The Optimal Capital Structure in Presence of Financial Assets*

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Abstract

Trade-off theories of capital structure describe how a firm chooses its leverage for a given set of assets. This paper studies how the predictions of such trade-off theories change if one accounts for the possibility that firms can invest in financial markets. In that case, the set of available assets is not given on the firm level, but depends on the characteristics of the market. In particular, new assets can be created by writing financial contracts. For arbitrary initial sets of assets and the corresponding optimal capital structures, I determine conditions under which the firm can decrease its leverage and its bankruptcy probability without a loss of value by use of 'integrated funds', which means by passively holding simple financial assets. This paper thus shows that, contrary to the usual notion, the trade-off theories of capital structure do not imply that capital requirements are (socially or privately) costly in the long run.

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1 Introduction

Capital requirements are a key instrument for the regulation of banks, and their potential costs are a key issue in debates about financial regulation. The serious[1] arguments for the existence of such costs in the long run[2] are based on theories that predict an optimal capital structure, which can be disturbed by capital requirements[3]. These theories deviate from Modigliani and Miller (1958) by describing a trade-off between the respective costs of equity and debt financing like, in the classic example, taxes and bankruptcy costs (see Modigliani & Miller (1963) or Kraus & Litzenberger (1973)). In such trade-off models the optimal capital structure depends on the characteristics of the firm assets[4]. And in their description of the firm problem, these models always take the set of available assets as given. If one accounts for the fact, however, that firms can invest in financial markets, the set of available assets is not given on the firm level, but depends on the characteristics of the financial market. Moreover, the set is not fixed, if new assets can be created by writing financial contracts. The aim of this paper is to show for several trade-off theories how their predictions about the optimal capital structure and the private costs of capital requirements change significantly, if one allows for such possibilities of investments in financial markets.

To put it differently, this paper analyze how the optimal capital structure of a firm is altered, when it 'integrates a fund'. This means I examine a firm that chooses its debt level in order to minimize frictions within the firm, while it can also choose to passively hold securities, which are issued in the same financial market in which the firm issues its own capital. I will restrict the set of possible securities to simple financial assets whose payoffs only depend on the overall cash flow of firms in the market (for instance, debt and equity claims or CDS). This implies that I do not consider a set of complete contracts that can condition on the processes within the firms and that could remove the frictions directly. Thus, the frictions and the capital structure of the firm matter. Yet, the possibility to invest in simple financial assets as described above has important consequences.

I will illustrate this for the trade-off between taxes and bankruptcy costs as well as for the trade-off between different types of agency costs highlighted by Jensen and Meckling (1976). And I will also address theories that are specific to banks and that try to explain their particularly high leverage - either by the disciplining role of demandable debt, as in Diamond and Rajan (2000), or by the special value of safe, 'money-like' debt claims, as in DeAngelo and Stulz (2015) or Gorton and Winton (2014).

[1]Admati et al. (2013) point out that some arguments about bank regulation are simply flawed.

[2]There can be (private as well as social) costs of equity increases in the short run because of a 'debt overhang problem', for instance, see Myers (1977), but these will not be subject of this paper.

[3]If capital requirements deviate from the optimal capital structure, banks incur private costs. And these can lead to social costs, if they impair the provision of credit and banking services to the economy. According to DeAngelo and Stulz (2015), capital requirements can cause social costs even directly by reducing the volume of 'money-like claims'. I will come back to this argument in more detail later.

[4]In case of the trade-off between taxes and bankruptcy costs, for instance, firms with less risky assets use more debt, because it reduces taxes while the expected costs of bankruptcy are small at the margin.

For these four theories of capital structure I will show: given any set of assets and the optimal capital structure that a firm would choose given these assets, the firm can reduce its leverage and its bankruptcy risk relative to this supposed optimum without a loss of value by the 'integration of a fund'. This means: by passively holding financial assets of the type described above, which are issued in the same market as its own debt and equity. Let me briefly preview why this result holds for the different theories of capital structure and under which conditions.

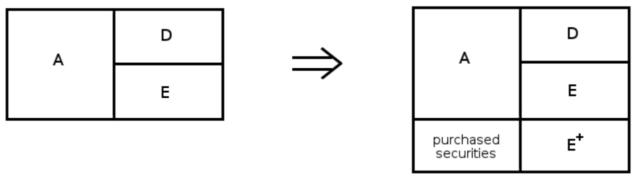

Figure 1: The 'integration of fund': A balance sheet with assets A (real and financial ones), which are financed with debt D and equity E, is enlarged by purchasing financial assets. This purchase can be financed by, for instance, issuing new equity E^+.

Let me start with the classic case of a trade-off between taxes and bankruptcy costs and consider the capital structure that is optimal for a firm with an arbitrary set of assets. Assume then that this firm issues more stocks in order to purchase financial assets in the same market, as illustrated in Fig. 1. The reduction of the firm leverage by means of such an 'integrated fund' can lead to efficiency gains rather than losses, if the resulting reduction of the bankruptcy probability and the related costs is larger than the increase in tax payments due to the additional equity. This holds true if the firm purchases assets that have a relatively high value in those states in which the value of the initial set of assets is too low to repay the debt. The sufficient condition for the result is thus one about the cash flow distribution of available financial assets and the result relies on a diversification argument that is similar to the one discussed in the literature about hedging, see Smith and Stulz (1985), for instance[5]. Before I discuss the availability of simple financial assets with such cash flow distributions as well as the empirical relevance of the results, let me point out how they extend to the other three theories of capital structure.

In case of a disciplining role of demandable debt, an 'integrated fund' has a similar effect as just described. The model of Diamond and Rajan (2000) basically describes a trade-off between costs that occur in case of a run (i.e. in case of bankruptcy) and the rent extraction by managers, which grows with the size of cash flows to equity holders (similar to taxes). A leverage reduction by means of an integrated fund hence leads to efficiency gains rather than losses, if the cash flow distribution of the purchased assets is such that the reduction in the first type of costs is larger than the increase in the second type.

In case of a trade-off between agency costs of debt (due to risk-shifting) and equity (due

[5] A similar mechanism has also been identified for mergers, see e.g. Lewellen (1971).

to reduced manager effort), as described by Jensen and Meckling (1976), an integrated fund neither increases nor decreases the firm value, as long as the payment scheme of the managers is not changed when the fund is integrated. If the payment of managers remains conditional on the performance of the initial set of firm assets (which they actually manage in contrast to the passively held financial assets), then their incentives to exert effort or to engage in risk-shifting remains unchanged. And since a passive fund has zero NPV in presence of a no-abritrage-condition, the same holds for the integrated fund in this case. Note that the alignment of the manager payment with the initial set of firm assets requires no information apart from the one about the overall values of the firms in the market, which determine the value of the simple financial assets held in the integrated fund.

The effect of an integrated fund on the provision of safe debt claims, finally, is simple: an integrated fund does not reduce the volume of debt issued by the firm[6], but it weakly increases the safety of the debt, because the additional cash flow from the purchased assets weakly increases the cash flow of the firm in each state. Thus, if there is a premium for issuing safe debt, the effect of integrated funds on the firm value is weakly positive.

To sum up, this preview has indicated that the only critical condition for the costless reduction of bankruptcy risk by means of integrated funds is the availability of financial assets with an appropriate cash flow distribution. An empirical investigation of the outstanding assets is insufficient to test the availibility of appropriate assets, since new financial assets can be created by writing contracts. As discussed in Section 6.1, a simple example for creating an asset with a beneficial cash flow distribution is selling a derivative that yields cash flows only in those states in which the cash flow of the firm is low.

Such an asset amounts to a 'capital insurance' of the firm. If it cannot be conditioned on externally given states, but has to be conditioned on the firm performance itself, there is the danger of moral hazard. An exploitation of such an insurance contract can be avoided, however, if it is provided by the owners of the firm themselves. This is possible, as discussed in Section 6.1, if the insurance is provided by an investment fund that also holds the entire equity of the firm. Such a construction has already been proposed by Admati et al. (2012), who call it 'liability holding company' (LHC).

They suggest LHCs as a way to increase the liability of bank owners (since bailouts are not provided by the public, but by the bank owners and their fund holdings), while the bank can maintain large amounts of debt in order to discipline its managers. This paper develops this idea of LHCs further by providing a systematic assessment of the 'costs' of such LHCs and of integrated funds in general. While Admati et al. (2012) focus on qualitative arguments in favor of LHCs, I study the effect of LHCs/integrated funds on

[6] Gorton and Winton (2014) and DeAngelo and Stulz (2015) already indicate that equity-financed purchases of securities do not reduce the level of safe debt and the related premium. But they either doubt that the stability of banks can be increased in this way or they estimate that huge parts of the capital markets had to be absorbed in order to increase the capital of banks significantly. In Section 7, however, I will illustrate that comparably small funds provide a significant increase of loss-absorbing capital even in case of the worst realizations of aggregate risk.

the firm value in an economic model of optimizing firms in equilibrium. And in contrast to them, I do not only address the disciplining role of debt, but I address the various theories of capital structure mentioned above.

The results of this paper seem to be at odds with empirical evidence. This remark does not refer to empirical studies about the costs and effects of changes in capital requirements like, for instance, Behn et al. (2016), because these studies look at the impact of changes in the short-run, whereas this paper is about long-run costs. The remark refers to the fact that firms do not use integrated funds or LHCs in order to reduce their bankruptcy risk to zero, though this would be optimal in presence of bankruptcy costs. In Section 6.3, I suggest the following explanation: If there is outstanding debt whose face value cannot be renegotiated, a change of capital structure with positive NPV leads to an asymmetric distribution of gains and losses: the gains (e.g. reduced expected bankruptcy costs) would accrue to the holders of the outstanding debt, while the owners would incur costs (e.g. higher taxes)[7] and hence have no incentive to implement this change. This obstacle to reaching a more efficient capital structure in presence of outstanding debt has been analyzed by Admati et al. (2016) in their description of the 'leverage ratchet effect' and it is reminiscent of the 'debt overhang problem'. The presence of such an obstacle, however, does not negate the fact that an altered capital structure with less bankruptcy risk does not imply costs in the long run, even if the trade-off theories hold true. And when all debt has been renewed and the reduced expected bankruptcy costs have been priced in, even the equity holders participate in the gains and become better off than in the initial state. In Sections 6.3 and 7, I will discuss how regulation can help to overcome the obstacle just mentioned and to achieve the transition to the weakly more efficient state with less bankruptcy risk.

Let me conclude the introduction with a remark about the aggregate effect of integrated funds in the equilibrium of the economy, which will be discussed in Section 5. The financial assets held in integrated funds or in LHCs have to be issued by other agents in the market and the overall level of cash flows in the economy does not change by rearranging them. Nevertheless, the fund structures can decrease the bankruptcy probability of firms in the aggregate. If a fund is added to an existing firm without changing its debt level, its bankruptcy risk never increases, but the additional cash flow from the assets in the fund can decrease the risk. At the same time, the solvency of the provider of these cash flows does not deteriorate only because the cash flows are sold to integrated funds rather than directly to the final recipients. To put it differently: Integrated funds allow firms to 'channel' cash flows from different sources through their balance sheets, where they have beneficial effects, before the final recipients receive these cash flows and consume them.

[7] As highlighted by Admati et al. (2016), this asymmetric distribution holds even in absence of frictions like taxes etc.. A reduction of bankruptcy risk always implies that the expected cash flow to holders of outstanding debt increases. And they do not pay for this increase, but they gain at the expense of the equity holders.

The **remainder of the paper** is organized as follows: Section 2 analyzes the trade-off between bankruptcy costs and taxes, and it also accounts for a premium for safe debt. Section 3 studies the theory about the disciplining role of demandable debt, and Section 4 addresses the trade-off between agency costs of debt and equity. The equilibrium of the model is determined in Section 5, before Section 6 discusses the availability of financial assets with beneficial characteristics (in 6.1 & 6.2) and the lack of empirical evidence for integrated funds (in 6.3). Section 7 concludes with stressing the implications for the regulation of banks.

The proofs of all Propositions and Lemmas are given in Appendix B.
The Corollaries are explained within the sections.

2 Taxes, Bankruptcy Costs, and a Premium for Safe Debt

This section addresses the classical trade-off between bankruptcy costs and taxes. In addition, a premium for safe claims is considered, as suggested by DeAngelo & Stulz (2013) and Gorton & Winton (2014). Following Gorton and Pennacchi (1990), riskless debt is useful as a means of payment and investors thus accept a discount on the interest rate of such claims. This premium is similar to the tax benefit of debt, only restricted to a certain subtype of debt.

Consider a firm owner who has a given set of productive assets that requires an investment $I = 1$ at $t = 0$ and that yields a stochastic cash flow $R \in \mathbb{R}^+$ at $t = 1$. Besides this investment in its productive assets, the firm can also 'integrate a fund', which means that it can buy a set S of financial assets in the same financial market in which it issues its own claims. I will comment on the choice of S later, but let us first assume that the composition of S is given and that the firm only chooses the amount s it invests in this portfolio at $t = 0$. The portfolio yields a stochastic cash flow $R_S \in \mathbb{R}^+$ at $t = 1$ per unit of s. The joint distribution of R and R_S is continuous and denoted as $\hat{f}$. The univariate marginal distribution of R is $f(R) := \int \hat{f}(R, R_S)\, dR_S$.

The firm finances its investments by issuing equity and two types of debt claims: senior debt with safe cash flow D_s at $t = 1$, and junior debt with face value D_r and default probability ϕ. In order to focus on the static capital structure of the firm, assume that it has no outstanding debt at $t = 0$. The bankruptcy probability of the firm is then

$$\phi(D_s, D_r, s) = \int \mathbf{1}_{\{R + s\, R_S < D_s + D_r\}}\, \hat{f}(R, R_S)\, dR_S\, dR\,.$$

Let us define the leverage $l(D_s, D_r, s)$ of the firm as the ratio of the face value of its debt over the expected cash flow of its assets:

$$l(D_s, D_r, s) = \frac{D_s + D_r}{E_{\hat{f}}\,[R + s\, R_S]}$$

If the debt level $D_s + D_r$ is kept fixed, an increase in the size of the integrated fund leads to a decrease of both, the leverage and the bankruptcy probability:

$$\frac{d}{ds} l(D_s, D_r, s) < 0 \;\forall\; s \in \mathbb{R}^+ \,, \quad \frac{d}{ds} \phi(D_s, D_r, s) \leq 0 \;\forall\; s \in \mathbb{R}^+,$$

and the second inequality is strict for some $s \in \mathbb{R}^+$ if $E_{\hat{f}} \left[\mathbf{1}_{\{R_S > 0\}} \mathbf{1}_{\{R < D_s + D_r\}} \right] > 0$.
In order to discuss the value of the claims and the value of the firm, assume that the debt and equity claims are priced in competitive markets with risk-neutral investors, where $1/r$ is the price at $t{=}0$ for one unit of expected cash flow at $t = 1$. (Appendix A discusses the robustness of the results to more general preferences of investors.) Assume that all agents can observe the firm's choice of capital structure and know $\hat{f}$ at $t = 0$.
With b denoting the costs in the event of bankruptcy[8], the value d_r of the junior debt at $t = 0$ is given as

$$d_r(D_s, D_r, s) = \frac{1}{r} \left(\big(1 - \phi(D_s, D_r, s)\big)\, D_r + \int \mathbf{1}_{\{R + s\, R_S < D_s + D_r\}} \,\big(R + s\, R_S - D_s - b\big)\, \hat{f}(R, R_S)\; dR_S\, dR \right)$$

and the expected cash flow of the junior debt at $t = 1$ is therefore $r \cdot d_r$.
If safe debt provides utility by serving as a means of payment, its value has two components: first, its expected cash flow, and second, the utility that safe financial claims provide to investors between $t = 0$ and $t = 1$. For conciseness, let us simply assume[9] that the latter is proportional to the face value D_s and equivalent to an expected cash flow $\lambda \cdot D_s$ at $t = 1$ with $\lambda > 0$. The value d_s of the safe debt claims at $t = 0$ is thus

$$d_s(D_s) = \frac{1}{r} \left(D_s + \lambda\, D_s \right) .$$

To discuss the tax benefit of debt, let us assume that the tax payments of the firm are given by $T(X_e)$ with $T'(X_e) > 0$ and X_e being the cash flow of the firm at $t = 1$ net of the debt payment in the event of solvency: $X_e = R + s\, R_S - D_r - D_s$. For simplicity, there are neither tax payments nor refunds in case of bankruptcy: $T(X_e) = 0$ for $X_e < 0$. The value e of the equity at $t = 0$ is the discounted expected residual cash flow net of taxes:

$$e(D_s, D_r, s) = \frac{1}{r} \left(\int \max\{R + s\, R_S - D_r - D_s, 0\}\, \hat{f}(R, R_S)\, dR\, dR_S - T^{exp}(D_r, D_S, s) \right) ,$$

[8]For simplicity, I assume that the bankruptcy costs b of the firm depend neither on R nor on $s\, R_S$. If one allowed for such dependencies, the explicit form of some relations (like Eq. (4), for instance) would change, but the qualitative results would remain the same.

[9] A microfoundation for this utility could be based on the possibility of asymmetric information during the period (between $t = 0$ and $t = 1$), when needs for trading and the exchange of claims arise. In that case, safe debt claims avoid either excepted losses from trading or costs for the acquisition of updated information about the assets that underlie the financial claims. Such a microfoundation, however, would neither change nor contribute anything to the results of this paper.

with $T^{exp}(D_r, D_S, s) := \int T\big(R + s\,R_S - D_r - D_s\big)\,\hat{f}(R, R_S)\,dR\,dR_S$. The value v_s of the firm with integrated fund at $t = 0$ is

$$\begin{aligned} v_s(D_r, D_s, s) =& \; d_r(D_r, D_s, s) + d_s(D_s) + e(D_r, D_s, s) \\ =& \; \frac{1}{r}\left(\int \big(R + s\,R_S\big)\,\hat{f}(R, R_S)\,dR\,dR_S - T^{exp}(D_r, D_S, s) - b\,\phi(D_s, D_r, s) + \lambda\,D_s\right) \end{aligned} \tag{1}$$

The 'net firm value' v, which is v_s net of the value of the purchased assets, is:

$$\begin{aligned} v(D_r, D_s, s) =& \; v_s(D_r, D_s, s) - \frac{1}{r}E_{\hat{f}}\left[s\,R_S\right] \\ =& \; \frac{1}{r}\left(\int R\,\hat{f}(R, R_S)\,dR\,dR_S - T^{exp}(D_r, D_s, s) - b\,\phi(D_s, D_r, s) + \lambda\,D_s\right) \end{aligned} \tag{2}$$

Assumption 1 *(no-arbitrage-condition)*
a) The expected cash flow per unit of s equals the risk-free market rate: $E_{\hat{f}}\left[R_S\right] = r$.
b) The outcome $R_S = 0$ has strictly positive measure.

The first assumption is imposed, because I want to study the purchase of financial assets in the same market in which the firm issues its own claims. The second assumption excludes financial assets that provide a safe cash flow. This assumption is only imposed to simplify further notation. If $R_S > 0$ in all states, the safe part of this cash flow would be priced in terms of the discounted rate $\frac{r}{1+\lambda}$, but this discount would net out with the discount on the claims that the firm would issue in order to finance the purchase of S.
The wealth of the initial firm owner at $t = 0$ is equal to v_s (the discounted value of the productive assets and the integrated fund) net of the value of claims that have to be sold to investors at $t = 0$ in order to finance both investments. The values of these claims have to add up to $I + s$ with $I = 1$. The decision problem of the wealth-maximizing initial firm owner is thus

$$\begin{aligned} &\max_{s\in\mathbb{R}^+,\, D_s\in[0,\overline{D}_s],\, D_r\in\mathbb{R}^+} \big(v^s(D_s, D_r, s) - (1+s)\big) = \max_{s\in\mathbb{R}^+,\, D_s\in[0,\overline{D}_s],\, D_r\in\mathbb{R}^+} \big(v(D_r, D_s, s) + \tfrac{1}{r}E_{\hat{f}}\left[s\,R_S\right] - (1+s)\big) \\ =& \max_{s\in\mathbb{R}^+,\, D_s\in[0,\overline{D}_s],\, D_r\in\mathbb{R}^+} \big(v(D_s, D_r, s) - 1\big)\,, \end{aligned} \tag{3}$$

with $\overline{D}_s := \min\big(R + sR_S \,|\, \hat{f}(R, R_S) > 0\big)$ being the lowest possible cash flow of the firm. The initial firm owner chooses those D_s, D_r and s that maximize the 'net firm value' $v(D_s, D_r, s)$. Before we discuss the full problem, let us examine the constrained optimization problems for given s and for given s and $D_r + D_s$:

Optimization Problem $P(s)$: $\max_{D_s\in[0,\overline{D}_s],\, D_r\in\mathbb{R}^+} v(D_s, D_r, s)$ for a given $s \in \mathbb{R}^+$

Optimization Problem $P(s, D)$: $\max_{D_s\in[0,\overline{D}_s],\, D_r\in\mathbb{R}^+} v(D_s, D_r, s)$ s.t. $D_s + D_r = D$, for a given $s \in \mathbb{R}^+$

In particular, let us consider a firm with an arbitrary set of assets and let us study the effect of integrating a fund into this firm while keeping its debt volume fixed:

Proposition 1

Consider the capital structure $(D_{s,0}, D_{r,0})$ *and the debt level* $D_0 := D_{s,0} + D_{r,0}$ *that a firm chooses in absence of an integrated fund, as it solves* $P(0)$*. Relative to this capital structure, a reduction of the leverage by means of an integrated fund increases the net firm value, if the purchased securities have a cash flow distribution that fulfills Eq.* (4)*:*

$\frac{d}{ds} v(D_{s,0}, D_{r,0}, s)\big|_{s=0} > 0$*, if*

$$b \; E_{\hat{f}}\left[R_S | R = D_0\right] \; f(D_0) \;\; > \int R_S \, T'(R + s\, R_S - D_0) \, \hat{f}(R, R_S) \, dR \, dR_S \, . \tag{4}$$

If one also accounts for the increase in the safe share of the debt D_0*, which means if one considers the solution* $(D_s^*(s, D_0), D_r^*(s, D_0))$ *of* $P(s, D_0)$*, then the result becomes:*

$$\frac{d}{ds} v(D_s^*(s, D_0), D_r^*(s, D_0), s)\bigg|_{s=0} > 0, \quad \textit{if} \tag{5}$$

$$b \, E_{\hat{f}}\left[R_S | R{=}D_0\right] f(D_0) + \lambda \, \min\left(R_S | \hat{f}(\underline{R}, R_S){>}0\right) > \int R_S \, T'(R{+}sR_S{-}D_0) \, \hat{f}(R, R_S) \, dR \, dR_S \, ,$$

with $\underline{R}$ *being the lower bound* $min(R|f(R) > 0)$ *for* R*.*

Given the optimal capital structure $(D_{s,0}, D_{r,0})$ of the benchmark case without fund, a decrease of leverage by a reduction of the debt level would reduce the firm value. A decrease of leverage by means of an integrated fund, in contrast, can increase the value, because it can be more efficient in decreasing the bankruptcy probability. The integration of a fund has two positive effects on the solvency of the firm: besides reducing the firm leverage (similar to a debt reduction), it can improve the diversification and can provide additional cash flows especially in those states in which the firm assets yield relatively low cash flows. Consequently, the decrease of the expected bankruptcy costs (given by the l.h.s. of Eq. (4)) can be larger than the increase of the tax payments due to the additional cash flow to equity holders (given by the r.h.s. of Eq. (4)).

If there is a premium for safe debt, there is a second positive effect of the integrated fund. If the cash flow from the purchased assets is greater than zero in all states in which the cash flow from the productive assets is at its minimum, the minimal cash flow of the firm increases. (This is possible in spite of Assumption 1 b, since the worst realizations of both sets of assets, $R_S = 0$ and $R = \underline{R}$, do not necessarily occur in a same state.) If the minimal cash flow increases, the firm with integrated fund can choose a higher level of safe debt, which implies a larger supply of safe claims for the investors and a larger premium for the bank owners[10]. An integrated fund never reduces the level of safe cash flow, as the value of the financial assets at $t = 1$ cannot be negative.

[10] This effect has already been indicated in Admati et al. (2013). Gorton and Winton (2014) neglect this effect in their analysis of the premium for safe debt, because they assume perfect correlation between

I would like to stress that Proposition 1 holds for any benchmark set of assets and the corresponding optimal capital structure. For each possible distribution of the cash flow R, Eq. (5) specifies a sufficient condition for the distribution of R_S, the cash flow of the purchased portfolio, such that a reduction of leverage by means of an integrated fund increases the firm value. This also holds for the set of firm assets that results from an integration of a fund. This means for $R + s\,R_S$ being the 'new benchmark set of assets', one can find another portfolio with an appropriate cash flow distribution, such that its integration increases the firm value further. One can derive the general statement:

Corollary 1
For every set of firm assets with continuous cash flow distribution $f(R)$ and corresponding optimal capital structure with strictly positive bankruptcy risk (i.e. $\phi(D_{s,0}, D_{r,0}, 0) > 0$), there exists a joint distribution $\hat{f}(R, R_S)$ with $E_{\hat{f}}[R_S] = r$ that fulfills Eq. (4) & Eq. (5).

This Corollary is proven by a simple example: Consider a financial asset that yields a cash flow $R_S = \frac{1}{m}(D_0 - R)$ in all states with $R \in [D_0 - \epsilon, D_0)$ and zero in all other states, with m chosen such that $E_{\hat{f}}[R_S] = r$. An infinitesimal investment in this asset would decrease the bankruptcy probability without causing any additional tax payments.
Let us now proceed with an extension of the firm problem by considering the choice of S. Should one assume that there are any restrictions for the choice of S or should one assume that assets with the optimal cash flow distribution are available for the firm? From a theoretical point of view, market participants should have an incentive to create such assets owing to the available gains. If they are in perfect competition, however, they should earn zero profits. I will make this more explicit in the equilibrium analysis in Section 5, where a set of dealers buys and sells financial assets. And in Section 6.1, I will indicate how such assets can be created from a practical point of view.
At this point, let us simply assume that the complete set of fairly priced Arrow-Debreu securities over the continuous set of states is offered on the financial market. With $\hat{\mathcal{F}} = \left\{\hat{f} \in \mathcal{C}_0(\mathbb{R}^+ \times \mathbb{R}^+) \,|\, E_{\hat{f}}[R_S] = r \wedge \int \hat{f}(R,0) dR > 0\right\}$, the decision problem of the initial firm owners becomes

$$\max_{\hat{f}\in\hat{\mathcal{F}},\, s\in\mathbb{R}^+,\, D_s\in[0,\overline{D}_s],\, D_r\in\mathbb{R}^+} v(D_s, D_r, s, \hat{f})\,, \quad \text{with } \overline{D}_s := \min\left(R + sR_S \,|\, \hat{f}(R, R_S) > 0\right)$$

$$\text{and } v(D_s, D_r, s, \hat{f}) = \frac{1}{r}\left(\int R\, f(R)\, dR + \lambda\, D_s - b\int \mathbf{1}_{\{R+s\,R_S<D_s+D_r\}}\, \hat{f}(R, R_S)\, dR_S\, dR - \int T\big(R + s\,R_S - D_r - D_s\big)\, \hat{f}(R, R_S)\, dR\, dR_S\right). \tag{6}$$

Corollary 2
a) The optimal choice of an unconstrained firm is a complete 'hedge of its firm assets' combined with the maximally possible level of debt. This means that it chooses s and $\hat{f}$

all issuers of financial claims. I will illustrate in Section 7 that their strict assumption is an inappropriate simplification, even if one considers the portfolio of banks in the worst crises.

such that $s\,R_S = \max\{\overline{R} - R, 0\}$ *for each realization of* R *and* $\overline{R} := \max(R|f(R) > 0)$, *while* $D_s + D_r = D_s = \overline{R}$.

b) [Alternative Scenario: Integrated Fund + Leverage Restriction]
If the firm is constrained in its choice of debt by an upper bound that equals its choice in absence of an integrated fund (i.e., if the constraint $D_s + D_r = D_0 = D_{s,0} + D_{r,0}$ *complements the problem in Eq.* (6)*), the constrained optimal choice is a 'capital insurance'. This means that the firm chooses* s *and* $\hat{f}$ *such that* $s\,R_S = \max\{D_0 - R, 0\}$ *for each* R.

Both, the hedge and the capital insurance, reduce the risk and the expected costs of bankruptcy to zero. The hedge also reduces the tax payments to the zero, while the capital insurance does not increase them. With D_0 being fixed, no reduction is possible, because the tax payments are equal to $\int T(R - D_0) f(R)\, dR\, dR_S$ in that case, and this term is independent of R_S. Finally, both, the hedge and the capital insurance, increase the premium for safe debt to the highest values that are possible in the respective cases: $\lambda\, D_0$ and $\lambda\, \overline{R}$. If one wanted to obtain a safe cash flow larger than $\overline{R}$ in the unconstrained case, it would require the purchase of financial assets whose cash flow R_S has a strictly positive lower bound. As mentioned above, this positive lower bound would imply that a certain component of the cash flow would be safe and would be priced accordingly in the financial market, such that the additional premium for safe debt in excess of $\overline{R}$ would net out.
These results prompt two questions: First, how can such financial assets be created from a practical point of view, when a specification of the cash flow in each state of the world might be unfeasible and when an unconditional 'capital insurance' or 'hedge' might induce moral hazard? I will indicate a simple way to overcome these obstacles in Section 6.1. Second, why do we not observe that firms use such hedges or capital insurances in order to reduce their bankruptcy risk to zero and to incur the supposed gains? In Section 6.3, I will argue that the lack of integrated funds might be due to frictions in a dynamic setting of the problem.

3 Disciplining Role of Demandable Debt

This section will show that the results obtained so far are not specific to the trade-off between bankruptcy costs and debt benefits, but apply to other trade-off theories as well. This shall be illustrated for a theory that tries to justify the strong leverage of the banking sector. Calomiris and Kahn (1991) and Diamond and Rajan (2000) have argued that a fragile funding structure with high levels of demandable debt can be optimal, because it disciplines the managers by the threat of 'runs' and reduces their possibilities to extract rents from the cash flow to investors.
The basic form of the firm problem is the same as before: A firm owner has productive assets that require an investment $I = 1$ at $t = 0$ and that yield a cash flow $R \in \mathbb{R}^+$ at

$t = 1$. In addition, the firm can 'integrate a fund', which means that it can choose to invest an amount s at $t = 0$ in a set S of financial assets, which are offered in the same market in which the firm issues its debt and equity. Again, I first take the portfolio S as given, before I discuss its selection at the end of the section. The joint distribution of R and the cash flow $R_S \in \mathbb{R}^+$ per unit of s at $t = 1$ is continuous and denoted as $\hat{f}$. In order to finance its investments, the firm issues debt, whose face value is denoted as D, and equity. There is no outstanding debt at $t = 0$ and all agents observe the firm's choice of capital structure and know $\hat{f}$ at $t = 0$.

The debt and equity holders do not receive the entire cash flow $R + s\,R_S$, because a part of it is lost either due to an extraction by managers or due to an inefficient liquidation in case of a run. In order to understand how the optimal capital structure balances these two types of costs, it is sufficient to know the state-contingent cash flows to the different stakeholders. Therefore, I only briefly summarize the story[11] presented in Diamond and Rajan (2000) in order to derive these state-contingent cash flows, before I focus on the choice of capital structure.

Assume that the firm is operated between $t = 0$ and $t = 1$ by managers who obtain special knowledge about the firm production. (In case of a bank, for instance, they establish relationships to the borrowers.) If the operation is not completed by the managers, but the debt or equity holders take over at $t = 1$ and liquidate it, the cash flow of the firm-specific assets declines from R to $R - l\,R$ with $0 < l < 1$. The same might hold true for the purchased assets, such that R_S declines to $(1 - l_S)R_S$ with $0 \leq l_S < 1$. It is unclear, however, whether the managers actually have a relative advantage in passively holding financial claims within the fund; and if they have it, whether the relative advantage is as strong as in the operation of the productive assets of the firm. In order to illustrate different cases, the results of the analysis will be stated for both, $l_S = 0$ and $l_S > 0$.

Threatening to withdraw their knowledge, managers are able to negotiate a rent with patient claim holders, which are the equity holders in this scenario. Debt in the form of depositors, in contrast, can prevent this rent extraction. The key characteristic of deposits is that, when they are withdrawn at $t = 1$, they are processed in order of arrival and are paid out at face value as long as the liquidation of assets provides some value. The depositors therefore immediately run, if their average cash flow is smaller than the face value D, either because $R + sR_S < D$ or because the managers attempt to extract some of their cash flow. Since the action of the depositors is immediate and uncoordinated, there is no chance for the managers to accomplish the extraction or to negotiate any other rent. The costs of this 'disciplining device' is the possibility of inefficient liquidations. The optimal

[11] The model focuses on the disciplining of the management by means of a fragile capital structure. It does not address the alleged potential of fragile funding structures to extract higher interest rates from the borrowers of banks. If one wanted to analyze comprehensively how the capital structure affects the extraction of cash flows from borrowers, one would need to go beyond Diamond and Rajan (2000), anyway. One would need to take into account, for instance, the reaction of borrowers to an increased extraction of rents that the fragile funding allows for (e.g. less entrepreneurial activity or evasion to alternative funding).

capital structure trades off the relative losses $lR + l_S\, sR_S$ from the 'runs' of depositors against the extraction of rents by managers. There are three types of states:

1. If $R + sR_S < D$, the depositors run and take hold of all assets. They only receive $R_l := (1-l)R + (1-l_S)s\,R_S$ due to an inefficient liquidation and neither managers nor equity holders get anything.

2. If $R_l < D \leq R + sR_S$, the depositors can be sure that they receive D. The managers do not dare to extract some of the cash flow to depositors, because they would lose access to the remaining cash flow $R + sR_S - D$. The equity holders do not take over the firm, because they could only obtain the cash flow R_l and would hence face a run. The distribution of $R + sR_S - D$ between managers and equity holders depends on the bargaining game between them. Let $b_e \in (0, 1)$ simply represent the share the managers get.

3. If $D \leq R_l$, the situation is similar to case 2. The depositors can be sure to get D and the equity holders and the managers bargain over the relative surplus that arises from keeping the managers. Since the equity holders could take over the firm without facing a run, the relative surplus is $lR + l_S\, sR_S$. Assume again that the managers get the share b_e.

To sum up, the state-contingent cash flows at $t = 1$ are[12]:

Pay-offs	depositors	equity	managers
1. $R + sR_S < D$	R_l	0	0
2. $R_l < D \leq R + sR_S$	D	$(1-b_e)\big(R+sR_S-D\big)$	$b_e \cdot \big(R+sR_S-D\big)$
3. $D \leq R_l$	D	$R+sR_S-D-b_e\cdot(l\,R+l_S\,sR_S)$	$b_e\cdot(l\,R + l_S\,sR_S)$

In order to discuss the value of the firm, assume again that the deposits and equity claims are priced in competitive markets with risk-neutral investors and a price $1/r$ at $t = 0$ for one unit of expected cash flow at $t = 1$. (Appendix A discusses the robustness of the results to generic preferences of investors.) Summing up the expected cash flows to depositors and equity holders at $t = 1$ and discounting them, one obtains the value $v_s(D, s)$ of the firm with integrated fund at $t = 0$:

$$\begin{aligned} v_s(D,s) =& \frac{1}{r}\left(\int_0^\infty (R + s\,R_S)\,\hat{f}(R,R_S)\,dR_S\,dR - L(D,s)\right), \quad \text{with} \\ L(D,s) =& \int b_e\,\big(l\,R + l_S\,sR_S\big)\,\mathbf{1}_{\{D\leq R_l\}}\,\hat{f}(R,R_S)\;dR_S\,dR \\ &+ \int b_e\,\big(R + sR_S - D\big)\,\mathbf{1}_{\{R_l\leq D\leq R+sR_S\}}\,\hat{f}(R,R_S)\;dR_S dR \\ &+ \int \big(l\,R + l_S\,sR_S\big)\,\mathbf{1}_{\{R+sR_S\leq D\}}\,\hat{f}(R,R_S)\;dR_S\,dR \end{aligned}$$

[12]The pay-off stated for the depositors is the one of the entire group of depositors, while the individual pay-offs vary in case of $R + sR_S < D$ due to the sequential order in processing the withdrawals.

The 'net firm value' $v(D,s)$, which is v_s net of the value of the purchased assets, is:

$$v(D,s) = v_s(D,s) - \frac{1}{r}\, s\, E_{\hat{f}}\left[R_S\right] = \frac{1}{r}\left(\int_0^{\infty} R\, \hat{f}(R,R_S)\, dR_S\, dR - L(D,s)\right)$$

The maximization of the (net) firm value is equivalent to the minimization of the expected losses $L(D,s)$. The optimal capital structure balances the expected extractions of cash flows by the management (given by the first and second term in $L(D,s)$) and the expected losses from runs of depositors (given by the third term in $L(D,s)$). The probability ϕ of a run (i.e. the bankruptcy probability) is $\phi(D,s) = \int \mathbf{1}_{\{R+sR_S \le D\}}\, \hat{f}(R,R_S)\; dR_S\, dR$. The leverage $l(D,s)$ of the firm shall again be defined as $l(D,s) = \frac{D}{E_{\hat{f}}[R+s\,R_S]}$.

Observation 1
If the debt level D is kept fixed, an increase in the size of the fund leads to a decrease of both, the leverage and the bankruptcy probability:

$$\frac{d}{ds} l(D,s) < 0 \;\forall\; s \in \mathbb{R}^+, \qquad \frac{d}{ds}\phi(D,s) \le 0 \;\forall\; s \in \mathbb{R}^+,$$

and the second inequality is strict for some $s \in \mathbb{R}^+$ if $E_{\hat{f}}\left[\mathbf{1}_{\{R_S>0\}}\mathbf{1}_{\{R<D\}}\right] > 0$.

To finance its investments, the firm has to sell deposits and equity at $t=0$ whose joint value is $I+s$ with $I=1$. Let us assume again that the firm buys the financial assets in the same competitive market in which it issues its debt and equity. Thus, **Assumption 1 a** applies: $E_{\hat{f}}\left[R_S\right] = r$. The problem of the wealth-maximizing initial firm owner is then

$$\begin{aligned} &\max_{D\in\mathbb{R}^+, s\in\mathbb{R}^+} \big(v_s(D,s) - (1+s)\big) = \max_{D\in\mathbb{R}^+, s\in\mathbb{R}^+} \big(v(D,s) + \frac{1}{r} s\, E_{\hat{f}}\left[R_S\right] - (1+s)\big) \\ &= \max_{D\in\mathbb{R}^+, s\in\mathbb{R}^+} \big(v(D,s) - 1\big) \;\Leftrightarrow\; \min_{D\in\mathbb{R}^+, s\in\mathbb{R}^+} L(D,s)\,. \end{aligned} \tag{7}$$

In order to study the impact of an integrated fund, let us start with a firm that has an arbitrary set of assets and let us then consider how its firm value changes when it integrates a fund. For this purpose, let us again define the constrained optimization problem $P(s) : \max_{D\in\mathbb{R}^+} v(D,s)$ for a given $s \in \mathbb{R}^+$.

Proposition 2
Consider the debt level D_0 that a firm would choose in absence of an integrated fund, as it solves $P(0)$. Relative to this capital structure, a reduction of the leverage by means of an integrated fund increases the net firm value, if the purchased securities have an appropriate cash flow distribution:

$\frac{d}{ds} v(D_0,s)\big|_{s=0} > 0$, *if*

$$l\, D_0\, f(D_0)\, E_{\hat{f}}\left[R_S | R{=}D_0\right] \;>\; b_e \int_{D_0}^{\frac{D_0}{1-l}} E_{\hat{f}}\left[R_S | R{=}R'\right] \cdot f(R')\, dR' \tag{8}$$

in case of $l_S = 0$. In case of $l_S > 0$, the condition in Eq. 8 becomes

$$\begin{aligned} l\, D_0\, f(D_0)\, E_{\hat{f}}\left[R_S | R{=}D_0\right] \;\geq\; & b_e \int_{D_0}^{\frac{D_0}{1-l}} E_{\hat{f}}\left[R_S | R{=}R'\right] \cdot f(R')\, dR' \\ & + l_S \left(\int_0^{D_0} E_{\hat{f}}\left[R_S | R{=}R'\right] f(R')\, dR' + \int_{\frac{D_0}{1-l}}^{\infty} b_e\, E_{\hat{f}}\left[R_S | R{=}R'\right] f(R')\, dR' \right). \end{aligned} \tag{9}$$

Given the optimal debt level D_0 of the benchmark case, a decrease of leverage by a reduction of debt would reduce the firm value. A decrease of leverage by means of an integrated fund, in contrast, can increase the value, if it alters the distribution of the cash flow of the firm in such a way, that it becomes more concentrated on a domain where the relative losses from rent extraction and inefficient liquidations are comparably small.
If the cash flow $R + s\, R_S$ of the firm is slightly larger than D, there is only a small residual cash flow from which the managers can extract rents (the losses are proportional to $R + s\, R_S - D$). If the cash flow is slightly smaller than D, in contrast, a run occurs and causes significant relative losses (that are linear in R and $s\, R_S$). In the benchmark case without integrated fund, it is therefore optimal for the firm to choose D such that the cash flow distribution is concentrated on the domain slightly above D (as illustrated on the left panel of Fig. 2). The additional cash flow $s\, R_S$ can decrease the expected losses relative to this benchmark, if the joint distribution of R and R_S is such that the distribution of the cash flow $R + s\, R_S$ becomes more concentrated on the domain slightly larger than D. This holds when the cash flow R_S is relatively large in those states in which it can prevent a run (i.e., the l.h.s. of Eq. (8) and Eq. (9) is large), while it is relatively small in those states in which it only increases the cash flow from which managers can extract rents or which is inefficiently liquidated (i.e., the r.h.s. of Eq. (8) and Eq. (9) is small). An example is depicted on the right panel in Fig. 2, where $\bar{f}(x) := \int \hat{f}(x - s\, R'_S, R'_S)\, dR'_S$ represents the probability distribution of the overall cash flow $x = R + s\, R_S$.
Proposition 2 shows that the possibility to decrease the leverage and bankruptcy risk of a firm without a loss of firm value by means of integrated funds does not only exist in case of the classical trade-off between taxes and bankruptcy costs, but it also exist in case of a trade-off between rent extraction by managers and inefficient liquidations. Owing to the similar structure of the two cases, the implications of the results are very similar, too:
Since Proposition 2 holds for any benchmark set of assets (which means it holds for any distribution f of the cash flow R), one can iteratively improve the firm value by integrating appropriate financial assets into the firm balance, until the probability of run approaches zero. Using the same exemplary financial asset as in Corollary 1 with $\epsilon \to 0$, one finds:

Corollary 3 *(analogue to Corollary 1)*
For every set of firm assets with continuous cash flow distribution $f(R)$ and corresponding optimal capital structure with strictly positive bankruptcy risk (i.e. with $\phi(D_0, 0) > 0$), there exists a joint distribution $\hat{f}(R, R_S)$ with $E_{\hat{f}}\left[R_S\right] = r$ that fulfills Eq. (8) & Eq. (9).

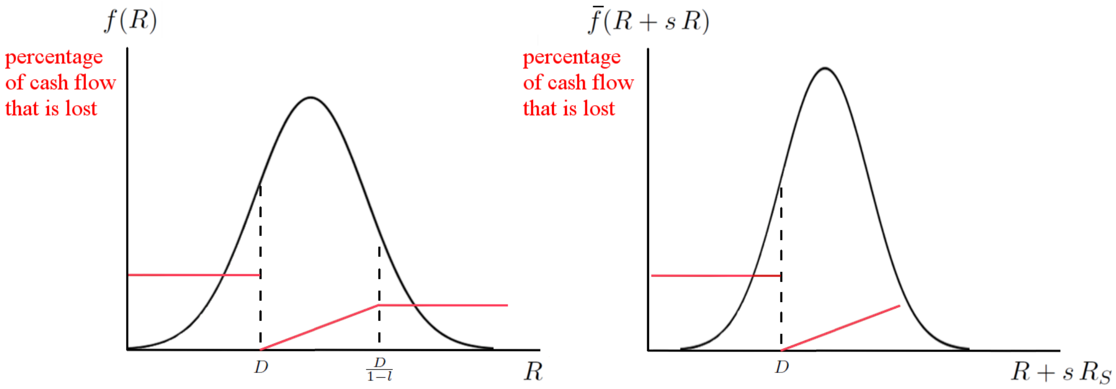

Figure 2: The left panel shows the optimal choice of D in the case without fund. It is chosen such that the distribution $f(R)$ is concentrated on the domain slightly above D, where the relative losses are comparably small. The right panel shows that the additional cash flow $s\,R_S$ can reduce losses and can increase the firm value, if the joint distribution of R and R_S is such that the overall cash flow $R + s\,R_S$ becomes more concentrated on the domain slightly above D. *(In the second graph, the second domain boundary is not indicated, because its position depends on R and R_S separately, not only on $R + sR_S$.)*

Owing to available gains from the integration of funds, there should be agents that create and offer assets with a beneficial cash flow distribution. (Section 5 makes this more explicit and Section 6.1 describes how such assets can be created, practically.) If this is true, one can study how the firm optimally chooses its portfolio, which means to study the following problem for $\hat{\mathcal{F}} := \{\hat f \in \mathcal{C}_0(\mathbb{R}^+ \times \mathbb{R}^+) \,|\, E_{\hat f}\,[R_S] = r\}$:

$$\max_{D_r \in \mathbb{R}^+,\, s \in \mathbb{R}^+,\, \hat f \in \hat{\mathcal{F}}} L(D, s, \hat f) \quad \text{with} \tag{10}$$

$$L(D,s,\hat f) = \int b_e\big(l\,R + l_S\,sR_S\big)\,\mathbf{1}_{\{D \le R_l\}}\hat f(R,R_S)dR_S dR + \int b_e\big(R + sR_S - D\big)\mathbf{1}_{\{R_l \le D \le R+sR_S\}}\,\hat f(R,R_S)dR_S dR$$
$$+ \int \big(l\,R + l_S\,sR_S\big)\,\mathbf{1}_{\{R+sR_S \le D\}}\,\hat f(R,R_S)dR_S\,dR$$

Corollary 4 *(analogue to Corollary 2)*
a) The optimal choice of an unconstrained firm is a complete 'hedge of its firm assets' combined with the maximally possible level of debt. This means that it chooses s and $\hat f$ such that $s\,R_S = \max\{\overline{R} - R, 0\}$ for each realization of R and $\overline{R} := \max(R|f(R) > 0)$, while $D = \overline{R}$.

b) [Alternative Scenario: Integrated Fund + Leverage Restriction]
If the firm is constrained in its choice of debt by an upper bound that equals its choice in absence of an integrated fund (i.e., if the constraint $D = D_0$ complements the problem in Eq. (6)), the constrained optimal choice is a 'capital insurance'. This means that the firm chooses s and $\hat f$ such that $s\,R_S = \max\{D_0 - R, 0\}$ for each realization of R.

As in the previous section, these choices of the balance sheet do not allow for any further improvement, since they reduce both, the losses from inefficient liquidations and the rent extraction by managers, to their lowest possible value (given the potential constraint). These results have been derived under the assumption that the cash flow of the financial asset can be specified for each state in the world, independent of the behavior of the managers. If writing complete contracts is not possible, but the 'capital insurance' or the 'hedge' have to be conditioned on the lack of cash flow, $D_0 - R$, they allow for moral hazard by the managers. I will present different possibilities how the firm can overcome this problem in Section 6.2. Finally, in Section 6.3, I will argue how frictions in a dynamic setting of the problem might explain why firms do not use integrated funds to reduce their bankruptcy risk to zero despite the available gains, and I will discuss the consequences for the debate about capital regulation in Section 7.

4 Trade-off between Risk-Shifting and Effort Reduction

In this section, I discuss the trade-off between agency costs of debt (in the form of risk-shifting) and agency costs of equity (in the form of a reduction in effort[13] by the managers) that has been described in Jensen and Meckling (1976). This trade-off differs from the previous ones, as it does not describe assets with a given distribution of cash flows, but assets whose cash flow is directly affected by the managers of the firm. The presentation of this case has two steps: first, the case of a generic firm without integrated fund is established as a benchmark, before the impact of an integrated fund is studied in the second part of this section.

4.1 Agency Costs of a Firm without an Integrated Fund

Assume again that a firm owner issues equity and debt claims in order to finance an investment $I = 1$ at $t = 0$ in a set of productive assets, when there is no outstanding debt. The cash flow of these assets at $t = 1$ depends on a basic cash flow R (with density f and upper bound $\overline{R}$) and on the behavior of the firm managers between $t = 0$ and $t = 1$:
1. The effort $c_m \in [0, \bar{c}_m]$ of the managers amplifies the cash flow, such that it becomes $\rho(c_m) \cdot R$, with $\frac{d}{dc_m}\rho > 0$. Exerting the effort c_m, the managers incur a disutility that is equivalent to a negative cash flow $-h(c_m)$ at $t = 1$, with $\frac{d}{dc_m}h > 0$. In order to incentivize the managers, the firm owner gives them a share $m \in [0, 1]$ of the firm equity at $t = 0$. (Later, I explain why the results also hold for a payment of managers with other claims.)
2. The managers can choose to operate a share $\alpha \in [0, 1]$ of the investment in a riskier way during the period. If this risky operation succeeds, which occurs with probability p, the cash flow of the firm at $t = 1$ is raised to $\rho\, R + \alpha \cdot \beta^{+}$. Otherwise, the cash flow is

[13] alternatively, 'effort' can be seen as the discipline to abstain from a misuse of firm resources

reduced to[14] $\rho R - \alpha \cdot \beta^-$. Assume that the risk-shifting is inefficient: $p\,\beta^+ < (1-p)\beta^-$. The risk-neutral managers choose c_m and α between $t=0$ and $t=1$, after the firm has issued debt with face value D. Their optimization problem is then

$$\max_{c_m\in[0,\bar{c}_m],\alpha\in[0,1]} \left(m\, E_f\left[\max\left\{0\,,\ \rho(c_m)R + \mathbf{1}_{\beta+}\alpha\beta^+ - (1-\mathbf{1}_{\beta+})\alpha\beta^- - D\right\}\right] - h(c_m)\right), \quad (11)$$

where $\mathbf{1}_{\beta+}$ identifies states with successful outcome of the risky project. The optimal choices c_m^* and α^* depend on m and D. The cash flow of the firm at $t=1$ is thus

$$X(R; D, m) := \rho\big(c_m^*(D,m)\big)R + \alpha^*(D,m)\cdot\left[\mathbf{1}_{\beta+}\beta^+ - (1-\mathbf{1}_{\beta+})\beta^-\right]\ . \quad (12)$$

Assume that all agents have complete information at $t=0$ and that the claims are again priced in competitive markets with risk-neutral investors, where $1/r$ is the price at $t=0$ for one unit of expected cash flow at $t = 1$. (Appendix A shows that the results are robust to more general preferences of investors.) The value d of the debt at $t = 0$ is then $d(D,m) = \frac{1}{r}\, E_f\left[\min\left\{D, X(R;D,m)\right\}\right]$. And the value of the equity at $t = 0$ is $e(D,m) = \frac{1}{r}\, E_f\left[\max\left\{0, X(R;D,m) - D\right\}\right]$. The value v of the firm at $t=0$ is the sum of the values of debt and equity net of the equity given to the managers:

$$v(D,m) = \frac{1}{r}\, E_f\left[X(R;D,m)\right] - m\, e(D,m)$$

To finance its investment, the initial firm owner has to sell debt and equity claims at $t=0$ whose joint value adds up to $I = 1$. The decision problem of the wealth-maximizing initial firm owner, who can choose D and m, is thus:

$$\max_{D\in\,[0,\overline{R}],\, m\in[0,1]} \left(v(D,\, m) - 1\right)$$

The firm value $v(D,m)$ depends on $X(R;D,m)$, which depends on the behavior of the managers who choose their optimal $c_m^*(D,m)$ and $\alpha^*(D,m)$ according to Eq. (11). Choosing D and m at $t = 0$, the initial firm owner takes this dependence into account and trades off the agency cost of debt against the agency cost of equity.

Lemma 1
For each $r > 0$, there is an optimal capital structure (D^*, m^*), which maximizes v.

The manager problem and the firm problem always have a solution, since both are optimizations of finite expressions over a compact set. Having a generic model that represents the managers' impact on productive firm assets and the trade-off between agency costs of equity and debt, let us now study the consequences of integrating a fund.

[14] In order to avoid uninformative case distinctions, assume that $\rho R - \beta^- > 0$ for all possible cases. One could allow for a dependence of β^- and β^+ on $\rho\, R$, but that would not change the results of this analysis.

4.2 The Effect of an Integrated Fund

The possibility to integrate a fund means again that the firm can choose to invest an amount s at $t = 0$ in a set S of financial assets, which are offered in the same market in which the firm issues its debt and equity. As before, I study the firm problem for a fixed composition of the portfolio S that yields R_S at $t = 1$ per unit of s, and $\hat{f}$ denotes the joint distribution with R.

The behavior of the managers might be influenced by an integrated fund, such the optimal choices c_m^* and α^* can depend on s. It seems to be reasonable, however, that the basic characteristics of the productive assets are not affected by financial assets held by the firm. I thus assume that the distribution f of the basic cash flow R as well as the function ρ, which describes the effect of effort on the output of the productive assets, are independent of s. Let us also assume for a moment that β^+ and β^-, the characteristics of an risky operation of the productive assets, are independent of the purchased financial assets, too. In Sections 6.1 and 6.3, I will discuss how an integrated fund might expand the possibilities for risk-shifting. Given these assumptions, the cash flow from the productive assets is

$$X(R; D, m, s) = \rho\big(c_m^*(D, m, s)\big)R + \alpha^*(D, m, s) \cdot \left[\mathbf{1}_{\beta^+}\, \beta^+ - (1 - \mathbf{1}_{\beta^+})\beta^-\right] . \qquad (13)$$

The impact of the integrated fund on the manager behavior (which means the form of $c_m^*(D, m, s)$ and $\alpha^*(D, m, s)$ as function of s) depends on the way in which the payment scheme of the managers is adjusted to the integration of a fund. While the firm has many degrees of freedom in choosing a scheme, I will only present a simple example here, for which the integrated fund has neither a positive nor a negative effect on the firm value.

Let us consider the case that managers receive the share m of the equity claims to the cash flow X from the productive assets of the firm. If the cash flow from the productive assets has priority (over the cash flow from the purchased financial assets) in repaying the firm debt, the decision problem of the managers during the period is

$$\max_{c_m \in [0, \bar{c}_m], \alpha \in [0,1]} \Big(m\, E_{\hat{f}}\left[\max\left\{0, \rho(c_m)R + \mathbf{1}_{\beta^+}\alpha\beta^+ - (1 - \mathbf{1}_{\beta^+})\alpha\beta^- - D\right\}\right] - h(c_m) \Big).$$

This problem is identical to the one in the benchmark case.

Observation 2

If the managers are paid with equity claims to the productive assets, then their behavior is independent of the integrated fund: $\alpha^*(D, m, s) = \alpha^*(D, m)$ *and* $c_m^*(D, m, s) = c_m^*(D, m)$. *Consequently, the cash flow from the productive assets is independent of the fund, too:* $X(R; D, m, s) = X(R; D, m)$.

The key idea behind this incentive scheme is to relate the payment of the managers to the part of the firm that depends on their behavior. This is the firm production that yields X. The cash flow R_S of purchased securities is independent of the managers of the firm, which simply holds the assets. The adjustment of the payment scheme is an example for the following, quite general point: although there are important relations between the capital structure of a firm and both, the incentives of agents in that firm and their control rights, these relations are not as strict as first highlighted in Jensen & Meckling (1976), but a firm has many degrees of freedom to shape these relations by writing better contracts with the involved agents. This has already been stressed by, for instance, Aghion & Bolton (1989), or in a similar circumstance as here, by Dybvig & Zender (1991) in their discussion of Myers & Majluf (1984).

The possibility to separate the manager behavior from the integrated fund is independent of the initial payment scheme of the managers. I have illustrated the case in which they only receive equity claims, but the same logic applies to any set of claims with which managers are paid. The structure and state-contingent payoffs of their claims can be maintained when a fund is integrated, if they continue to refer to the firm production.

Although the integrated fund does not change the behavior of the managers, it has an impact on the solvency of the firm. In states in which the cash flow X from the firm production is too small to repay the firm debt D, a sufficiently large cash flow $s\,R_S$ from the fund can avoid bankruptcy. The bankruptcy probability ϕ is thus given as $\phi(D,m,s) = \int \mathbf{1}_{\{X(R;D,m,s)+s\,R_S\,<\,D\}}\,\hat{f}(R,R_S)\,dR\,dR_S$. And the integrated fund also affects the firm leverage, which is again defined as $l(D,m,s) = \frac{D}{E_{\hat{f}}[X(D,m,s)+s\,R_S]}$.

Observation 3

If D is kept fixed and the managers are paid with a share m of equity claims to the productive assets, then an increase in the size of an integrated fund leads to a decrease of both, the leverage and the bankruptcy probability:

$$\frac{d}{ds}l(D,m,s) < 0 \;\forall\; s \in \mathbb{R}^+, \qquad \frac{d}{ds}\phi(D,m,s) \leq 0 \;\forall\; s \in \mathbb{R}^+$$

and the second inequality is strict for some $s \in \mathbb{R}^+$ if $E_{\hat{f}}\left[\mathbf{1}_{\{R_S>0\}}\mathbf{1}_{\{X(R;D,m)<D\}}\right] > 0$.

The additional cash flow $s\,R_S$ also affects the value of the debt claims at $t = 0$, which becomes $d(D,m,s) = \frac{1}{r}\,E_{\hat{f}}\left[\min\left\{X(R;D,m,s) + s\,R_S, D\right\}\right]$. The value e' of the equity of the overall firm (firm production plus integrated fund) is equal to the value of the expected cash flow from the firm production and the integrated fund net of the expected debt payments and the expected cash flow to the managers:

$$\begin{aligned} e'(D,m,s) &= \frac{1}{r}\,E_{\hat{f}}\left[\max\left\{0, X(R;D,m,s) + s\,R_S - D\right\}\right] - m\,e_X(D,m,s)\,, \\ \text{with}\;\; e_X(D,m,s) &= \frac{1}{r}\,E_{\hat{f}}\left[\max\left\{0, X(R;D,m,s) - D\right\}\right]\;. \end{aligned} \tag{14}$$

The value $v_s(D,m,s)$ of the firm with integrated fund is the joint value of d and e':

$$v_s(D,m,s) = \frac{1}{r}\left(E_{\hat{f}}\left[X\right](D,m,s) + s\,E_{\hat{f}}\left[R_S\right]\right) - m\,e_X(D,m,s)\,.$$

The 'net firm value' v, which means v_s net of the value of the financial assets, is:

$$v(D,m,s) = v_s(D,m,s) - \frac{1}{r}\, s\, E_{\hat{f}}\left[R_S\right] = \frac{1}{r} E_{\hat{f}}\left[X\right](D,m,s) - m\, e_X(D,m,s)\,. \qquad (15)$$

To finance its investments, the firm has to sell debt and equity at $t = 0$ whose joint value is $I + s$ with $I = 1$. If the firm buys the financial assets in the same competitive market in which it issues its debt and equity, then **Assumption 1 a** applies again: $E_{\hat{f}}\left[R_S\right] = r$. The decision problem of the wealth-maximizing initial firm owner is then

$$\max_{D\in\mathbb{R}^+, m\in[0,1], s\in\mathbb{R}^+} \big(v_s(D,m,s) - (1+s)\big) = \max_{D\in\mathbb{R}^+, m\in[0,1], s\in\mathbb{R}^+} \big(v(D,m,s) + \frac{1}{r} s\, E_{\hat{f}}\left[R_S\right] - (1+s)\big)$$
$$= \max_{D\in\mathbb{R}^+, m\in[0,1], s\in\mathbb{R}^+} \big(v(D,m,s) - 1\big) \qquad (16)$$

Proposition 3
Consider a firm without integrated fund ($s \equiv 0$) whose optimal capital structure is (D_0, m_0). If this firm can integrate a fund and pays its managers with claims to the productive firm assets, then its optimal capital structure (D^*, m^*, s^*) is given by

$$D^* = D_0\,, \quad m^* = m_0\,, \quad \textit{and } s^* \textit{ being an abitrary element of } \mathbb{R}^+\,.$$

Consequently, an increase in the size of the integrated fund and a corresponding decrease of the firm leverage has no effect on the optimized net firm value:

$$v(D^*, m^*, s^*) = v(D_0, m_0, 0)\ \forall\ s^* \in \mathbb{R}^+.$$

The proposition follows directly from the fact that the firm problem is effectively independent of the fund, when the manager payment remains aligned with the firm production. The firm can thus increase its equity to any level without a reduction of its firm value.
To sum up, this section has shown that a key result of the previous sections also holds for the trade-off between agency costs of debt and equity: the integration of a fund allows for a decrease of leverage and bankruptcy risk without a loss of firm value. In contrast to the cases discussed before, this result does not depend on an appropriate cash flow distribution of the financial assets, but on an appropriate payment scheme for the managers. Given the payment scheme discussed here, integrated funds do not increase the firm value, as in the previous sections, but they just maintain the value. Further research, however, might show that more refined payment schemes perhaps allow for an increase.

5 Equilibrium

This section will conclude the analysis of the model by studying the equilibrium of a set of firms that choose their capital structure and their integrated funds as described above.

This analysis will show that all firms in the set can simultaneously increase their firm value and decrease their bankruptcy risk by means of integrated funds, although the underlying set of productive assets in the economy is fixed and finite. In order to illustrate the effects of integrated funds, I will first introduce a benchmark equilibrium with firms that can only invest in their productive assets, before I add the possibility of integrated funds.

5.1 The Equilibrium of the Benchmark Case

Assume that there is a continuum $J = [0, 1]$ of firms and each firm $j \in J$ maximizes its firm value v_j by choosing a vector of choice variables as described in the previous sections. The vector is (D_s, D_r) for the trade-off between taxes and debt benefits; it is (D) for the trade-off between liquidation losses and rent extraction; and it is (D, m) for the trade-off between agency costs of debt and equity. The optimally chosen shall be denoted as x_j. Let us assume that a firm is active if the investment of the firm has strictly positive value for the firm owner at $t = 0$, which means if $v_j(x_j) - 1 > 0$; and it is inactive for $v_j(x_j) - 1 < 0$. For $v_j(x_j) = 1$, the owner is indifferent between being active or being inactive.

There is a continuum of investors and, in accordance with the previous sections, I assume that all investors are risk-neutral. The financial market can consequently be characterized by the demand and supply of generic claims to expected cash flows at $t = 1$. The types of these claims and their cash flow distributions do not matter. This demand and supply, measured by the value of the claims at $t = 0$, shall be denoted by $\mathcal{I}^d$ and $\mathcal{I}^s$. Concerning $\mathcal{I}^d$, let us simply assume that the continuum of investors has an aggregate demand for financial claims which is continuous and monotonically increasing in r: $\mathcal{I}^d = \mathcal{I}^d(r)$ with $\frac{d}{dr}\mathcal{I}^d(r) > 0$ and $\mathcal{I}^d(0) = 0$. These characteristics can be derived from saving-consumption-decisions of households, but the additional structure would not provide any further insights.

For each specification of the model discussed in the previous sections, the firm value v depends on the risk-free interest rate r through the pricing factor $\frac{1}{r}$, but x_j is independent of r (as shown in the proof of Lemma 2). The firm value will thus be denoted as $v_j(x_j; r)$ in this section. Staying with the normalization $I = 1$ of the funding that each firm $j \in J$ requires at $t = 0$, the aggregate supply $\mathcal{I}^s(r)$ of securities by the firms at $t = 0$ is[15]:

$$\mathcal{I}^s(r) = \int_J \mathbf{1}_{\left\{v_j(x_j;r)\geq 1\right\}} \, dj \, . \tag{17}$$

Lemma 2

$\mathcal{I}^s(r)$ is continuous[16] and monotonically decreasing in r with $\lim_{r\to\infty} \mathcal{I}^s(r) = 0$.

As mentioned, $v_j(x_j; r)$ depends on r only through the discount factor $1/r$, which is

[15]To be more precise, the supply function $\mathcal{I}^s(r)$ can be multi-valued, since the firm owners are indifferent about being active or inactive for $v_j(x_j; r) = r$. Consequently, $\mathcal{I}^s(r)$ maps to all values in the interval between $\int_J \mathbf{1}_{\left\{v_j(x_j;r)>1\right\}} dj$ and $\int_J \mathbf{1}_{\left\{v_j(x_j;r)\geq 1\right\}} dj$.

[16]As mentioned in Footnote 15, $\mathcal{I}^s(r)$ might be multi-valued at some r. It is yet continuous at these points in the sense of multi-valued functions, which means it is upper-hemicontinuous as well as lower-hemicontinuous.

continuous and monotonically decreasing in r. With a continuum of firms, these properties of $v_j(x_j;r)$ also apply to $\mathcal{I}^s(r)$.

Lemma 3

There is a unique interest rate r^* for which the financial market clears with $\mathcal{I}^d(r^*)=\mathcal{I}^s(r^*)$.

The existence of a unique equilibrium follows directly from the continuity and monotonicity of supply and demand. Having established this benchmark case, the next subsection will study the effect of integrated funds on an aggregate level.

5.2 Equilibrium with Integrated Funds

The equilibrium of the benchmark case shall serve as reference point in this section. For that purpose, all parameters of the benchmark equilibrium will be denoted by a subscript 0.

While the next section will address the practical problem of creating financial assets with beneficial cash flow distributions, let us impose a simplifying assumption here:

Assumption 2

There is a continuum $\mathcal{D} = [0,1]$ of profit-maximizing, risk-neutral dealers with complete information at $t = 0$, who purchase debt and equity from the firms and sell derivatives (whose payoffs are conditional on the cash flows of the firms in the market) to firms and investors in perfect competition, while they have no own wealth at $t=0$.

I assume that the cost of writing a simple derivative contract are negligibly small. The structure of the interdependent decision problems is as follows. For given r, the dealers, who anticipate the decision problems of the firms, demand equity and debt from the firms and offer financial assets to them. The firms solve their decision problems as described in the previous sections, including the possibility to integrate a fund by buying assets from the dealers. Given perfect competition, the dealers earn no profits and the prices of the financial assets equal their discounted expected cash flows.
The demand for financial assets by the firms depends on the capital structure theory that describes v_j. If agency costs determine the optimal capital structure and the firm chooses the payment scheme that has been discussed in 4.2, then a firm is indifferent about the integration of a fund. If one of the other two trade-off theories discussed above applies, then an unconstrained firm will demand a combination of financial assets that add up to a complete hedge of its productive assets. Let us focus on this case for the remainder of this section. In order to simplify the discussion, let us impose:

Assumption 3

The cash flow R_j of the productive assets of each firm $j \in J$ has a strictly positive and finite lower bound $\underline{R}_j$ as well as a positive and finite upper bound $\overline{R}_j$.

Other assumptions would be equally useful, since the purpose of the assumption is mainly to ensure that there is a strictly positive minimal cash flow in each possible state. If this holds, it is feasible that all firms in the economy integrate the optimal set of financial assets (which amounts to a complete hedge), as we will see in the following.

There are infinitely many ways how the competitive dealers buy claims from firms and offer securities to them and to the external investors, which all add up to an optimal set of financial contracts. An optimal set of financial contracts means that it reduces the costs from frictions within the firms to zero, such that no additional financial asset can improve the net firm value any further. For simplicity, I will illustrate such optimal sets of contracts by a particular example with two large dealers, denoted as $\mathcal{D}_1$ and $\mathcal{D}_2$, which represent subsets of the competitive dealers.

Consider the case that the dealer $\mathcal{D}_1$ buys the share $\frac{R_j}{\overline{R}_j}$ of the debt issued by all firms $j \in J^+ := (\frac{1}{2}, 1] \subset J$. This investment yields a nonvanishing cash flow in each possible state, which allows to engage in the following operations. Each firm $j \in J^- := [0, \frac{1}{2}] \subset J$ optimally chooses $D = \overline{R}_j$ and demands a set of financial assets that yields $\overline{R}_j - R_j$ in each state. By this choice the firm can avoid any costs due to taxes/rent extraction or due to bankruptcy costs/losses from liquidations. Since each single firm $j \in J^-$ is infinitesimally small relative to the aggregate cash flow that $\mathcal{D}_1$ receives from its share of the debt of firms in J^+, it is feasible that $\mathcal{D}_1$ offers the hedge demanded by a single firm $j \in J^-$. If $\mathcal{D}_1$ does not only offer the hedge to this firm, but if it also buys the share $1 - \frac{R_j}{\overline{R}_j}$ of the debt of this firm, then this two-sided deal with the firm j does not decrease the cash flow that the dealer can sell to investors. Basically, the cash flow from the dealer 'flows throw' the firm and reduces the frictions therein, before the dealer 'collects' it again, in addition to a share of the cash flow from the productive assets of that firm.

Since there is no loss of cash flow by this two-sided deal, the dealer can offer it to all firms in J^-. And these firms demand it, since it allows for a reduction of their costs (like bankruptcy costs, etc.) to zero. As a part of these two-sided deals with the firms in J^-, the dealer $\mathcal{D}_1$ buys a large part of the cash flow their productive assets. It can finance these purchases by selling claims to its aggregated cash flow to external investors. Basically, the the dealer acts like a fund that purchase debt claims from many different firms and sells hedges to them. As mentioned, I assume perfect competition between the dealers, such that $\mathcal{D}_1$ earns no profits and the purchased and sold state-contingent cash flows net out in the aggregate. The gains from the reduction of the frictions within the firms accrue to the firm owners and the external investors, as we will see below.

The example is completed by the second set of firms and the second dealer $\mathcal{D}_2$. It holds the share $\frac{R_j}{\overline{R}_j}$ of the debt of firms $j \in J^-$, which provides a nonvanishing cash flow in each possible state. This allows to engage in the same two-sided deals with firms that have been described above (i.e., selling a hedge plus purchasing the share $1 - \frac{R_j}{\overline{R}_j}$ of debt), but $\mathcal{D}_2$ trades with the firms in J^+. As a result, all firms in J are completely hedged, choose maximal debt financing, and are able to avoid all costs that are due to the frictions described in the previous sections.

Let us now study the aggregate supply and demand of financial assets that results from these optimal choices of firms and dealers. Owing to the risk-neutrality of all agents and the perfect competition of the dealers, it is sufficient to describe the demand and supply of generic claims to expected cash flows at $t = 1$, independent of their cash flow pattern. The aggregate demand and supply, measured in terms of the value of the claims at $t = 0$, shall be denoted as $\mathcal{I}^d$ and $\mathcal{I}^s$, again. The supply of claims by an active firm $j \in J$, which chooses to integrate a fund with size s_j, equals $(1 + s_j)$. The aggregate supply of financial assets by the dealers shall be denoted as $\mathcal{I}^s_D(r)$. The overall supply of financial assets at $t = 0$ is thus

$$\mathcal{I}^s(r) = \int_J (1 + s^*_j)\, \mathbf{1}_{\left\{v_j(x^S_j\,;\,r)\geq 1\right\}}\, dj \; + \mathcal{I}^s_D(r)\,, \tag{18}$$

where x^S_j denotes the optimally chosen vector of variables in the firm problem that allows for an unconstrained choice of the integrated funds. Besides the above mentioned components, the vector also contains[17] s^*_j and $\hat{f}^*_j$. The demand for financial claims by the external investors is the same as in the benchmark case, and shall be denoted as $\mathcal{I}^d_{inv}(r)$ here. In addition, there is the aggregate demand of the dealers, which shall be denoted $\mathcal{I}^d_D(r)$. And each active firm $j \in J$ demands the amount s_j of financial assets. The total demand is therefore

$$\mathcal{I}^d(r) = \mathcal{I}^d_{inv}(r) + \int_J s^*_j\, \mathbf{1}_{\left\{v_j(x^S_j\,;\,r)\geq 1\right\}}\, dj \; + \mathcal{I}^d_D(r). \tag{19}$$

It is useful to distinguish between the gross supply and demand stated in the Eqs. (18) and (19) and the net supply and demand, $\mathcal{I}^{s,n}$ and $\mathcal{I}^{d,n}$, in which the claims held between firms and dealers are netted out. The net supply represents the volume of the firm investments in their productive assets, and the net demand represents the volume of financial claims held by external investors. Since the dealers are unable to earn profits in perfect competition, the value of the financial claims that they offer equals the value of the securities that they hold: $\mathcal{I}^d_D(r) = \mathcal{I}^s_D(r)$. Furthermore, the value of the financial assets demanded by the firms is equal to the funding they need to buy them ($s_j = s_j$). Consequently, the net demand and supply of claims are given as

$$\mathcal{I}^{s,n}(r) := \; \mathcal{I}^s(r) - \mathcal{I}^d_D(r) - \int_J s^*_j\, \mathbf{1}_{\left\{v_j(x^S_j\,;\,r)\geq 1\right\}}\, dj \;\; = \;\; \int_J \mathbf{1}_{\left\{v_j(x^S_j\,;\,r)\geq 1\right\}}\, dj \tag{20}$$

$$\mathcal{I}^{d,n}(r) := \; \mathcal{I}^d(r) - \mathcal{I}^s_D(r) - \int_J s^*_j\, \mathbf{1}_{\left\{v_j(x^S_j\,;\,r)\geq 1\right\}}\, dj \;\; = \;\; \mathcal{I}^d_{inv}(r) \tag{21}$$

Lemma 4

The net supply $\mathcal{I}^{s,n}(r)$ is continuous and monotonically decreasing in r, and it is weakly larger than in the benchmark case without integrated funds (which is described in Eq. (17)):

$$\mathcal{I}^{s,n}(r) \geq \mathcal{I}^s_0(r) \text{ for all } r > 0\,.$$

[17] While s^*_j and $\hat{f}^*_j$ separately depend on r, the optimal choice of the combination $(s, \hat{f})$ is always given by the condition $s\, R_S = \max\{\overline{R} - R, 0\}$, which is independent of r. Consequently, the optimized firm value $v(x^S_j; r)$ depends on r only through the discounting factor $\frac{1}{r}$, not through the choice of the x^S_j.

If there is a non-zero measure of firms in J that are inactive in the benchmark equilibrium, but that are able to raise their net firm value v above 1 owing to the possibility to integrate a fund, then the supply $\mathcal{I}^s(r)$ as well as the net supply $\mathcal{I}^{s,n}(r)$ of financial claims increase relative the benchmark case. The net firm value is unaffected by integrated funds in case of the trade-off between agency costs, as it is described in Section 4. However, if one of the trade-offs described in the Sections 2 and 3 applies, then the net firm value increases for firms that have a strictly positive bankruptcy probability in the benchmark case. By buying the appropriate assets provided by the dealers, these firms can reduce the expected bankruptcy/liquidation costs and can raise their value. In contrast, it is impossible that a firm loses value due to the possibility to integrate a fund.

Proposition 4
There is a unique market-clearing interest rate r^* with $\mathcal{I}^d(r^*)=\mathcal{I}^s(r^*) \wedge \mathcal{I}^{d,n}(r^*)=\mathcal{I}^{s,n}(r^*)$. This rate (which is the expected cash flow to investors at $t = 1$ per unit of investment at $t = 0$) as well as the aggregate volume $\mathcal{I}^{s,n}(r^*)$ of firm investments in their productive assets are weakly larger than in the benchmark equilibrium: $r^* \geq r_0$ and $\mathcal{I}^{s,n}(r^*) \geq \mathcal{I}_0^s(r_0)$.

While integrated funds weakly increase the net supply of expected cash flows, the net demand by external investors is the same as in the benchmark case. As a consequence, the equilibrium interest rate as well as the net supply in equilibrium weakly increase relative to the benchmark case. The net supply of claims is equivalent to the aggregate volume of investments in productive assets and the rate r^* is equivalent to the expected cash flow to investors at $t = 1$ per unit of investment at $t = 0$.
Although integrated funds weakly increase the value of all firms for all $r > 0$, these increases might be differently strong. This implies that the impact of integrated funds on the wealth of the initial firm owners can be ambiguous due to equilibrium effects. If there are firms that are inactive in the benchmark case, but whose value rises above 1 owing to integrated funds, then the initial owners of these firms benefit, as their wealth increases and becomes larger than zero. The resulting increase of active firms, however, raises the investment volume and the demand for funding, which implies an increase of the interest rate. While the investors benefit from this increase of expected cash flows, it also raises the funding costs of those firms that are already active in the benchmark equilibrium. If some of these firms have comparably small benefits from an integration of a fund, such that these gains are smaller than the increase of the interest rate, then the wealth of the initial owners of these firms decreases relative to the benchmark case. These relative changes, however, are only due to the increased efficiency of other firms that compete for the same financing and that benefit from integrated funds relatively strongly. In any case, the aggregate volume of firm production increases. It is given by the aggregate volume of expected cash flows from the productive assets of active firms at $t = 1$, and the expected cash flow per active firm is $r \cdot v_j$.

Corollary 5

The overall cash flow to firm owners and external investors weakly increases owing to integrated funds:

$$\int_J r^* \cdot v_j(x_j^S; r^*)\,\mathbf{1}_{\left\{v_j(x_j^S\,;\,r^*)\geq 1\right\}}\,dj \geq \int_J r_0 \cdot v_j(x_j; r_0)\,\mathbf{1}_{\left\{v_j(x_j\,;\,r_0)\,dj\right\}}\,dj\,.$$

As illustrated in the previous sections, the optimized firm value $v_j(x_j^S; r)$ is proportional to $\frac{1}{r}$ and the expected cash flow $r \cdot v_j(x_j^S; r)$ is thus independent of r for each firm. The number of active firms, however, increases due to integrated funds, because $\mathcal{I}^{s,n}(r^*) \geq \mathcal{I}_0^s(r_0) \Leftrightarrow \int_J \mathbf{1}_{\left\{v_j(x_j^S\,;\,r^*)\geq 1\right\}}\,dj \geq \int_J \mathbf{1}_{\left\{v_j(x_j\,;\,r_0)\,dj\right\}}\,dj$, as shown in Proposition 4. The composition of the set of active firms can change, as some firms might drop out due to an increased r, but they are replaced by firms with a larger optimized firm value $v_j(x_j^S; r)$, which means firms with more efficient productive assets and larger expected cash flows.
To sum up, this section has shown that there is an equilibrium of firms that integrate funds in order to reduce their bankruptcy risk and to increase their firm value. The optimal choices and the corresponding interconnected sets of financial contracts are feasible despite a fixed and finite set of underlying productive assets. In the aggregate, the use of integrated funds by firms (weakly) increases the efficiency of the economy and the aggregate volume of investments in productive assets. The following sections will now conclude the paper with a discussion of, first, potential obstacles to the implementation of integrated funds, and second, the implications of the results for the regulation of banks.

6 Discussion

The analysis of the previous sections suggests that firms, whose choice of capital structure is determined by the trade-offs discussed above, should use integrated funds in order to reduce their bankruptcy probability to zero. This obviously contrast with the empirical evidence. There might be three reasons: first, the available financial assets do not have an appropriate cash flow distribution; second, the frictions described by the trade-off theories have little relevance for the choice of capital structure; third, there are other frictions that limit the use of integrated funds.
In Section 6.1, I give a rather simple example how a financial asset with beneficial cash flow distribution can be created for any firm. This possibility casts doubt on the first explanation. The second explanation contradicts the evidence that shows that frictions described by the trade-off theories have some impact on the choice of financing, see e.g. Heider and Ljungqvist (2015). Therefore, I believe that the lack of integrated funds is due to an additional friction, which complements the trade-off theories. In Section 6.3, I will highlight such an additional friction, which is similar to the ones that cause the debt overhang problem (Myers (1977)) and the 'leverage ratchet effect' (Admati et al. (2016)). This means that it occurs in a dynamic setting, when the capital structure shall be changed in presence of outstanding debt.

6.1 Financial Assets with Beneficial Cash Flow Distributions

The previous sections have identified sufficient conditions for the existence of efficiency gains owing to the integration of funds. Apart from the separation of the manager payment from these funds, there are two conditions concerning the joint distribution of the cash flows from the productive firm assets and the purchased financial assets. They are given in Eq. (5) and Eq. (9) for the respective trade-off theories. A key question is whether the financial markets can provide securities that satisfy these conditions?

Appropriate securities have to yield comparably high cash flows in states in which the firm without fund would become bankrupt. One could test for each firm whether there are outstanding assets in the financial markets that have an appropriate cash flow distribution given the cash flow distribution of that firm. This would be an extensive exercise, which would yet not provide a conclusive answer to the question, because additional financial assets can simply be created by writing contracts. I will therefore provide an alternative answer to the question by giving a simple example how financial assets with beneficial cash flow distributions can be created.

As shown in Sections 2 and 3, the financial claim that reduces the probability of a bankruptcy or a run most efficiently without any increase of tax payments or possibilities for rent extraction is an asset that yields the state-contingent cash flow $\max\{D - R, 0\}$, with D being the face value of firm debt and R being the cash flow from the productive firm assets. This financial claim is effectively a capital insurance. If the firm can buy financial assets of this type, it has an incentive to choose D as high as possible in order to minimize the taxes or rent extraction. In the remainder of this section, I will focus on the case of a capital insurance, but the arguments remain valid in the limit of a vanishing equity level, which means in the limit of maximizing D.

From a practical point of view, it might be difficult to predict the cash flow R for all relevant states of the world. If this is the case, the insurance contract cannot be conditioned on exogenously given events, but has to be conditioned on the lack of cash flow itself, which means the difference $D - R$. Such an insurance contract, however, leads to moral hazard, if the cash flow R can be altered by the firm.

This problem can be overcome if the insurance is offered by those who own the firm. In the following, I will describe this possibility and indicate how it prevents an exploitation of the insurance despite possibilities for risk-shifting in the firm (similar to the problem in Section 4). The moral hazard problem due to rent extracting managers, as described in Section 3, is a bit more complicated and will be dicussed in the subsequent section.

The way how a capital insurance can be provided to a firm without a possibility for risk-shifting is depicted in Fig. 3. The entire equity of the firm is held by its owners through a fund, which also has other securities in its portfolio. And this fund sells a capital insurance to the firm whose equity it holds. If the equity holders of the firm decided to engage in risk-shifting, they would not shift risk to the debt holders, but to the insurance providers

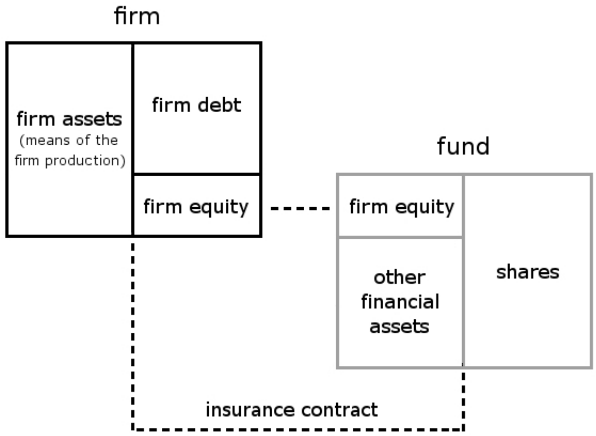

Figure 3: Possibility to provide a capital insurance to a firm without creating moral hazard.

- which means that they would shift the risk to themselves.

One might wonder how the managers of the firm behave in presence of such an insurance. As shown in Section 4, an integrated fund do not increase the risk-shifting by the managers, as long as their payment can be separated from the fund and can be conditioned on the performance of the actual firm production. This assumption seems to be particularly convincing in the case that the integrated fund only consists of an insurance, whose cash flow should easily be distinguishable from the actual revenue of the firm.

There is another possibility of risk shifting, however, in which equity holders can engage: they might reduce the size of the fund or might change the composition of its portfolio, such that the insurance loses the ability to provide sufficient cash flows when an insured event occurs. The next subsection will discuss this problem and its contribution to the empirical lack of integrated funds in more detail.

As described in the previous sections, the capital insurance leads to a reduction of costs within the firm. And on the level of the fund, the provision of the insurance does not cause any losses. Since (passive) funds are just a set of financial contracts that transmit cash flows (in contrast to firms, which create cash flows), it is common practice that they are not subject to corporate taxation. And being financed by selling shares, there are no additional bankruptcy costs related to the fund.

One might wonder if the absorption of the entire firm equity into a fund, in which its cash flow is 'mixed' with the cash flows from other securities, would distort the preferences of investors. This should not be a concern, if there is a sufficiently large set of financial contracts, such that the investors can decompose their joint, mixed cash flow into the components they value the most. They are in fact weakly better off than in the benchmark case without integrated funds, because these funds weakly increase the available cash flow in each possible state, which means that the size of each component is weakly larger than

in the benchmark case. Appendix A discusses this issue in some more detail.
It is interesting that the firm-fund structure, which I derive as a way to obtain efficiency gains, is effectively the same as the 'liability holding companies' (LHCs) that Admati et al. (2012) suggest in the context of bank regulation. They propose LHCs with the aim to counteract negative incentives due to implicit bailout guarantees. Some opponents of capital regulation argue that the choice of capital structure by banks would not be driven by such guarantees, but mainly by the trade-offs discussed above. The result of this paper is: if this is true, banks should actually welcome the establishment of LHCs, as they allow for private efficiency gains.

6.2 The Moral Hazard Problem of an Insurance Contract in Presence of Rent Extraction

Let us assume that the story about rent extracting managers and a disciplining effect of demandable debt has some empirical relevance. If the capital insurance of a firm cannot be conditioned on exogenously given states, but has to be conditioned on the lack of cash flow itself, there is the danger of an unrestricted rent extraction by the managers. If the cash flow of the firm production is smaller than the face value D of the deposits, neither the equity holders of the firm (who have nothing to lose) nor the depositors (who are protected by the capital insurance) have an incentive to monitor the managers. The managers can thus increase the rent extraction in these states without any constraint.
There are (at least) two different solutions, depending on whether the depositors and the managers can collude. If they cannot, a small modification of the capital insurance can solve the problem (if one follows the logic of Diamond & Rajan). Consider an insurance that does not only yield $D - R^n$, but $D - R^n + g(D - R^n)$ in every state with $R^n < D$, where R^n is the 'net cash flow' of the firm, which means the cash flow R from its productive assets minus the rent extraction by managers. And $\frac{d}{dx}g(x) < 0$ with $g(x) \geq 0$ for all $x \in [0, D]$. Furthermore, the additional cash flow $g(D-R^n)$ in case of an insured event, which increases in R^n, shall accrue to the depositors who do not run. Running depositors simply receive their share in D. Given this kind of insurance contract, the debt holders maintain an incentive for monitoring. They can threaten the managers with a run, if their premium $g(D-R^n)$ decreases too strongly due to the rent extraction. Since running depositors simply receive D, the value of keeping the managers in states with $R < D$ (when the insurance becomes effective) is $g(D-R)$. Bargaining over this continuation value, the depositors are in the same position that the theory of Diamond & Rajan assigns to equity holders. Assuming that they also behave in the same way, they allow the managers to get a share b_e of this continuation value. By choosing a function g with values slightly above zero, one can minimize the extraction of rents from the capital insurance. Moreover, with $b_e\, g(x)$ close to zero for all $x > 0$, the managers have no incentive to trigger an insured event (by trying to extract so much that R^n falls below D) in states with $R > D$, where

they can extract the rent $b_e(R-D)$. As result, the capital insurance leads to an expected loss $\int_0^D b_e\, g(D-R)\, f(R)\, dR$, but (for a sufficiently small g) this loss is smaller than the gains from preventing liquidations, which are $\int_0^D l\, Rf(R)\, dR$.

If the depositors and the managers can collude, however, this modified insurance contract cannot suppress the moral hazard, because the overall gains for managers plus depositors from exploiting the insurance (by extracting X) are larger than the costs: $\left|\frac{d}{dX}X\right| > \left|\frac{d}{dX}g(D-R-X)\right|$ for g close to zero. In that case, the modification g of its insurance is useless and the fund can simply provide a normal capital insurance (that yields $\max\{D-R, 0\}$). If the fund also holds the firm equity, however, it still has a disciplining device owing to the power to replace the managers. As in states with $R > D$, the fund (i.e., the equity holders) can bargain with the managers over the continuation value of keeping the managers in states with $D > R$. If the equity holders took over the firm, the resulting losses from inefficient liquidations would increase the insurance payments that are necessary to pay out the depositors. The value of keeping the managers is thus the avoidance of these losses, which are the same that would occur in case of runs. Since managers can only obtain a share of this value in the bargaining process, the losses from the exploitation of the capital insurance are smaller than the losses from runs that are avoided by this insurance. Consequently, the capital insurance leads to efficiency gains, even if the managers can exploit this insurance and can collude with the depositors.

6.3 Obstacles to Integrated Funds

In order to understand why firms do not use integrated funds (and reduce their bankruptcy risk to zero) despite the available efficiency gains, it is useful to consider the process of changing the capital structure. In contrast to the assumption used in the analysis of the trade-offs theories, a firm usually has outstanding debt. In that case, a problem arises that has been highlighted by Admati et al. (2016) in their description of the 'leverage ratchet effect': If the face value of this outstanding debt cannot be renegotiated, the owners of the firm will not chose a project (or a change of the capital structure) that has a positive NPV owing to its reduction of expected bankruptcy costs. The reason is the asymmetric distribution of gains and losses: the benefits accrue to the debt holders while the owners/equity holders incurs costs. These costs can be higher taxes, for instance, but equity holders lose even in absence of such frictions, as highlighted by Admati et al. (2016). A reduction of the bankruptcy risk always implies that the cash flow to holders of outstanding debt increases in some states. If the face value of their debt is not adjusted, but their debt contract is fixed, they gain at the expense of the equity holders. This problem applies to integrated funds, as their efficiency gains are solely due to a reduction of expected bankruptcy costs.

If a firm could commit to the establishment of an integrated fund at a future point in time, the pricing of debt that is rolled over or newly issued could account for the reduction of the bankruptcy risk at this future point. As a consequence, the firm owners could participate in the gains from the integrated fund and would thus have an incentive to establish it in the long run. However, once the firm owners have incurred their part of the gains in the form of adjusted debt prices, they have an incentive to reduce the integrated fund or to choose its portfolio in such a way, that risk is shifted to the debt holders. There are so many degrees of freedom related to an investment in financial assets at a future point in time (since the set of available assets as well the cash flow distribution of these assets constantly evolve), such that it might be impossible to credibly commit to the future characteristics of an integrated fund. The consequence of this inability is that debt holders cannot fully trust in the safety of their claims and thus do not accept debt prices that account for prospective integrated funds and that allow to share the gains with the firm owners.

7 Implications for the Regulation of Banks

The results of this paper have important implications for the debate about the regulation of banks. There is the widespread notion that capital requirements for banks, which are intended to improve the stability of the financial sector, entail some costs. First, they are supposed to cause private costs for banks due to a deviation from their privately optimal choice of financing; and second, they are supposed to cause social costs - either indirectly, because the private costs for banks impair their provision of credit and other services to the economy, or directly, because the requirements allegedly reduce the volume of socially beneficial 'money-like' claims.

There are plausible arguments for private costs in the short run, when capital requirements are raised quickly and equity increases transfer wealth from equity holders to the holders of outstanding debt, as described in Admati et al. (2016). And these private costs can lead to social costs, when the bank owners prefer to comply with increased capital requirements by liquidating assets or by forsaking new NPV projects. The arguments for (both, private and social) costs of capital requirements in the long run, in contrast, are usually based on the trade-off theories discussed in this paper. This paper has shown, however, that these theories actually allow for a decrease of the leverage and bankruptcy probability of banks without any social or private costs, if one takes into account that banks can invest in financial assets and can 'integrate a fund'. In fact, the integration of a fund in order to reduce bankruptcy risk can even provide private and social gains.

Such beneficial reductions of the bankruptcy risk depend on the availability of assets with an appropriate distribution of cash flows. In Section 6.1, I have pointed out how simple financial assets with an appropriate distribution can be created. This possibility is depicted in Fig. 6.1 and it is effectively the same as the liability holding companies (LHCs) suggested by Admati et al. (2012). A regulation of banks that takes LHCs into consideration could therefore reduce the bankruptcy risk of banks without any social or private costs, but rather with social and private gains.

To be precise, one type of private costs would actually accrue: the loss of the subsidies that

banks get from the government in form of implicit bailout guarantees. But as long as one does not want to subsidize banks in this way, one should not be concerned about strong increases of capital requirements[18] for banks in the long run. And high capital levels in the long run also reduce the probability of capital shortages and the related problems of adjustment in the short run, which I mentioned above.
Capital regulation based on integrated funds or LHCs faces a problem similar to the one discussed in the previous subsection: It has to ensure that the size of the funds and their compositions are such, that the cash flows from the purchased securities are large enough in those states in which the banks need them to avoid bankruptcy. As mentioned before, the banks might exploit their discretion about the fund portfolio for the purpose of risk-shifting. In this case, however, this is no obstacle to the establishment of the fund, as it can be enforced by regulation. And the problem is usually not a risk-shifting from equity to debt holders, but a risk-shifting from equity holders to the government, which takes the risk of the debt holders by either explicitly or implicitly insuring the majority of the bank debt. This is the standard problem of capital regulation, which tries to alleviate the risk-shifting by setting risk-weights for different types of financial assets and by enforcing a sufficient size of the capital buffers. But independent of its results, this 'regulatory game' is only about the amount of implicit subsidies that banks can extract; it is not about efficiency losses due to capital requirements.

Let me conclude with brief **estimates for the ability of integrated funds/LHCs to absorb losses**, for the exemplary case that they invest in relatively risky assets:
Let us look at corporate bonds and their decline during financial crises. The weighted average of default rates of all corporate bonds rated by Moody's[19] peaked at 8.424 % in 1933 and reached its second highest value 5.422 % in 2009. One can thus expect that a fund which issues equity claims in order to purchase an amount X of debt claims can provide a capital insurance worth $(1-\delta_D)X$ even in very bad states with $\delta_D = 0.1$ as conservative estimate for the discount factor. This is a conservative estimate, since positive recovery rates are ignored and corporate bonds are a relatively risky type of debt.
Let us now consider a scenario in which the loss-absorbing capital of US banks shall be increased by 5% of their assets by means of LHCs that invest in bonds. This would double the amount of loss-absorbing capital in banks, if they comply with the leverage ratio that is imposed by the current regulation, which is in the range of $3-5\%$. Given the discount factor δ_D and an aggregate volume A_{agg} of bank assets, the volume V_{abs}^D of bonds that the banks would need to absorb is $V_{abs}^D = \frac{1}{1-\delta_D} \cdot 0.05 \cdot A_{agg}$. Take the example of the US banks

[18] To be precise, an increase of capital requirements based on LHCs would not increase the required amount of equity within the balance sheet of the bank itself, but it would impose requirements for the size of the funds that insure the banks.

[19] see `http://efinance.org.cn/cn/FEben/Corporate%20Default%20and%20Recovery%20Rates,1920-2010.pdf`

in December 2012[20]: According to the FDIC[21] the aggregate volume of assets in insured US banks was $A_{agg} = \$\,14.5\,tn$. This means that the banks would need to absorb bonds worth $V^D_{abs} = \$\,0.8\,tn$ in order to double their capital buffer.

In order to get an appropriate impression of this volume, it should be compared to the volume of bonds available on the market. In case of the US market in December 2012, the volume of outstanding bonds was $\$\,36.6\,tn$ according to SIFMA[22], Using information from Hanson et al. (2015) and the FDIC[23], one can subtract the volume of bonds already held by banks. As a result, the volume of bonds that are not held by banks and that could be purchased by them is at least $V^D_{ext} = \$\,33.8\,tn$. This means that only 2.4% of the available bonds would need to be purchased by integrated funds/LHCs in order to double the capital buffers of banks that can absorb losses even in the worst states of the economy.

[20] a recent date for which all data is very easily accessible

[21] see `https://www.fdic.gov/bank/statistical/stats/2012dec/industry.pdf`

[22] see `http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/CM-US-Bond-Market-SIFMA.xls?n=13061`

[23] Hanson et al. (2015) state that 20.8 % of the assets of the banks in their sample were securities. Since debt is only a part of this set, the given estimate for the volume of bonds already held by banks is an upper bound.

A Generalized Preferences of Investors

In order to analyze the robustness of the results to generalized preferences of investors, let us study the same model(s) as before, but let us change the pricing of the debt and equity. Let Σ denote the set of all possible states at $t = 1$, in which the assets yield state-contingent cash flows $R(\sigma)$ and $R_S(\sigma)$. To simplify the discussion, let us assume that $f(x) := \int_\Sigma \mathbf{1}_{\{R(\sigma)=x\}} d\sigma$ is continuous in x. Assume furthermore that the equity and debt claims issued by the firms can be held by investors through a series of funds provided in a perfectly competitive capital market without entry or contracting costs. Consequently, these funds earn zero profits and are structured such that the diverse preferences of the investors are satisfied optimally. This implies that the prices of debt and equity claims are given by their decomposition into Arrow-Debreu securities and by the prices $p(\sigma)$ of these securities at $t = 0$, with $0 \le p(\sigma) < \infty$. See Hellwig (1981) for a more detailed discussion of such decompositions of financial claims into state-contingent securities. The assumption of perfect capital markets does not contradict the purpose of this paper, which is the analysis of optimal capital structures on the firm level. The paper critically discusses trade-off theories that deviate from the Modigliani-Miller Theorem because of frictions within firms, not because of frictions within the capital markets.

Let us now study the value of a firm given this generalized pricing of cash flows, and let us start with the **trade-off between taxes, bankruptcy risk and a premium for safe debt**. Since all steps in the derivation of the firm value remain the same, apart from the pricing kernel, the expressions in Eqs. (1) and (2) simply become

$$\begin{aligned} v_s(D_r, D_s, s) &= \lambda\, D_s + \int_\Sigma \Big(R(\sigma) + s\, R_S(\sigma) - T\big(R(\sigma) + sR_S(\sigma) - D\big) - b\, \mathbf{1}_{\{R(\sigma) + s\, R_S(\sigma) < D\}} \Big) p(\sigma)\, d\sigma\,, \\ v(D_r, D_s, s) &= v_s(D_r, D_s, s) - \int_\Sigma s\, R_S(\sigma) p(\sigma)\, d\sigma \\ &= \lambda\, D_s + \int_\Sigma \Big(R(\sigma) - T\big(R(\sigma) + sR_S(\sigma) - D\big) - b\, \mathbf{1}_{\{R(\sigma) + s\, R_S(\sigma) < D\}} \Big) p(\sigma)\, d\sigma\,. \end{aligned}$$

with $D = D_r + D_s$. The utility that investors incur from safe debt and the corresponding premium λD_s are not state-contingent, and the premium is thus accounted as separate term. If the productive assets require a fixed investment I at $t = 0$, the optimization problem of a firm owner, who wants to maximize the value of cash flows that it can sell, is again $\max_{s \in \mathbb{R}^+,\, D_s \in [0, \overline{D}_s],\, D_r \in \mathbb{R}^+} v(D_s, D_r, s)$ with $\overline{D}_s := \min\big(R + sR_S \mid \hat{f}(R, R_S) > 0\big)$. The cost of buying a portfolio S of financial assets with size s equals the value $\int_\Sigma s\, R_S(\sigma) p(\sigma)\, d\sigma$ of the state-contingent cash flows of these assets.

The integration of a fund leads to the same trade-off like in the risk-neutral case. The fund increases the cash flows that are taxed, but it reduces the risk of bankruptcy and it might increase the level of safe debt that can be issued. The result stated in Proposition 1 thus remains valid, if one accounts for the generalized pricing. This means that the condition stated in Eq. (4) becomes

$$\lim_{s\to 0}\int_\Sigma b\,\mathbf{1}_{\{R(\sigma)<D_0\}}\,\mathbf{1}_{\{R(\sigma)+s\,R_S(\sigma)\geq D_0\}}\,p(\sigma)\,d\sigma \;>\int_\Sigma R_S(\sigma)\,T'\big(R(\sigma)+s\,R_S(\sigma)-D_0\big)\,p(\sigma)\,d\sigma\,.$$

And the condition in Eq. (5) becomes

$$\lim_{s\to 0}\int_\Sigma b\,\mathbf{1}_{\{R(\sigma)<D_0\}}\mathbf{1}_{\{R(\sigma)+sR_S(\sigma)\geq D_0\}}\,p(\sigma)d\sigma+\lambda\,\min\big(R_S|R{=}\underline{R}\big)>\int_\Sigma R_S(\sigma)\,T'\big(R(\sigma)+sR_S(\sigma){-}D_0\big)p(\sigma)d\sigma$$

with $\underline{R}$ being the lower bound $\min\big(R(\sigma)|\sigma\in\Sigma\big)$ for R. This result has implications that are completely analogous to the case with risk-neutral pricing. For each benchmark set of assets and corresponding optimal capital structure with positive bankruptcy probability, there exist financial assets with a cash flow distribution that fulfills the conditions stated here. This is illustrated by the example of an asset (denoted as A_{ex}) that yields $R_S(\sigma)=\frac{1}{m(\epsilon)}(D_0-R(\sigma))$ for all $\sigma\in\Sigma$ with $R(\sigma)=[D_0-\epsilon,D_0)$ and zero in all other states, where $m(\epsilon)$ is a normalization parameter. This means that there exist an extension of the firm balance sheet that reduces the bankruptcy risk below the optimum of benchmark case, while it increases the firm value. Furthermore, if the firm can choose its portfolio, it will optimally choose to invest in a 'capital insurance', which yields $s\,R_S=\max\{D{-}R,0\}$. And as long as there is no constraint to the debt level, it will increase D to $\max\big(R(\sigma)|\sigma\in\Sigma\big)$, such that the insurance becomes a complete 'hedge' of the productive assets.

The results for the two other specifications of the model can be generalized in the same way. In case of the **trade-off between rent extraction and liquidation losses**, the problem of the firm owner in presence of state-contingent pricing is[24] $\min_{D\in\mathbb{R}^+,s\in\mathbb{R}^+}L(D,s)$, with

$$\begin{aligned}L(D,s)=&\int_\Sigma b_e\,\big(l\,R+l_S\,sR_S\big)\,\mathbf{1}_{\{D\leq R_l\}}\,p(\sigma)\,d\sigma+\int_\Sigma b_e\,\big(R+sR_S-D\big)\,\mathbf{1}_{\{R_l\leq D\leq R+sR_S\}}\,p(\sigma)\,d\sigma\\&+\int_\Sigma\big(l\,R+l_S\,sR_S\big)\,\mathbf{1}_{\{R+sR_S\leq D\}}\,p(\sigma)\,d\sigma\,,\end{aligned}$$

and all cash flows of the assets are state-contingent: $R=R(\sigma)$, $R_S=R_S(\sigma)$, $R_l=R_l(\sigma)$. Proposition 2 remains valid under generalized preferences, if condition Eq. (9) is replaced by its generalized version, which is:

$$\begin{aligned}\lim_{s\to 0}\int_\Sigma l\,R\,\mathbf{1}_{\{R<D_0\}}\,\mathbf{1}_{\{R+s\,R_S\geq D_0\}}\,p(\sigma)\,d\sigma\;\geq\;&\int b_e\,R_S\,\mathbf{1}_{\{(1-l)R\leq D\leq R\}}\,p(\sigma)\,d\sigma\\&+l_S\left(\int R_S\,\mathbf{1}_{\{R\leq D\}}\,p(\sigma)\,d\sigma+\int b_e\,R_S\,\mathbf{1}_{\{D\leq(1-l)R\}}\,p(\sigma)\,d\sigma\right).\end{aligned}$$

The condition in Eq. (8) is just a special case of this condition with $l_S=0$. Again, for each benchmark set of assets and corresponding optimal capital structure with positive

[24]For simplicity, the bargaining game (i.e., the parameter b_e) is assumed to be independent of the state-contingent preferences of the agents.

bankruptcy risk, there is a possibility to simultaneously decrease the bankruptcy risk and to increase the firm value by means of an integrated fund. This is exemplified by the financial asset A_{ex} with a sufficiently small ϵ. And again, a 'capital insurance'/'hedge' is the optimal financial asset to purchase.

Finally, in case of a **trade-off between agency costs of debt and equity** (as described in Section 4), the robustness of the results with respect to generalized preferences of the investors is straight-forward. If the firm has chosen an optimal capital structure given its productive assets and has aligned the payment scheme/the incentives of the managers with the firm production, then the integration of a fund has no effect on the behavior of the managers, independent of the pricing of the state-contingent cash flows. If the fund is integrated without an increase of the debt level, the bankruptcy risk of the firm decreases. This implies that the state-contingent cash flows of the different types of claims change (for instance, the debt holders receive higher payments in states in which the firm would have been bankrupt), but there are no losses and the overall sum of cash flows remains the same in each possible state. As long as the 'mixing' of cash flows due to integrated fund can be decomposed by appropriate financial contracts, there is no distortion of the investors' preferences due to integrated funds.

B Proofs

B.1 Proposition 1

The derivative of the net firm value $v(D_r, D_s, s)$ w.r.t. s is

$$\frac{d}{ds}v(D_r, D_s, s) = -\frac{1}{r}\int R_S\, T'(R + s\, R_S - D_r - D_s)\hat{f}(R, R_S)\, dR\, dR_S - \frac{1}{r}\, b\, \frac{d}{ds}\phi(D_r, D_s, s)$$

With $D = D_r + D_s$, the derivative of the bankruptcy probability is:

$$\begin{aligned}
\frac{d}{ds}\phi(D_r, D_s, s) &= \frac{d}{ds}\int_0^D \int_0^{\frac{1}{s}(D-R)} \hat{f}(R, R_S)\, dR\, dR_S \\
&= -\int_0^D \frac{1}{s^2}(D-R)\, \hat{f}\left(R, \frac{1}{s}(D-R)\right) dR = \int_{\frac{1}{s}D}^0 R'\hat{f}(D - sR', R')dR' \\
\lim_{s\to 0}\frac{d}{ds}\phi(D_r, D_s, s) &= -\int_0^\infty R'\hat{f}(D, R')dR' = -f(D)\, E_{\hat{f}}\left[R_S | R = D\right]
\end{aligned}$$

Plugging the derivative of ϕ into the derivative of v and evaluating it at $(D_s = D_{s,0}, D_r = D_{r,0}, s = 0)$, one finds that $\frac{d}{ds}v(D_{s,0}, D_{r,0}, s)\big|_{s=0} \geq 0$, if

$$b\ E_{\hat{f}}\left[R_S | R = D_0\right]\ f(D_0) - \int R_S\, T'(R + s\, R_S - D_0)\, \hat{f}(R, R_S)\, dR\, dR_S \geq 0\,.$$

While the bankruptcy probability does not depend on the composition of D, safe debt earns a premium λ. Consequently, the firm will always choose the highest possible value for D_s, which is the lowest possible realization of $R + s\, R_S$. The derivative of this value w.r.t. s evaluated at $s = 0$ is $\min\left(R_S | R = \underline{R}\right)$. Accounting for this increase in the level of safe debt and the related premium, one has $\frac{d}{ds}v(D_s^*(s, D_0), D_r^*(s, D_0), s)\big|_{s=0} \geq 0$, if

$$b\ E_{\hat{f}}\left[R_S | R = D_0\right]\ f(D_0) + \lambda\ \min\left(R_S | R = \underline{R}\right) - \int R_S\, T'(R + s\, R_S - D_0)\, \hat{f}(R, R_S)\, dR\, dR_S \geq 0\,.$$

B.2 Proposition 2

Computing the derivative $\frac{d}{ds}v(D, s) = -\frac{1}{r}\frac{d}{ds}L(D, s)$ yields:

$$\begin{aligned}
\frac{d}{ds}L(D, s) = &-\int_0^D \frac{D-R}{s^2}\left(l\, R + l_S\, (D-R)\right)\, \hat{f}\left(R, \frac{D-R}{s}\right)\ dR \\
&+ \int_0^D \int_0^{\frac{1}{s}(D-R)} l_S\, R_S\ \hat{f}(R, R_S)\ dR_S dR\ + \int_0^{\frac{D}{1-l}} \int_{\frac{1}{s}(D-R)}^{\frac{D-(1-l)R}{s(1-l_S)}} b_e\, R_S\ \hat{f}(R, R_S)\ dR_S dR \\
&+ \int_0^\infty \int_{\frac{D-(1-l)R}{s(1-l_S)}}^\infty b_e\, l_S\, R_S\ \hat{f}(R, R_S)\ dR_S dR
\end{aligned}$$

Terms that cancel out are not displayed. Applying the same substitution of the integration variable as in the proof of Proposition 1, one can write the derivative $\frac{d}{ds}L$ for $\lim_{s\to 0}$ as

$$
\begin{aligned}
& -\int_0^\infty R'\, l\, D\, \hat{f}(D,R')\, dR' + \int_0^D \int_0^\infty l_S\, R_S\ \hat{f}(R,R_S)\ dR_S dR \\
& \quad + \int_D^{\frac{D}{1-l}} \int_0^\infty b_e\, R_S\ \hat{f}(R,R_S)\ dR_S dR + \int_{\frac{D}{1-l}}^\infty \int_0^\infty b_e\, l_S\, R_S\ \hat{f}(R,R_S)\ dR_S dR \\
& = -l\, D\, E_{\hat{f}}\left[R_S|R{=}D\right]\cdot f(D)\ + \int_0^D l_S\, E_{\hat{f}}\left[R_S|R{=}R'\right]\cdot f(R')\, dR' \\
& \quad + \int_D^{\frac{D}{1-l}} b_e\, E_{\hat{f}}\left[R_S|R{=}R'\right]\cdot f(R')\, dR' + \int_{\frac{D}{1-l}}^\infty b_e\, l_S\, E_{\hat{f}}\left[R_S|R{=}R'\right]\cdot f(R')\, dR'
\end{aligned}
$$

The net firm value $v(D,s)$ increases with s at $s=0$, if this expression is negative. The second and fourth term are zero for $l_S = 0$. The statement in Proposition 2 is given by comparing the negative first term with the remaining positive terms for both cases, $l_S = 0$ and $l_S > 0$.

B.3 Lemma 1

Starting with the manager problem given in Eq. (11), the choice set $[0,\bar{c}_m]\times[0,1]$ is compact and the objective function is bounded from below by $-h(\bar{c}_m)$ and from above by

$$
E_f\left[\rho(\bar{c}_m)R + \mathbf{1}_{\beta^+}\alpha\beta^+\big(\rho(\bar{c}_m)R\big)\right] < E_f\left[\rho(\bar{c}_m)R + (1-\mathbf{1}_{\beta^+})\alpha\beta^-\big(\rho(\bar{c}_m)R\big)\right] < 2\rho(\bar{c}_m)E_f\left[R\right] < \infty\,.
$$

Consequently, for each m and D, the manager problem always has a finite solution (c_m^*, α^*). It is possible that several choices are equally optimal for the managers. Let us simply assume that managers choose each of these absolute maxima with equal probability in such cases. Concerning the firm problem, all terms in the objective function $v(D,m)$ are bounded from above and below for all $r > 0$, since this holds for $X(R;D,m)$ as implicitly shown above. And because the choice variables (D,m) are defined on the compact set $[0,\overline{R}]\times[0,1]$, the optimization problem is an optimization of a bounded expression over a compact set and thus has a solution.

B.4 Proposition 3

As already noted, the proposition follows directly from the fact that v is effectively independent of s, because $X(R;D,m,s)$ is effectively independent of s.

B.5 Lemma 2

The three specifications of v in the Sections 2, 3, and 4 are as follows: (in absence of an integrated fund, which means for $s=0$, the expressions for the firm value simplify)

1. The trade-off between taxes, bankruptcy costs and a premium for safe debt:

$$v(D_s, D_r; r) = \frac{1}{r}\left(\int R\, f(R)\, dR - \int T\big(R - D_r - D_s\big)\, f(R)\, dR - b \int \mathbf{1}_{\{R<D_s+D_r\}}\, f(R)\, dR + \lambda\, D_s\right)$$

2. The trade-off between rent extraction and liquidation losses:

$$\begin{aligned} v(D; r) = \; & \frac{1}{r}\Bigg(\int R\, f(R)\, dR - \int b_e\, l\, R\; \mathbf{1}_{\{D\le R_l\}}\, f(R) dR - \int b_e\, \big(R - D\big)\, \mathbf{1}_{\{R_l\le D\le R\}}\, f(R)\; dR \\ & - \int l\, R\; \mathbf{1}_{\{R\le D\}}\, f(R)\, dR\Bigg) \end{aligned}$$

3. The trade-off between agency costs of debt and equity:

$$\begin{aligned} & v(D, m; r) = \frac{1}{r}\, E_f\left[X(R; D, m)\right] - m\, \frac{1}{r}\, E_f\left[\max\left\{0, X(R; D, m) - D\right\}\right] \quad \text{with} \\ & X(R; D, m) := \rho\big(c_m^*(D, m)\big)R + \alpha^*(D, m)\cdot\big[\mathbf{1}_{\beta^+}\beta^+ - (1 - \mathbf{1}_{\beta^+})\beta^-\big] \quad \text{where } c_m^* \text{ and } \alpha^* \text{ solve} \\ & \max_{c_m\in[0,\bar{c}_m], \alpha\in[0,1]}\Big(m\, E_f\left[\max\big\{0\, ,\, \rho(c_m)R + \mathbf{1}_{\beta^+}\alpha\beta^+ - (1-\mathbf{1}_{\beta^+})\alpha\beta^- - D\big\}\right] - h(c_m)\Big) \end{aligned}$$

In all three cases, the firm value depends on r only through the factor $\frac{1}{r}$, which is continuous and monotonically decreasing in r for $r > 0$. Consequently, all specifications of v are continuous and monotonically decreasing in r. This includes $v(D, m; r)$, because $E_f\left[\max\left\{0, X(R; D, m) - D\right\}\right] \le E_f\left[X(R; D, m)\right]$ and $m \le 1$, and one of these inequalities has to be strict, since $m = 1$ and $D = 0$ would imply that the entire cash flow goes to the managers, which is never an optimal solution.
Since v is continuous and monotonically decreasing in r for each possible vector of choice variables, this also holds at the respective optimal vector x. With $\mathcal{I}^s(r)$ mapping to all values in the interval between $\int_J \mathbf{1}_{\left\{v_j(x_j;r)>1\right\}}\, dj$ and $\int_J \mathbf{1}_{\left\{v_j(x_j;r)\ge 1\right\}}\, dj$, the supply function $\mathcal{I}^s(r)$ is also monotonically decreasing and continuous in the sense of multi-valued functions.
Both, $v(D_s, D_r; r)$ and $v(D; r)$ are bounded from above by $\frac{1}{r}\int R\, f(R)\, dR$, and Lemma 1 shows that $E_{\hat{f}}\left[X\right]$ is bounded from above by a value that is independent of r. Consequently, $\lim_{r\to\infty} v(x; r) = 0$ for each firm, which implies $\lim_{r\to\infty}\mathcal{I}(r) = 0$.

B.6 Lemma 3

As shown in the proof of Lemma 2, the supply function $\mathcal{I}^s(r)$ is continuous and monotonically decreasing with $\lim_{r\to\infty}\mathcal{I}^s(r) = 0$. Having a supply function $\mathcal{I}^d(r)$ that is continuous and strictly monotonically increasing with $\mathcal{I}^d(0) = 0$, there is exactly one r for which $\mathcal{I}^d(r) = \mathcal{I}^s(r)$.

B.7 Lemma 4

The proof is analogous to the one of Lemma 2. In all three specifications of the model, the firm value $v(x_j^S\,;\,r)$ depends on r only through the continuous and monotonically decreasing discount factor $\frac{1}{r}$, as can simply be seen in the respective descriptions of the firm problem in the Eqs. (6), (10), and (16) (with the latter making references to the Eqs. (15) and (14)). Consequently, the supply function $\mathcal{I}^{s,n}(r) = \int_J \mathbf{1}_{\left\{v_j(x_j^S\,;\,r)\geq 1\right\}}\,dj$ is also monotonically decreasing and continuous in the sense of multi-valued functions.
Since $v_j(x_j^S; r) \geq v_j(x_j; r)$ for all $j \in J$ and all $r > 0$, it holds that $\mathcal{I}^{s,n}(r) = \int_J \mathbf{1}_{\left\{v_j(x_j^S\,;\,r)\geq 1\right\}}\,dj \geq \int_J \mathbf{1}_{\left\{v_j(x_j\,;\,r)\geq 1\right\}}\,dj = \mathcal{I}_0^s(r)$ for all $r > 0$.

B.8 Proposition 4

The proof of the unique equilibrium rate r^* is identical to the proof in Lemma 3 with $\mathcal{I}^{s,n}$ and $\mathcal{I}^{d,n}$ in addition to $\mathcal{I}^s$ and $\mathcal{I}^d$. The market clearing of the net demand and supply is equivalent to the market clearing of the gross demand and supply, since the financial contracts held between the agents net out. The fact, that the equilibrium rate and the equilibrium volume are weakly larger than in the benchmark case, follows directly from $\mathcal{I}^{d,n}(r) = \mathcal{I}_0^d(r)$ and $\mathcal{I}^{s,n}(r) \geq \mathcal{I}_0^s(r)$, which has been shown in Lemma 4.

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