}\Delta p_{0,\tau}\right\}\left(\Delta p_{i,\tau}\Delta p_{0,\tau}\right)^2\right|}{\sum_{\tau=1}^{mT}\left(\Delta p_{0,\tau}\right)^4}\right)^{\frac{1}{2}}$$

The indicator functions $1_{\left\{\left|\Delta p_{i,\tau}+\Delta p_{0,\tau}\right| \le \theta_{i+0,\tau}\right\}}$, $1_{\left\{\left|\Delta p_{i,\tau}-\Delta p_{0,\tau}\right| \le \theta_{i-0,\tau}\right\}}$ and $1_{\left\{\left|2\Delta p_{0,\tau}\right| \le \theta_{2\times 0,\tau}\right\}}$ are introduced to filter out jumps when calculating the continuous beta. The intraday jumps are detected using the LM-ABD procedure for the three considered processes. The market index is constructed by attributing an equal weight to each fund. We fix the sampling frequency of intraday returns used in the estimation of the continuous and discontinuous betas at 5 minutes, with returns spanning from 9:35 am to 4:00 pm for each trading day. We calculate both betas on a monthly basis based on the data from the previous month (about 1617 intraday returns for each fund per month). Figure 5 Panels A, B and C show the variation of the monthly continuous and discontinuous betas respectively, of SPY, EFA and EEM. We find that the monthly estimated betas (continuous and discontinuous) are statistically significant for three funds. The discontinuous betas are in average higher than continuous betas for three funds. Both diffusive and jump risks are significantly different from zero and thus they are systematic. Moreover, market jumps seem to be more sensitive in average than continuous market moves. These graphs also suggest that the continuous betas are less dispersed than discontinuous betas across time and funds. This result is confirmed by the values of the standard deviation of continuous and discontinuous betas that we find for each fund (detailed figures are reported in

Table 9). Figure 6 shows the autocorrelograms of diffusive and jump betas averaged across three funds. This figure suggests that the degree of persistence of continuous beta is higher than the discontinuous beta for at least the first three orders of autocorrelation, which is consistent with the evidence found by Barndorff-Nielsen and Shephard (2004, 2006), Andersen et al. (2007), and Bollerslev et al. (2015).

### *5.4. The impact of asset cojumps on optimal portfolio composition*

This section studies the composition of the optimal international portfolio in MV and CVaR frameworks and determines how the demand of foreign assets varies in function of the number of cojumps between domestic and foreign assets. The set of instruments to invest in is composed of the three funds previously introduced. We use weekly historical returns (a total of 294 observations for each fund) to run the MV and CVaR optimization. The weekly returns are calculated as the log difference of closing prices. The optimization procedure is performed each week using a rolling window of about 50 weekly returns (one year) that immediately precede the optimization day.

We need to minimize the portfolio risk (standard deviation or CVaR) for a given level of expected return to find the optimal portfolio weights. The composition of the optimal portfolio is thus a function of the target expected return. The set of optimal portfolios obtained for different level of expected returns is called the efficient frontier. We choose to study one particular portfolio from the efficient frontier that maximizes the expected return per unit of risk, known as the tangency portfolio. This portfolio has the advantage of being more diversified than the global minimum risk portfolio, which is often concentrated on assets that minimize the risk of the portfolio. Figure 7 Panel A shows the variation of the optimal proportion of the foreign assets (EFA and EEM) for MV and CVaR approaches. The portfolio is composed of one domestic asset (SPY fund) and two foreign assets (EFA and EEM). We first remark that MV and CVaR approaches almost lead to the same portfolio compositions during the period of study. Figure 7 Panels B and C suggest standard deviation and CVaR are thus equivalent risk measures when they are calculated using our historical data. The figures also show that the standard deviation or the CVaR of the domestic asset (SPY) are often lower than those of foreign funds (EFA and EEM). Indeed, the variability of foreign assets includes both the changes of the stock prices and the exchange rates.

Once the composition of the optimal portfolio is determined, we will show how cojumps between domestic and foreign assets affect the benefit from international diversification. Our hypothesis posits that a higher intensity of cojumps between domestic and foreign assets leads to an increase of the correlation between these assets. The benefit from internationally diversifying is thus reduced and therefore, the domestic investor will be encouraged to invest more on domestic assets. To confirm our analysis, we first calculate the correlation between the daily intensity of cojumps and the optimal proportion of foreign assets that we find using MV and CVaR approaches. The main results for the MV approach are summarized in Table 10.[1]The correlation between the demand of foreign assets and the daily intensity of cojumps between the domestic asset (SPY in our study) and each of foreign assets is negative. We find a correlation of -0.46 for EFA and -0.39 for EEM. If we take the cojumps that involve the three funds, we find a negative correlation of around -0.53, which indicates that the proportion of foreign assets is more sensitive to the cojumps that involve three funds than the cojumps between two funds. The correlation is more negative for negative cojumps than positive cojumps. The correlations between the demand of foreign assets and the negative cojumps between SPY and EFA, SPY and EEM, and all three are -0.43, -0.41, and -0.50 compared to -0.28, -0.17, and -0.22 for positive cojumps of the same pair. We also perform a test of significance of the Pearson correlation with a null hypothesis of no correlation present between two variables against an alternative hypothesis that there is linear correlation present. The correlation is statistically significant for positive, negative and all markets cojumps, which supports the hypothesis of the existence of negative correlation between the number of cojumps between domestic and foreign assets and the demand of the foreign assets in the international portfolio composition.

The results thus far have indicated that the impact of asset cojumps on the demand of foreign assets differs from one asset to another. Indeed, the proportion of foreign assets in the portfolio seems to be more sensitive to the number of intraday cojumps of the domestic asset with the fund that proxies the developed markets (EFA). The proportion of foreign assets is reduced the most when jumps in three markets occur simultaneously.

We also study the relationship between the daily intensity of cojumps and the optimal demand of the foreign assets by performing a linear regression. The objective is to determine if the variation of the proportion of the foreign assets in the portfolio could be explained by the variation of the number of assets cojumps. We apply the OLS to provide an estimate of the regression coefficient. Table 11 summarizes the results of the linear regression. We find a regression coefficient of -1.81 (-1.61 and -2.59, respectively) when the daily intensity of

[1]The results of the CVaR approach are not presented because MV and CVAR approaches lead to almost the same optimal portfolio composition. The results are available upon request.

cojumps between SPY/EFA (SPY/EEM and SPY/EFA/EMM) is used as explanatory variable. All of these coefficients are statistically different from zero with the p-value inferior to e-11. This means that if the average daily intensity of cojumps between SPY and EFA increases by 1%, the optimal proportion of the foreign assets will decrease by -1.81%. The cojumps that involve the three funds have a greater effect as an increase of 1% of the daily intensity of cojumps of three assets leads to a reduction of -2.59% of the proportion of foreign assets in the portfolio.

## 6. Conclusions

In this paper, we study how asset jumps and cojumps affect the benefit from the international diversification. Using a nonparametric intraday jump detection technique developed by Lee and Mykland (LM) and Anderson et al. (ABD), we find that international equity funds have tendency to be involved in systematic cojumps. We further show that these jumps are transmitted both in time (in the same market) and in space (across markets). The high degree of interconnection between jumps in equity markets suggests that the jump risk is rather systematic and thus couldn't be eliminated by the diversification. By disentangling and estimating the sensitivity towards systematic diffusive and jump risks, we show evidence that systematic jump risk is significant and thereby should be taken into consideration by investors when selecting the composition of their portfolios among all available assets.

Studying the link between the composition of the optimal international portfolio (in the sense of Mean-Variance and Mean-CVaR approaches) and the cojumps between domestic and foreign assets, we find that a domestic investor is encouraged to reduce the proportion of his wealth invested in foreign assets when the number of cojumps increases. In fact, the correlation of asset returns increases with the intensity of cojumps and, by consequence, the benefit from diversifying the portfolio abroad is extremely reduced when the cojumps of international equity indices are frequent.

This work opens interesting perspectives for future research. It would be of great interest to broaden the scope of this study by including a larger number of international equity indices and studying the impact of asset cojumps on the demand of foreign assets for a larger panel of countries. It is also interesting to see if the mechanisms of jumps transmission studied in this paper are also valid for individual country markets.

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**Appendix A: Intraday volatility pattern**

It is widely documented (Wood et al. (1985) and Harris (1986)) that intraday returns show a systematic seasonality over the trading day, also called the U-shaped pattern. The intraday volatility is particularly higher at the open and the close of the trading than the rest of the day. To minimize the effects of intraday volatility on our jump detection test we modify our procedure by rescaling intraday returns with a volatility jump robust corrector introduced by Bollerslev and al (2008).

The rescaled $\hat{r}_{t,i}$ is defined by:

$$\hat{r}_{t,i} = \frac{r_{t,i}}{\varsigma_i}$$

where:

$$\varsigma_i^2 = \frac{M \sum_{t=1}^{T} |r_{t,i-1}|^{\frac{1}{2}} |r_{t,i}| |r_{t,i+1}|^{\frac{1}{2}}}{\sum_{t=1}^{T} |r_{t,1}||r_{t,2}| + \sum_{t=1}^{T} \sum_{i=2}^{M-1} |r_{t,i-1}|^{\frac{1}{2}} |r_{t,i}| |r_{t,i+1}|^{\frac{1}{2}} + \sum_{t=1}^{T} |r_{t,M-1}||r_{t,M}|}$$

, for i = 2,.., M-1

$$\varsigma_i^2 = \frac{M \sum_{t=1}^{T} |r_{t,i}||r_{t,i+1}|}{\sum_{t=1}^{T} |r_{t,1}||r_{t,2}| + \sum_{t=1}^{T} \sum_{i=2}^{M-1} |r_{t,i-1}|^{\frac{1}{2}} |r_{t,i}| |r_{t,i+1}|^{\frac{1}{2}} + \sum_{t=1}^{T} |r_{t,M-1}||r_{t,M}|}$$

, for i = 1

$$\varsigma_i^2 = \frac{M \sum_{t=1}^{T} |r_{t,i-1}||r_{t,i}|}{\sum_{t=1}^{T} |r_{t,1}||r_{t,2}| + \sum_{t=1}^{T} \sum_{i=2}^{M-1} |r_{t,i-1}|^{\frac{1}{2}} |r_{t,i}| |r_{t,i+1}|^{\frac{1}{2}} + \sum_{t=1}^{T} |r_{t,M-1}||r_{t,M}|}$$

, for i =M

**Table 1: Summary statistics of jump occurrences and jump sizes**

The number of total, positive (percentage) and negative (percentage) detected jumps are reported. The mean, standard deviation, skewness and kurtosis of jump sizes are shown. Results reported below are obtained using LM-ABD procedure with a critical value $\theta = 4$.

| | SPY | EFA | EEM |
|---|---|---|---|
| Intraday jumps | 1119 | 1114 | 1024 |
| Positive jumps | 475 (42%) | 495 (44%) | 455 (44%) |
| Negative jumps | 644 (58%) | 619 (56%) | 569 (56%) |
| Mean (jump return) | -4.3e-04 | -2.5e-04 | -2.1e-04 |
| Standard deviation | 0.0048 | 0.0047 | 0.0058 |
| Skewness | -0.59 | 0.27 | -0.006 |
| Kurtosis | 14.00 | 8.40 | 15.60 |

**Table 2: Summary statistics of jump occurrences at day level**

The number of days with no jumps, one jump, and two jumps up to more than 5 jumps are reported. The last row shows the percentage of days with at least one jump.

| | SPY | EFA | EEM |
|---|---|---|---|
| 0 | 843 | 843 | 879 |
| 1 | 357 | 365 | 353 |
| 2 | 139 | 147 | 128 |
| 3 | 75 | 50 | 60 |
| 4 | 30 | 37 | 26 |
| 5 | 12 | 13 | 9 |
| More than 5 | 12 | 13 | 13 |
| At least one jump | 42% | 42% | 40% |

**Table 3: Summary statistics of cojump occurrences**

The number of total (percentage of cojumps compared to the total number of detected jumps), positive and negative detected co jumps between SPY and EFA (column 1), SPY and EEM (column 2), EFA and EEM (column 3) and SPY, EFA and EEM (column 4)are reported.

| | SPY / EFA | SPY/EEM | EFA/EEM | SPY/EFA/EEM |
|---|---|---|---|---|
| Intraday cojumps | 586 (53%) | 510 (50%) | 458 (45%) | 366 (36%) |
| Positive cojumps | 242 | 203 | 193 | 144 |
| Negative cojumps | 343 | 306 | 265 | 221 |

**Table 4: Summary statistics of cojump occurrences at day level**

The number of days with no cojumps, one cojump, two cojumps up to more than 4 cojumps. The last row shows the percentage of days with at least one cojump.

| | SPY/EFA | SPY/EEM | EFA/EEM | SPY/EFA/EEM |
|---|---|---|---|---|
| 0 | 1071 | 1130 | 1147 | 1208 |
| 1 | 282 | 233 | 233 | 193 |
| 2 | 70 | 61 | 57 | 39 |
| 3 | 27 | 28 | 19 | 20 |
| 4 | 12 | 11 | 6 | 5 |
| More than 4 | 3 | 5 | 6 | 3 |
| At least one cojump | 27% | 23% | 22% | 18% |

**Table 5: Summary statistics of jump returns aggregated at day level**

The mean, standard deviation, skewness and the kurtosis of the sum of intraday jump returns for each day of our study period are reported. We only take days with at least one intraday jump to calculate these statistics.

| | SPY | EFA | EEM |
|---|---|---|---|
| Mean | -7. 7e-04 | -4.4e-04 | -4.0e-04 |
| Standard deviation | 0.0061 | 0.0059 | 0.0056 |
| Skewness | -0.28 | 0.82 | 0.97 |
| Kurtosis | 11.70 | 11.30 | 14.60 |

**Table 6: Summary statistics of diffusive returns aggregated at day level**

The mean, standard deviation, skewness and the kurtosis of the sum of intraday diffusive returns for each day of our study period are reported. We also add the overnight return to the sum of continuous movements of the price for each day.

| | SPY | EFA | EEM |
|---|---|---|---|
| Mean | 5. 45e-04 | 1.75e-04 | 1. 07e-04 |
| Standard deviation | 0.015 | 0.018 | 0.024 |
| Skewness | -0.22 | -0.06 | 0.11 |
| Kurtosis | 12.30 | 11.00 | 13.40 |

**Table 7: Diffusive returns correlation matrix**

The correlation matrix of diffusive returns of the three ETFs: SPY, EFA and EEM is shown. Diffusive returns are calculated for each day as the sum of intraday diffusive returns plus the overnight return.

| Diffusion component | SPY | EFA | EEM |
|---|---|---|---|
| SPY | — | | |
| EFA | 0.89 | — | |
| EEM | 0.88 | 0.89 | — |

**Table 8: Maximum likelihood estimation of the bivariate Hawkes model**

The table below shows the results of the maximum likelihood estimation of the bivariate Hawkes model for SPY/ EFA (panel A), SPY/EEM (panel B) and EFA/EEM (panel C). The values of the estimate, standard error, z- statistic and p-value are reported for each parameter of the bivariate model. ***, **, and * represent 0.1 percent, 1 percent and 5 percent significance levels, respectively.

**Panel A: SPY / EFA**

| | Estimate | Std. Error | z value | Pr(z) |
|---|---|---|---|---|
| $\lambda_{1,\infty}$ | 1.6305e-03 | 6.4929e-05 | 25.1122 | < 2.2e-16*** |
| $\lambda_{2,\infty}$ | 1.5445e-03 | 6.4217e-05 | 24.0505 | < 2.2e-16 *** |
| $\beta_1$ | 4.2957e-02 | 5.2288e-03 | 8.2154 | < 2.2e-16 *** |
| $\beta_2$ | 1.9230e-02 | 1.6585e-03 | 11.5942 | < 2.2e-16 *** |
| $\alpha_{1,1}$ | 1.2004e-02 | 1.8520e-03 | 6.4816 | 9.074e-11 *** |
| $\alpha_{1,2}$ | 1.7864e-03 | 5.5554e-04 | 3.2156 | 0.001302 ** |
| $\alpha_{2,2}$ | 3.7166e-03 | 4.9845e-04 | 7.4563 | 8.896e-14 *** |
| $\alpha_{2,1}$ | 2.4015e-03 | 4.6676e-04 | 5.1452 | 2.673e-07 *** |

-2 log L: 31320.47

**Panel B: SPY/EEM**

| | Estimate | Std. Error | z value | Pr(z) |
|---|---|---|---|---|
| $\lambda_{1,\infty}$ | 1.5452e-03 | 6.4580e-05 | 23.9268 | < 2.2e-16*** |
| $\lambda_{2,\infty}$ | 1.4443e-03 | 6.3741e-05 | 22.6583 | < 2.2e-16 *** |
| $\beta_1$ | 1.8364e-02 | 1.6053e-03 | 11.4395 | < 2.2e-16 *** |
| $\beta_2$ | 1.8342e-02 | 1.6588e-03 | 11.0573 | < 2.2e-16 *** |
| $\alpha_{1,1}$ | 4.0540e-03 | 5.3480e-04 | 7.5804 | 3.445e-14*** |
| $\alpha_{1,2}$ | 2.1598e-03 | 4.6120e-04 | 4.6829 | 2.828e-06 *** |
| $\alpha_{2,2}$ | 3.8814e-03 | 5.5318e-04 | 7.0164 | 2.277e-12 *** |
| $\alpha_{2,1}$ | 2.2278e-03 | 4.2694e-04 | 5.2181 | 1.808e-07 *** |

-2 log L: 30228.68

**Panel C: EFA/EEM**

| | Estimate | Std. Error | z value | Pr(z) |
|---|---|---|---|---|
| $\lambda_{1,\infty}$ | 1.5471e-03 | 6.2625e-05 | 24.7045 | < 2.2e-16*** |
| $\lambda_{2,\infty}$ | 1.4194e-03 | 6.2419e-05 | 22.7400 | < 2.2e-16 *** |
| $\beta_1$ | 2.6309e-02 | 2.7030e-03 | 9.7335 | < 2.2e-16 *** |
| $\beta_2$ | 2.5798e-02 | 2.7876e-03 | 9.2544 | < 2.2e-16 *** |
| $\alpha_{1,1}$ | 4.0065e-03 | 5.2480e-04 | 7.6342 | 2.272e-14 *** |
| $\alpha_{1,2}$ | 2.1582e-03 | 4.7037e-04 | 4.5882 | 4.470e-06 *** |
| $\alpha_{2,2}$ | 3.7661e-03 | 5.5861e-04 | 6.7418 | 1.564e-11 *** |
| $\alpha_{2,1}$ | 2.5773e-03 | 4.4460e-04 | 5.7969 | 6.754e-09 *** |

-2 log L: 30103.98

**Table 9: Summary statistics of continuous and discontinuous betas**

The mean and the standard deviation (between brackets) of the continuous and discontinuous betas are reported. These betas are calculated monthly during the period covered by our study.

| | SPY | EFA | EEM |
|---|---|---|---|
| Continuous beta | 0.935 | 0.926 | 1.126 |
| | (0.074) | (0.099) | (0.102) |
| Discontinuous beta | 0.967 | 0.967 | 1.123 |
| | (0.095) | (0.112) | (0.123) |

**Table 10: Pearson's product-moment correlation**

The table provides the correlation between the daily intensity of cojumps and the optimal proportion of the foreign assets (EFA+EEM) calculated using the MV (CVaR) approach. It shows results for all possible combinations of all, positive and negative cojumps between the domestic asset and two foreign assets. It also provides the results of the Pearson significance test.

| | Estimate (Correlation) | Confidence interval (95%) | T-stat | P-value |
|---|---|---|---|---|
| SPY/EFA | -0.46 | [-0.55, -0.35] | -8.00 | 4.95 e-14*** |
| Positive SPY/EFA | -0.28 | [-0.39, -0.16] | -4.56 | 7.87 e-06*** |
| Negative SPY/EFA | -0.43 | [-0.53, -0.33] | -7.53 | 9.93 e-13*** |
| SPY/EEM | -0.39 | [-0.50, -0.29] | -6.77 | 9.68 e-11*** |
| Positive SPY/EEM | -0.17 | [-0.29, -0.05] | -2.73 | 0.006** |
| Negative SPY/EEM | -0.41 | [-0.50, -0.29] | -6.83 | 6.26 e-11*** |
| SPY/EFA/EEM | -0.53 | [-0.62, -0.43] | -9.76 | <2.2 e-16*** |
| Positive SPY/EFA/EEM | -0.22 | [-0.33, -0.09] | -3.46 | 0.0006*** |
| Negative SPY/EFA/EEM | -0.50 | [-0.59, -0.40] | -8.96 | <2.2 e-16*** |

**Table 11: Linear regression**

The optimal demand of the foreign assets in the studied portfolio is regressed on the number of cojumps. The cojumps are between SPY and EFA, SPY and EEM, and SPY, EFA, and EEM. Panels A, B, and C report the results for all, negative, and positive cojumps, respectively. ***, **, and * represent 0.1 percent, 1 percent and 5 percent significance levels, respectively.

**Panel A: All cojumps**

| | Coefficient | Estimate | St.dev | T-stat | P-value |
|---|---|---|---|---|---|
| SPY/EFA | α | 1.23 | 0.09 | 13.37 | <2.2 e-16*** |
| | β | -1.81 | 0.22 | -8.00 | 4.9 e-14*** |
| | | | | | |
| SPY/EEM | α | 1.06 | 0.08 | 12.60 | <2.2 e-16*** |
| | β | -1.61 | 0.23 | -6.70 | 9.7 e-11*** |
| | | | | | |
| SPY/EFA/EEM | α | 1.15 | 0.06 | 16.90 | <2.2 e-16*** |
| | β | -2.59 | 0.26 | -9.76 | <2.2 e-16*** |

**Panel B: Negative cojumps**

| Negative cojumps | Coefficient | Estimate | St.dev | T-stat | P-value |
|---|---|---|---|---|---|
| SPY/EFA | α | 1.07 | 0.07 | 13.92 | <2 e-16*** |
| | β | -2.38 | 0.31 | -7.53 | 9.9 e-13*** |
| SPY/EEM | α | 0.92 | 0.06 | 14.54 | <2 e-16*** |
| | β | -2.00 | 0.29 | -6.84 | 6.2 e-11*** |
| SPY/EFA/EEM | α | 0.91 | 0.04 | 18.95 | <2 e-16*** |
| | β | -2.73 | 0.30 | -8.96 | <2 e-16*** |

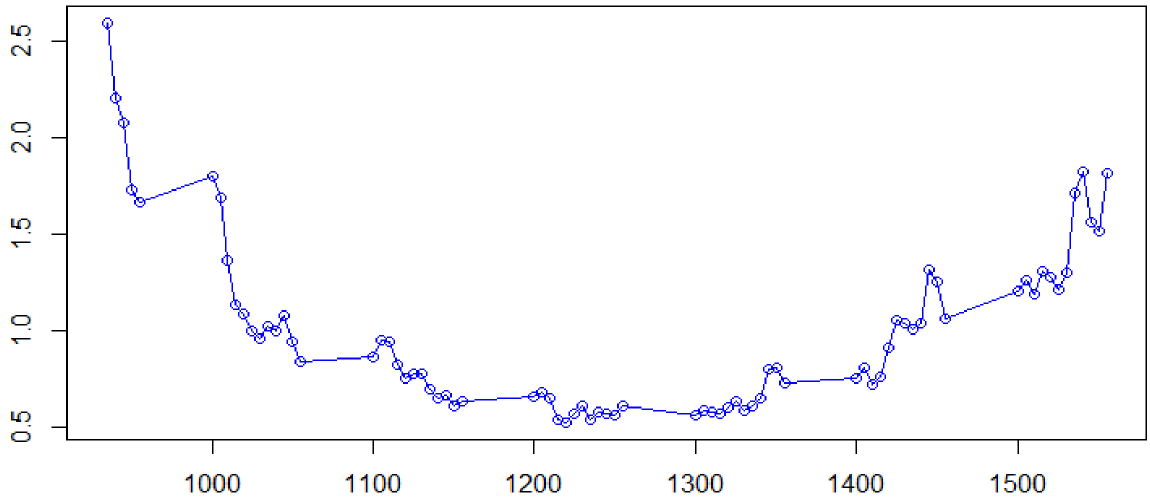
**Panel C: Positive cojumps**

| Positive cojumps | Coefficient | Estimate | St.dev | T-stat | P-value |
|---|---|---|---|---|---|
| SPY/EFA | α | 0.86 | 0.08 | 10.69 | < 2 e-16*** |
| | β | -2.21 | 0.48 | -4.56 | 7.8 e-16*** |
| SPY/EEM | α | 0.73 | 0.08 | 8.37 | 4.3e-15*** |
| | β | -1.72 | 0.62 | -2.73 | 0.006** |
| SPY/EFA/EEM | α | 0.77 | 0.08 | 9.65 | 2.52e-16*** |
| | β | -2.72 | 0.78 | -3.45 | 0.0006*** |

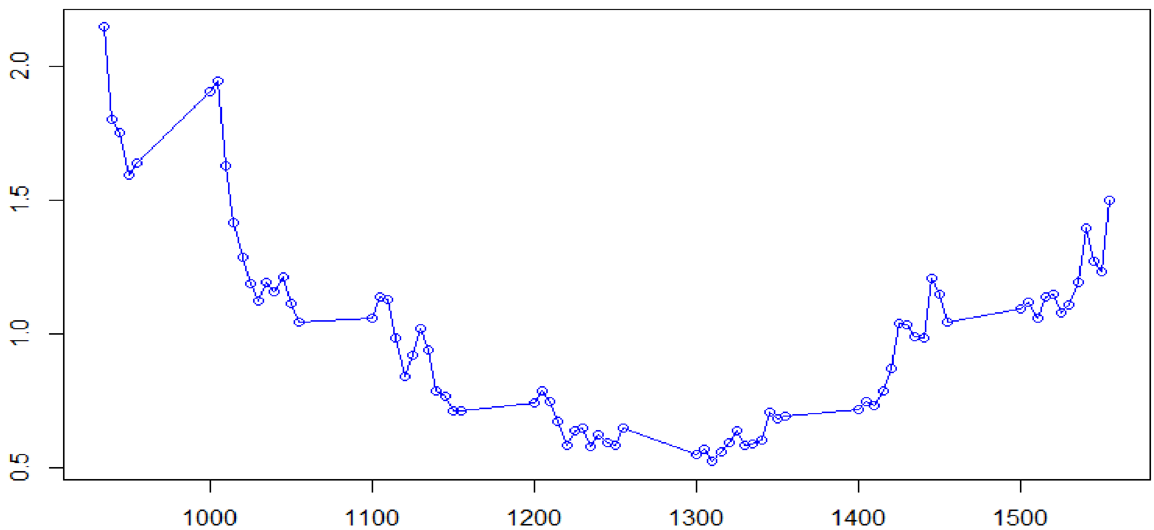
**Figure 1 : Volatity pattern corrector**

This figure shows the variation of the volatility pattern corrector during the trading hours (from 9:30 AM to 15:55AM). The intraday volatility is high at the opening and the closing of the market with a remarkable jump around 10:00 AM, which corresponds to the usual time of announcements.

**SPY Volatility Corrector**




**EFA Volatility Corrector**



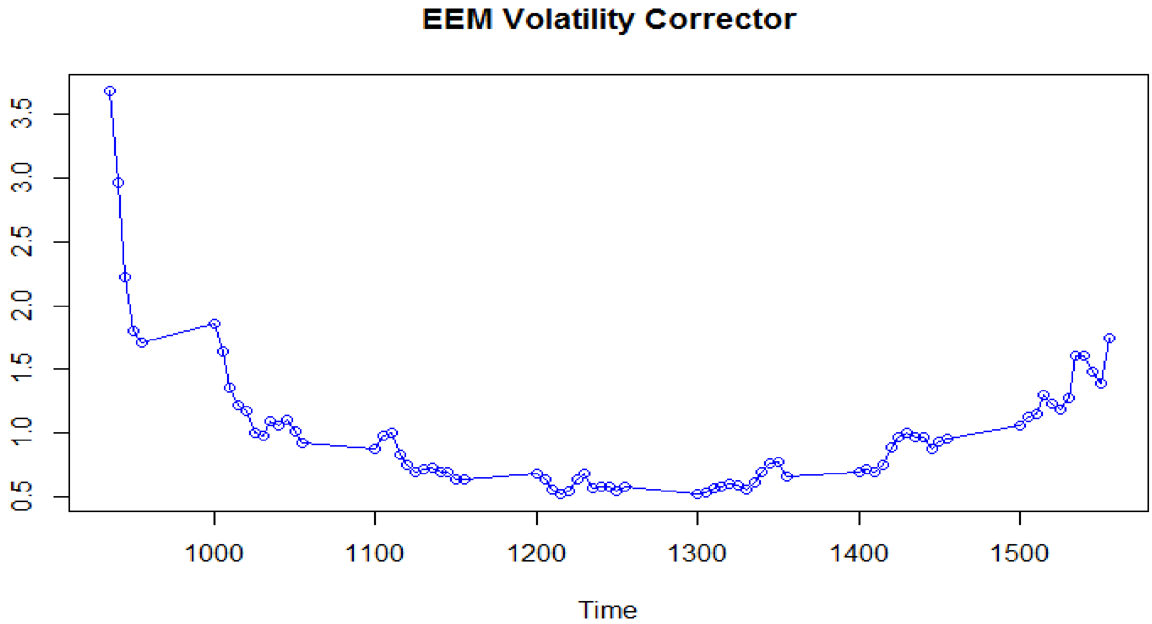
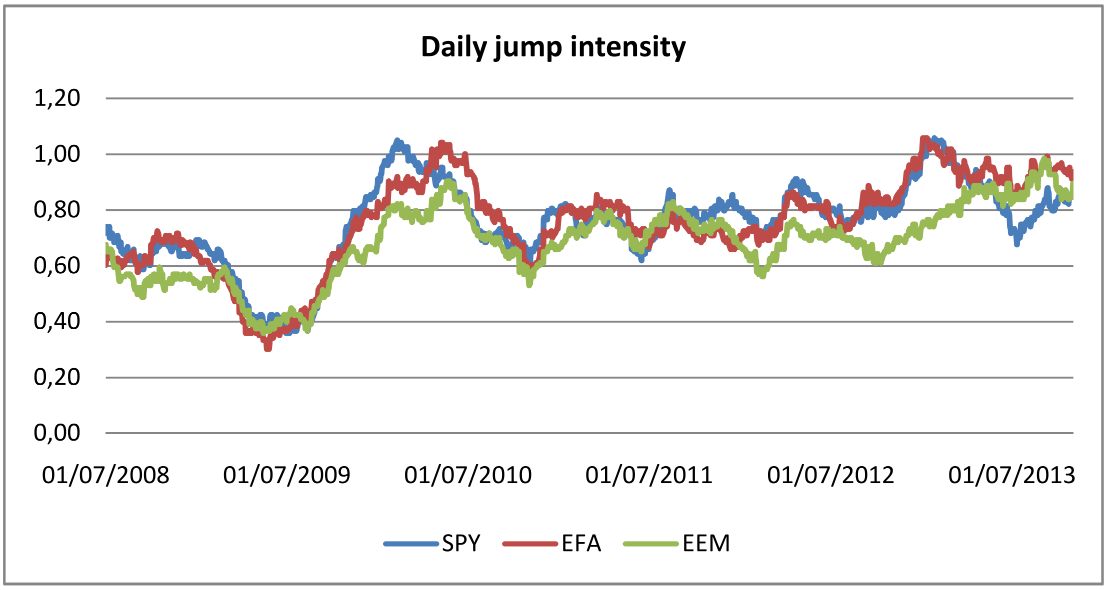



**Figure 2 and 3: Jump and cojump occurences**

These figures show the variation of the daily jump and cojump intensities (see definition in part 4.1) of the three funds from January 2008 to October 2013. These time-varying jump intensities are obtained for a rolling six month window of observations.



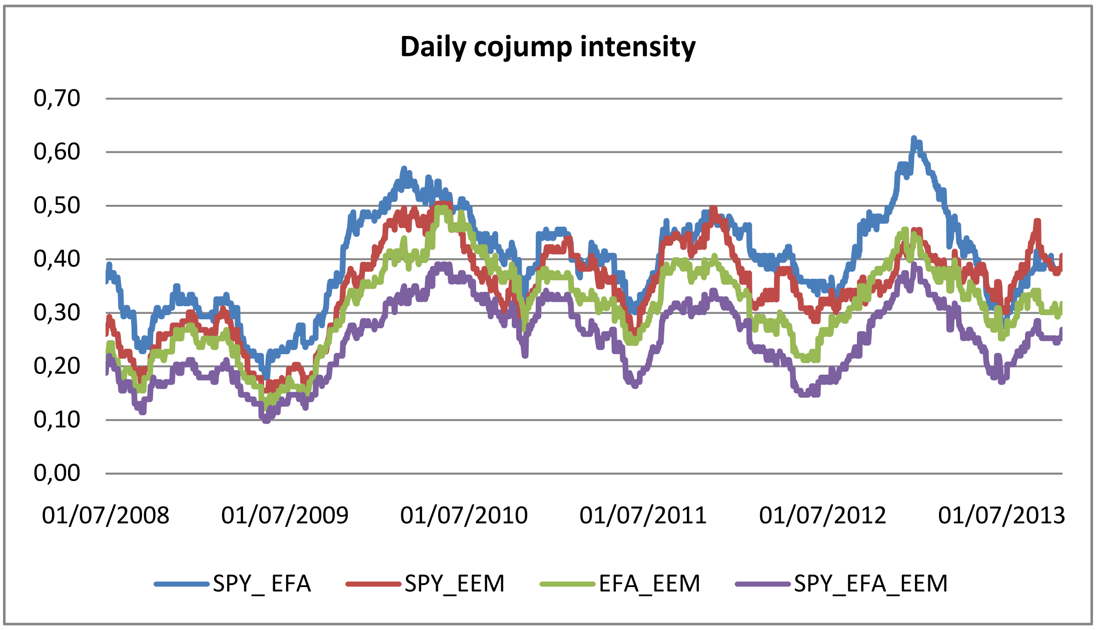



**Figure 4: Time and space clustering of intraday jumps**

This figure shows the arrival times of intraday jumps of the three funds from April 2010 (574th day of the sample) to November 2010 (715th day of the sample).

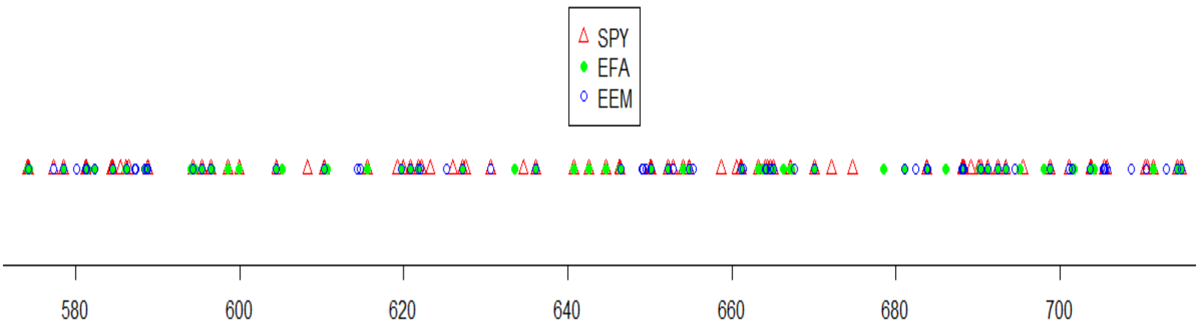
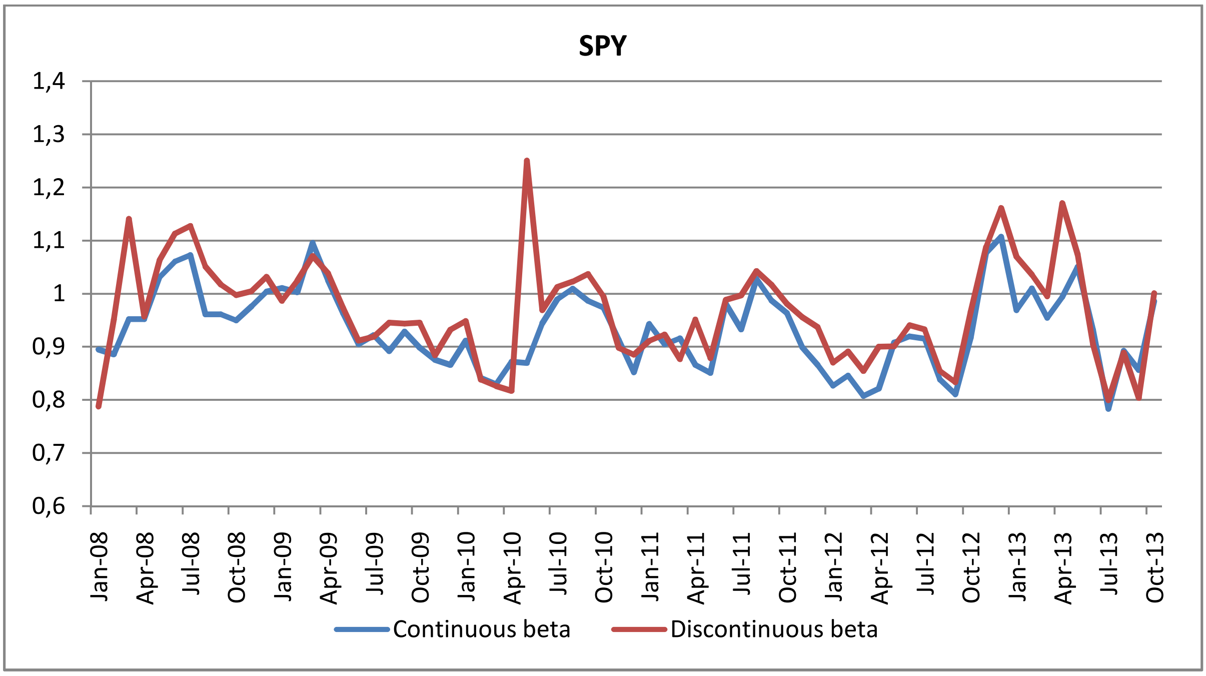


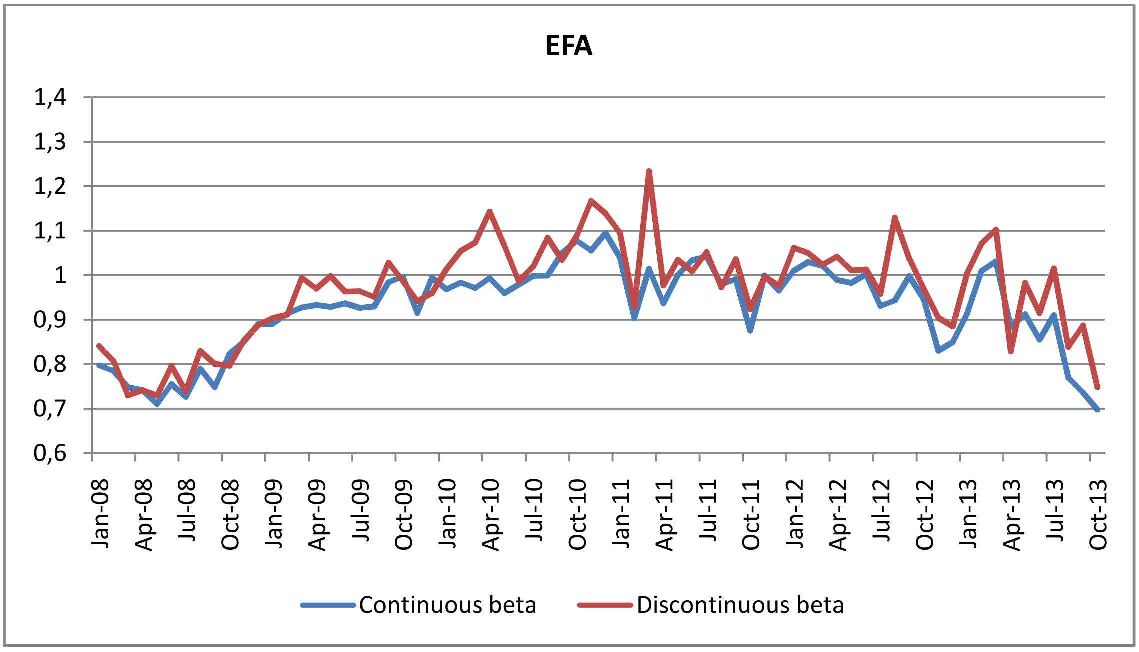
**Figure 5: Systematic diffusive and jump risks**

Panels A, B and C show the variation of the monthly continuous and discontinuous betas respectively, of SPY, EFA and EEM. See Section 5.3 for the definition of continuous and discontinuous betas.

**Panel A: SPY**



**Panel B: EFA**




**Panel C: EEM**

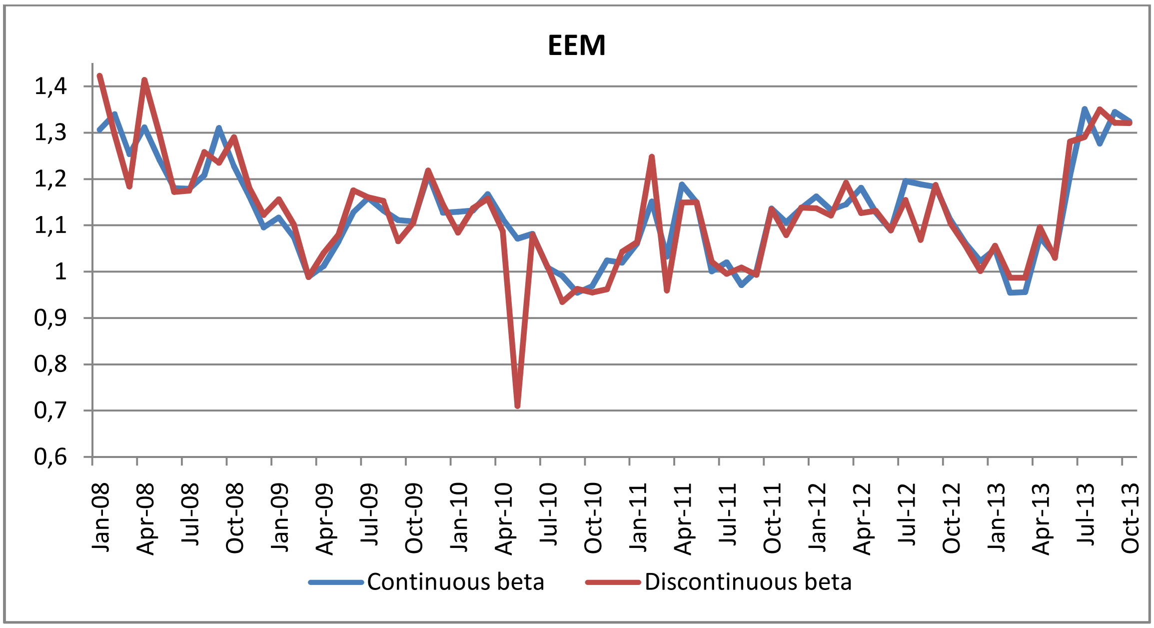


**Figure 6 : Autocorrelograms of diffusive and jump betas**

The figure shows the autocorrelograms of the continuous and discontinuous betas averaged across three funds.

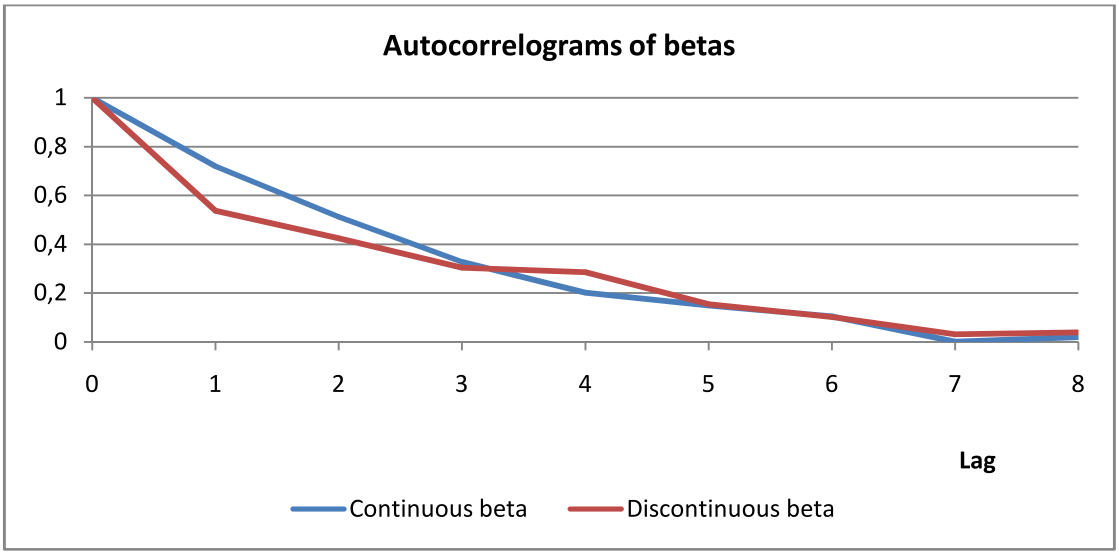
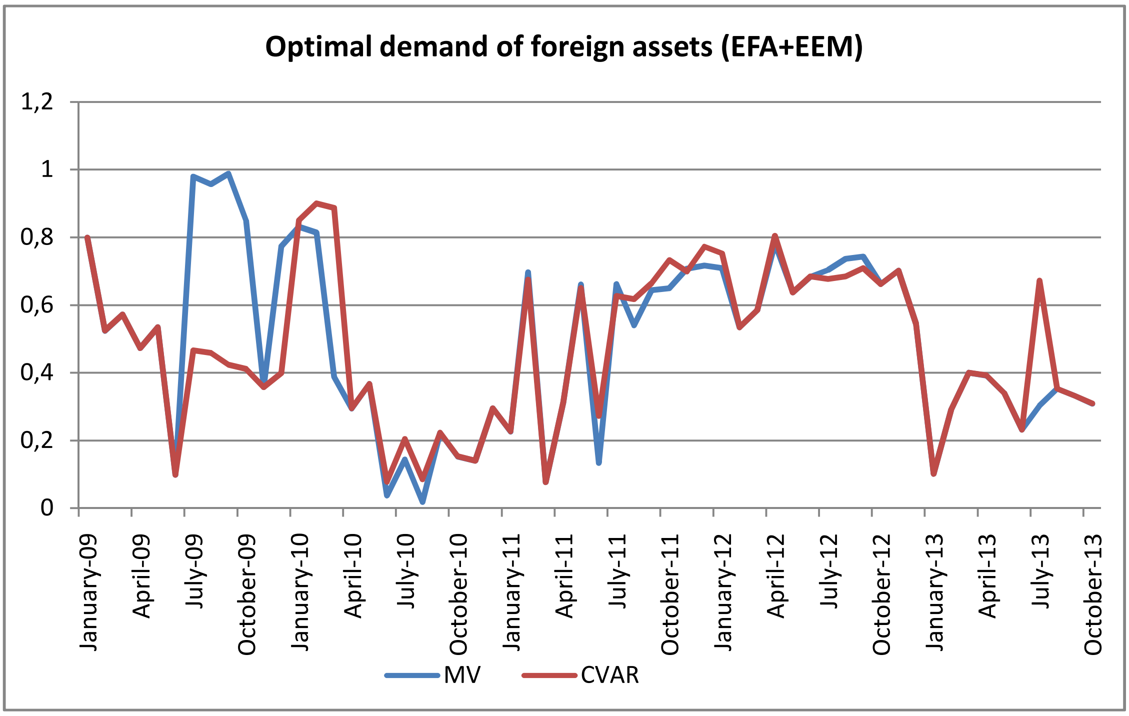


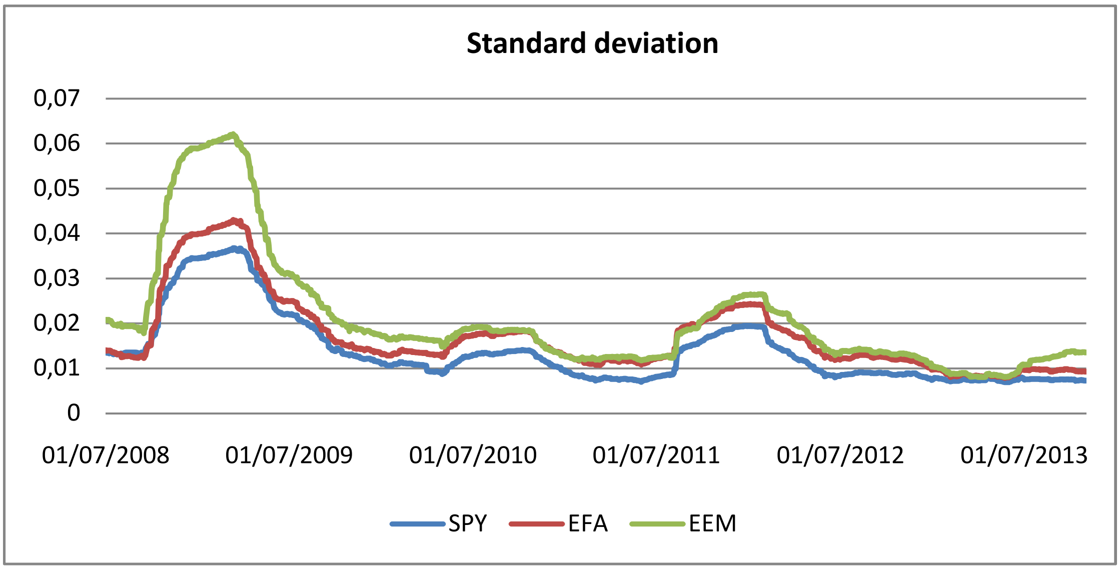
**Figure 7 : The variation of the optimal proportion of foreign assets**

Panel A shows the variation of the optimal proportion of the foreign assets (EFA and EEM) for MV and CVaR approaches. The optimal portfolio composition is determined in a monthly basis, using a one-year rolling window of weekly returns. The portfolio is composed of one domestic asset (SPY fund) and two foreign assets (EFA and EEM). Panel B and C presents moving standard deviation and absolute value of the CVaR, respectively of SPY, EFA and EEM.These variations are obtained for a rolling six-month window of daily returns.

**Panel A: Optimal demand of foreign assets under MV and CVaR**



**Panel B: Standard deviation of SPY, EFA and EEM**




**Panel C: CVaR of SPY, EFA and EEM**