The Time-Varying Risk of Macroeconomic Disasters*

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Abstract

The rare disasters model of asset prices suggests stock market variations reflect persistent fluctuations in the probability of a large decline in consumption. This paper estimates this probability from macroeconomic data alone, using a dataset of 42 countries over more than a century. We find that disaster risk is volatile and persistent, strongly correlates with the dividend-price ratio, and forecasts stock returns. Our evidence suggests that disaster risk can rationalize the equity premium and risk-free rate puzzles, the excess volatility puzzle, and the predictability of aggregate stock market returns by the dividend-price ratio. A variable disaster model calibrated with our risk estimates confirms these results under standard assumptions. While former works support the plausibility of disaster risk hypothesis, we provide direct evidence that disaster risk can rationalize price fluctuations.

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Keywords: rare disasters, equity premium, return predictability, state-space model

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Many puzzles in finance arise from the inability of models to reconcile, quantitatively, asset returns and macroeconomic risks. A classic example is the equity premium puzzle: average stock returns are far too high to be explained by the observed risk in consumption (Mehra and Prescott, 1985). Another well-known example is that stock returns can be predicted by valuation ratios such as the dividend-price ratio (Campbell and Shiller 1988b). Likewise, stock market volatility is too high to reflect forecasts of future dividends (Shiller, 1981).

These puzzles might be resolved if one assumes that financial markets compensate for the risk of infrequent but severe macroeconomic disasters. Barro (2006) uses international data from the last 100 years to document several episodes of large declines in consumption growth. These macroeconomic disasters occur with an annual frequency of about 3% and are strong enough to generate a sizable equity premium, as hypothesized by Rietz (1988). A first generation of models assumes that disasters arise with a constant probability (see e.g. Barro 2009; Barro and Jin 2011; Nakamura et al. 2013). The next generation of papers shows that allowing the risk of disaster to vary over time generates excess volatility and predictability (Gabaix, 2012; Gourio, 2012; Wachter, 2013) and also solves puzzles related to the markets for bonds, options, and currencies (Gabaix, 2012; Gourio, 2013; Farhi and Gabaix, 2016; Hasler and Marfe, 2016).[1] Variable disasters also offer the possibility of connecting macroeconomic aggregates with asset prices in production economies (Gabaix, 2011; Gourio, 2012; Kilic and Wachter, 2015; Isoré and Szczerbowicz, 2016).

This paper estimates the time-varying probability of a macroeconomic disaster. Our approach follows closely the seminal work of Barro (2006), who finds that the average number of disasters across countries and across time in the past century—in other words, the unconditional probability of a macroeconomic disaster—is high enough to rationalize the unconditional equity premium. We go one step further by estimating an econometric model under which the risk of a disaster varies over time. We document that the

[1] Another strand of research maintains a constant probability of disasters but assumes that the representative agent learns about the model parameters or states (Weitzman 2007, Koulovatianos and Wieland 2011, Du and Elkamhi 2012, Lu and Siemer 2016, Johannes et al. 2016). There is also a burgeoning literature on the implication of disagreements about the likelihood of rare disasters (see e.g. Chen et al. 2012; Piatti 2015).

conditional probability of a disaster varies over time and is strongly correlated with the conditional equity premium. This step is critical for assessing how well disaster risk can rationalize why asset prices are volatile and why valuation ratios forecast future returns.

Formally, we use a latent variable approach to infer the probability of a macroeconomic disaster over the next year. Estimating this probability is essentially a filtering problem. The econometrician observes the number of disasters occurring over a given year, which is a noisy signal of the true (unobserved) disaster probability. While assuming that this probability follows an exogenously specified time-series process, we infer the time-varying probability via a (non-Gaussian) Kalman filter.

We find that disaster risk is time varying, persistent, and a good predictor of future disasters—not only at the aggregate level but also at the country level. For example, although the United States experienced only four disasters in our sample period, our filtered probability forecasts these disasters significantly with a slope of nearly 1. That probability also correlates strongly with the stock market dividend-price ratio, which is a reasonable proxy for the conditional equity premium (van Binsbergen and Koijen 2010). The implications are that our estimated probability effectively captures not only US disaster risk but also the time-series dynamics of the equity premium. It is noteworthy that the disaster probability predicts returns in post-war data, which indicates that disaster concerns drive the equity premium even when no disasters actually occur (as in the post-war US economy).

We embed our estimated probability of a macroeconomic disaster in a variable disaster model (Gabaix, 2012; Wachter, 2013). Namely, we consider a discrete-time economy, populated by a representative agent with recursive preferences, in which asset prices are derived from the dynamics of aggregate consumption. Consumption growth is subject to rare disasters, which occur with a time-varying probability. The model is of exponential affine type, so it can be solved with standard techniques and yields simple, closed-form solutions. We use this model as a laboratory to assess both the qualitative and quantitative effects of our measure of time-varying disaster probability—that is, our only state variable.

While we only use macroeconomic data to calibrate our model, we find that it can generate a large and volatile equity premium—along with a low risk-free rate—under conservative preferences. We find that time-varying disaster risk is crucial in generating a quantitatively large and volatile equity premium as well as volatile returns. Our model can therefore rationalize both the high predictability of stock returns and the low predictability of consumption growth in actual data.

This paper contributes to the rapidly growing literature on disaster risk. Our approach extends the work of Barro (2006), Barro and Jin (2011), Nakamura et al. (2013), and Barro and Jin (2016), who use international consumption data to compute the (constant) probability of a disaster. Our paper is also related in spirit to Berkman et al. (2011), who use the number and severity of international political crises to proxy for disaster risk, and Colacito et al. (2016), who use survey data to construct a measure of time-varying skewness. Chen et al. (2016) studies the structural sources of the dividend yield's variation and find that long-run risks and habit leave considerable room (80% of variance) for additional factors; that finding is consistent with results derived using our measure of disaster risk. Another suitable vehicle for estimating disaster risk is option prices. Backus et al. (2011) extract market crash risk premia from index options, but they assume a constant disaster probability. Kelly and Jiang (2014), Farhi et al. (2015), Gao and Song (2015), Seo and Wachter (2015), and Siriwardane (2015) all assume time-varying disaster risk and infer tail risk premia from option prices and from the cross section of stock returns. Manela and Moreira (2017) construct a proxy for the VIX using front-page articles of the *Wall Street Journal*, which they relate to disaster concerns. We show in Section II.G that our measure of disaster risk also correlates with the VIX.

To the best of our knowledge, this paper is the first that uses consumption data in constructing a measure of time-varying disaster risk. Our approach is crucial for assessing quantitatively the ability of consumption-based models to rationalize asset pricing puzzles. Another benefit of macroeconomic data is their availability over long time periods, which enables us to identify a source of risk that is more persistent than the risks present in options. In particular, we offer empirical support to the idea (previously advanced in

Lettau et al. 2008) that the slow decline in the dividend-price ratio can be explained by a persistent decrease in macroeconomic risk.

The rest of this paper is organized as follows. Section I introduces the theoretical framework and our main research question. In Section II we present the data and our estimation strategy before discussing the estimation results. Section III describes our asset pricing model of time-varying rare disasters and reports its predictions. We conclude in Section IV.

I. Theoretical Framework

A. Consumption Dynamics

We model log consumption growth in country $i = 1, \ldots, N$, where $\Delta c_{i,t+1} \equiv \log(C_{i,t+1}/C_{i,t})$, as follows:

$$\Delta c_{i,t+1} = \mu_i + \sigma_i \varepsilon_{i,t+1} + v_{i,t+1}; \tag{1}$$

here $\varepsilon_{i,t+1}$ and $v_{i,t+1}$ are two mutually independent shocks. The first shock is a standard normal random variable, and the second shock captures rare consumption disasters. We model $v_{i,t+1}$ as a compound Poisson shock: $v_{i,t+1} = J_{i,t+1}\mathbf{1}_{\Delta n_{i,t+1}>0}$, where $n_{i,t+1}$ is a Poisson counting process such that $\Delta n_{i,t+1} > 0$ describes a disaster event occurring at time $t+1$. Let $\pi_{i,t}$ denote the probability that the economy i encounters a disaster in in $t+1$:[2]

[2]We commit a slight abuse of notation since $\pi_{i,t}$ is only an approximation of the conditional probability of a disaster on the unit time interval (i.e., yearly). The Poisson counting process $n_{i,t+1}$ has intensity $\pi_{i,t}$. The exact conditional probability of a single disaster occurring over the horizon τ is therefore $\pi_{i,t}\tau\exp(-\pi_{i,t}\tau)$, while the probability that a disaster does not occur is $\exp(-\pi_{i,t}\tau)$. Hence the residual probability that more than a single disaster occurs is $1-\exp(-\pi_{i,t}\tau)(1+\pi_{i,t}\tau)$. According to our estimates, the latter value is about 0.05% when $\pi_{i,t}$ is at its steady state and less than 1% when $\pi_{i,t}$ is at its 99th percentile. So to facilitate terminology and ease the notation, we shall use $\pi_{i,t}$ for the conditional disaster probability and $\sum_i \Delta n_{i,t+1}$ (instead of $\sum_i \mathbf{1}_{\Delta n_{i,t+1}>0}$) for the number of disasters.

$$\pi_{i,t} = \varrho_i \pi_t, \quad \text{where } \varrho_i > 0 \text{ for all } i \text{ and } \frac{1}{N}\sum_i \varrho_i = 1. \tag{2}$$

We assume that π_t follows a discretized square-root process:

$$\pi_{t+1} - \bar{\pi} = \rho(\pi_t - \bar{\pi}) + \nu\sqrt{\pi_t}u_{t+1}, \tag{3}$$

where $\bar{\pi} > 0$, $0 < \rho < 1$, and $\nu > 0$ are constants and where u_t is a standard normal random variable uncorrelated with ε_t. Finally, disaster size $J_{i,t+1}$ follows a shifted gamma distribution with moment-generating function given by

$$\varphi(u) = e^{-u\theta}(1 + u\beta)^{-\alpha}, \tag{4}$$

where disasters have support on $(-\infty, -\theta)$ and where the mean and variance are equal to $-(\theta + \alpha\beta)$ and $\alpha\beta^2$, respectively.

Because macroeconomic disasters occur infrequently, any meaningful inference from the data requires that we pool information from many countries. Toward that end, we assume that the country-i disaster probability is the product of a country-specific component ϱ_i and a time-varying component π_t. The latter corresponds to the average disaster probability across countries, which we refer to as "disaster probability" (or simply "disaster risk") in the rest of the paper. By construction, country-specific probabilities $\pi_{i,t}$ are perfectly correlated with π_t. Because country-specific disasters follow Poisson distributions, the sum of observed disasters in $t + 1$ also follows a Poisson distribution:

$$\Delta n_{t+1} \equiv \sum\nolimits_i \Delta n_{i,t+1} \sim \text{Poisson}\Big(\sum\nolimits_i \pi_{i,t}\Big) = \text{Poisson}(N\pi_t). \tag{5}$$

Equation (5) indicates that we can estimate π_t by pooling information across countries; that outcome is fortunate because using country-specific probabilities is not feasible given the low number of disasters in any given country. Although we cannot estimate $\pi_{i,t}$ from individual countries, we expect π_t to have forecasting power for country disasters.

We confirm this expectation in Section II.F and also show that, for most countries, the individual probabilities $\pi_{i,t}$ differ little from π_t. We assume that our disaster-related parameters (i.e., $\bar{\pi}$, ρ, μ, θ, α, and β) are common across countries and also time invariant; at the same time, we allow the parameters that are unrelated to disasters (μ_i and σ_i) to vary across countries. Because we focus primarily on the US economy, we estimate these parameters with US data.

These model dynamics are fairly standard and have been shown to be capable of capturing a number of asset pricing regularities (Gabaix, 2012; Wachter, 2013). Our model is most closely related to the continuous-time model of Wachter (2013). Gabaix (2012) calibrates a richer model that allows for movements in the disaster probability and in the expected disaster size, and alternative specifications have also been proposed in the literature. For example, Barro and Jin (2011) consider different laws for disaster size, such as single and double power laws. Gabaix (2011) and Gourio (2012) introduce time-varying disaster risk in production economies. More complex disaster paths have been considered by Gourio (2008), Nakamura et al. (2013), and Hasler and Marfe (2016) such as unfolding consumption declines and subsequent recovery as well as consumption declines leading to economic regime changes (Branger et al., 2015). We intentionally keep the model simple, tractable, and parsimonious in order to focus on the time-series relationship between disaster probability and equilibrium asset prices—the core of our empirical analysis.

B. Asset Prices and Disaster Risk

We next discuss how disaster risk matters for asset prices. It is natural to express the log dividend-price ratio using Campbell and Shiller's (1988a) approximation:

$$d_t - p_t = \kappa_0 + \mathbb{E}_t \sum_{j=1}^{\infty} \kappa_1^j [r_{t+j} - \Delta d_{t+j}]; \tag{6}$$

here d_t is log of the economy-wide dividend, p_t is the log of the asset price, κ_0 and κ_1 are log-linearizing constants, and r_t is the continuously compounded return on the market portfolio. To simplify the notation, we reason at the level of a representative country and assume that $\pi_{i,t} = \pi_t$.[3] We establish in Section III that the log equity premium is affine in the conditional probability of a disaster; that is,

$$\log \mathbb{E}_t[e^{r_{d,t+1}}] - r_{f,t} = \begin{array}{c}\text{CCAPM}\\ \\ \text{premium}\end{array} \; + \; \pi_t \; \times \; \left(\begin{array}{c}\text{Disaster size}\\ \\ \text{premium}\end{array} \; + \; \begin{array}{c}\text{Disaster probability}\\ \\ \text{premium}\end{array} \right). \tag{7}$$

The first term is due to consumption volatility in normal times and is proportional to the relative risk aversion of the representative agent; the second term compensates for a disaster in consumption and is proportional to the current level of π_t. Each of these terms obtains under expected utility (e.g., with CRRA preferences). The third term compensates for the time-varying nature of disaster risk and obtains under non-expected utility; such compensation is positive and proportional to π_t when the representative investor prefers an early resolution of uncertainty (Epstein and Zin, 1989). In other words, investors fear uncertainty in their future wealth due to fluctuations in π_t. Under both expected and non-expected utility, Eq. (7) implies that if π_t changes over time then it should forecast excess returns. Equivalently, we show in Section III that (by Eq. (6)) the log dividend-price ratio is affine in π_t:

$$d_t - p_t = A_0 + A_\pi \pi_t; \tag{8}$$

it follows that the log dividend-price ratio should be perfectly correlated with disaster risk. Of course, we do not expect Eq. (7) to hold exactly in the data and so (8) should include an error term. These circumstances motivate our regressions of the dividend-price ratio on disaster risk: a leading empirical question is the economic magnitude of that relation, and answering that question is our paper's main contribution.

[3] Figure 3a shows that this assumption cannot be rejected in most countries, including the United States.

II. Disaster Risk over the Twentieth Century

The measurement of macroeconomic disaster risk is central to this paper. Our empirical strategy is to follow the prior literature in pooling information from the largest possible number of countries and years. Hence this section begins by presenting our international data set. We then explain our measure of macroeconomic disasters and comment on the twentieth century's international disasters. Next we describe our econometric approach to extracting disaster risk, after which our maximum likelihood estimates—and our key results on the co-movement between disaster risk and the conditional equity premium—are presented.

A. Data

This paper uses an updated version of the international panel on per capita consumption expenditures that was constructed by Robert Barro and José Ursúa and described in Barro and Ursúa (2010). The sample covers 42 countries, for some of which data is available since the early nineteenth century. We extend the data set to the years 2010–2015—and thereby include the Great Recession—by using the World Bank's *World Development Indicators*. Modeling a time-varying disaster probability requires a fairly large number of countries each year in order to obtain meaningful estimates; we therefore start our analysis in 1900, which leaves us with a minimum of 20 countries per year. Our data set hence covers the 1900–2015 period, which subsumes the original time frame of Barro (2006). The data comprises 25 OECD countries, 14 countries from Latin America and Asia, as well as Egypt, Russia, and South Africa.[4]

[4]Some observations are missing for Austria, Singapore, and Malaysia during at least one of the world wars. In these cases, we use a cubic spline function to interpolate missing observations. This method is conservative because it tends to smooth consumption during these troubled periods and leads us, in the cases of Singapore and Malaysia, to treat the years around World War II as non-disasters.

B. Macroeconomic Disaster Events

We define disaster events as large declines in consumption during a given year. A country i experiences a disaster in year t if log consumption falls by 2 standard deviations (SD) from its long-term growth path:

$$\Delta c_{i,t} < \text{mean}(\Delta c_{i,t}) - 2 \times \text{SD}(\Delta c_{i,t}). \tag{9}$$

This choice conforms with prior research in the literature, which defines disasters as peak-to-trough declines in consumption (or GDP) that exceed a fixed cutoff value of 10% or 15%. Barro (2006), for example, considers a panel of 35 countries and documents 60 episodes of GDP contractions exceeding 15% in the twentieth century. We depart from this approach in two ways. First, we define disasters as one-year events. Although it is natural to think of disasters as events that unfold over several years, we need to measure disaster risk in calendar time. When disasters do unfold over several years, they are observed only at the end of the trough; in contrast, our approach requires the identification of a disaster in real time. Measuring disasters as one-year events is also consistent with our modeling assumption that disasters occur instantaneously (Constantinides, 2008; Donaldson and Mehra, 2008; Julliard and Ghosh, 2012). Second, we define disasters as a 2-SD contraction; this definition contrasts with previous research, most of which use the same cutoff percentage for all countries. Our approach reflects that consumption tends to be more volatile in some countries than others, and the resulting country-specific cutoff ensures that disaster events are equally probable, a priori, across countries. We choose 2 standard deviations as our baseline cutoff because doing so identifies disasters that are about as rare as in the prior literature. However, in Section II.H we document that our results are not sensitive to that cutoff value and present disaster probabilities based on cutoffs of 2.5 and 3 standard deviations.

We find that our approach produces macroeconomic disasters that are economically similar to disasters measured as peak-to-trough contractions. We obtain 147 disasters, which corresponds to an average disaster probability of 3.8%.[5] Although we define disasters as one-year events, our approach generates disasters with a quite large average size of 17.3%. Barro and Ursúa (2008), who use a similar data set and a peak-to-trough definition of disasters, report a 3.6% annual probability of disaster and a typical size of 22% over an average duration of 3.6 years. By and large, our disasters also coincide with those identified by Barro and Ursúa. For the period during which the two samples overlap, we find 128 disasters, among which 99 are identified as disasters in Barro and Ursúa. Put differently: conditional on being in a one-year disaster, our approach has a $99/128 \approx 77.3\%$ probability of identifying a peak-to-trough disaster.

C. *Estimation*

We now describe the econometric framework. Our goal is to estimate π_t, the annual probability of a macroeconomic disaster over the next year in a typical country, as well as the parameters governing that probability's dynamics. The remaining parameters, which correspond to the size distribution of disasters and to consumption in normal times, are estimated in a straightforward way using maximum likelihood techniques. Here we provide a general overview of the estimation procedure; readers are referred to Appendix A for a detailed description.

We assume that π_t follows the square-root process given by Eq. (3). Although this probability π_t is latent, we observe disasters $\Delta n_{i,t}$ as they occur across countries and time. In light of our assumption that individual disaster probabilities are perfectly correlated, we focus on the number of disasters occurring in a given year t, or Δn_t.[6] According to Eq. (5), Δn_t follows a Poisson distribution with intensity $N\pi_t$. Eqs. (3) and (5) together define a filtering problem: the true disaster probability is unobserved but we can still measure it, albeit with noise. Hence our procedure aims to remove this noise in order to

[5] We report the list of disasters in Table A.I.

[6] Our econometric model does not account for the increase, over time, in the number of countries for which we have data. To correct for that shortcoming, we divide the number of disasters at time t by the number of countries for which data is available in t and then multiply that quotient by the total number of countries $N = 42$. The result is then rounded to the nearest integer.

recover an estimate of π_t and of the parameters in Eq. (3).

Standard filtering techniques require Gaussian observations and that the latent variable is represented as an autoregressive process. Because our observations instead obey a Poisson distribution, we follow Durbin and Koopman (1997) and approximate the non-Gaussian model by a traditional linear Gaussian model. In their approach, $N\pi_t$ is assumed to follow a log-autoregressive process. That differs from our assumption of a square-root process, but it does ensure a positive probability of disaster. In order to estimate a square-root process, we proceed in two steps. First, we estimate a log-autoregressive disaster probability using the Durbin and Koopman (1997) methodology. In the second step, we recover the square-root–process probability and parameters by way of a simple OLS regression. The two probabilities are close, in practice, so little information is lost in this process (see Figure 5 in Section II.H). We simulate 1,000 bootstrap samples in order to assess statistical significance.

Our estimation procedure requires that observations be drawn from an exponential family distribution, such as the Poisson distribution. It is natural to ask whether our key empirical results depend on this assumption. Equivalently, one might wonder whether our definition of individual country disaster is sensible given our goal of estimating π_t. Of course, merging country results (or, for example, treating each US state separately) would lead to different estimates. One could argue that World War II (WWII) was a single world event, but we treat such large disasters as comprising multiple observations—an approach that leads us to conclude that disaster risk was higher around WWII. One could also argue that disasters are of different intensities and should be weighted accordingly, which is similar in effect to adopting alternative distributional assumptions about disaster risk. Thus a scholar who assumes that disaster risk depends on both local and global factors will treat WWII as a single global event. Yet taking that view would severely limit our power to detect time variation in disaster risk. In the same vein, we could replace the conditional Poisson disaster distribution by a distribution with a fatter tail. Such distributions (e.g., the Conway–Maxwell–Poisson distribution) are not of the exponential family and so do not have closed-form solutions for the likelihood function. Nonetheless, Section II.H offers an indirect demonstration of our results' robustness to the distributional assumption. For example, we find that the results are not changed when we use filtering with different cutoff values for defining a macroeconomic disaster or use different data sets. We also present results based on the Gaussian Kalman filter. It is worth emphasizing that the implications of our paper do *not* depend on π_t. We show in Section II.F that the dividend-price ratio remains a good predictor of macroeconomic disasters even when the regressions control for π_t. It follows that much of the variance in the dividend-price ratio is related to disaster risk and that disaster risk varies over time. This latter finding is clearly one that does not depend on a precise estimate of disaster risk.

D. *Estimation Results*

Table I presents the point estimates, standard errors, and 95% confidence intervals for the consumption process parameters defined by Eqs. (1)–(4). Disaster parameters are estimated by pooling information across countries, although estimates for consumption growth in "normal times" are based on US data. Our first key result is that disaster risk is both persistent ($\rho = 0.82$) and volatile, with a scale parameter ν of about 0.10. These two parameters contribute to the unconditional volatility of disaster risk, which is $\sqrt{\nu^2 \bar{\pi}/(1-\rho^2)} = 3.3\%$. Interestingly, the point estimates are close to the corresponding (continuous-time) parameters in Wachter (2013). Wachter chooses a more persistent $\rho = 0.92$ (using our notation), which is set to match the autocorrelation of the price-dividend ratio.[7] The scale parameter is lower than in our setup ($\nu = 0.067$). Altogether, these parameters imply a very similar volatility of disaster risk (3.2%). The mean disaster size is $\theta + \alpha\beta = 17.3\%$ with a 9.3% standard deviation (the minimum disaster size is $\theta = 4.6\%$). We plot the distribution of consumption declines, along with the fitted density, in Figure 1. Consumption growth parameters in "normal times" belong to the usual range of values: the long-term consumption growth rate is 2.0%, and consumption

[7] The persistence of scaled price variables varies according to how they are defined. In our sample, the autocorrelation of the dividend-price ratio is 0.80, while the autocorrelation of the price-dividend ratio is 0.93. When measured in logs, the autocorrelations are both equal to 0.89.

volatility is 2.7%.

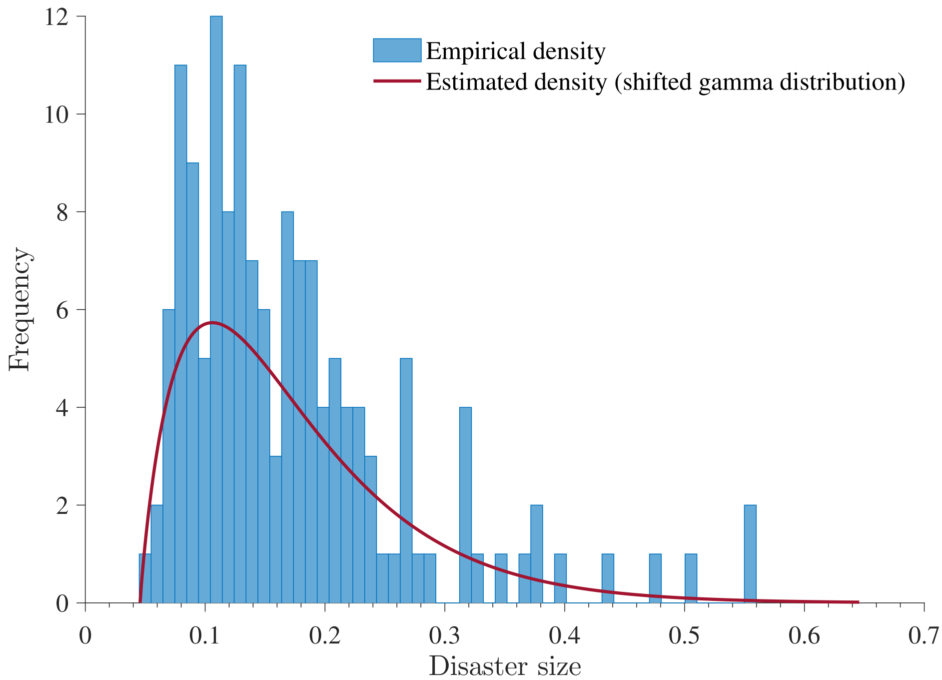


Figure 1: Size Distribution of Macroeconomic Disasters
The figure plots the empirical density of consumption disasters alongside the density estimated from the shifted gamma distribution. The estimated parameters of the fitted distribution are given in Table I.

E. Disaster Risk and the Dividend-Price Ratio

Our second key result is the strong correlation between π_t and the dividend-price ratio.[8] Figure 2 plots the share $\Delta n_t/N$ of countries encountering a disaster, the estimated disaster probability π_t, and the S&P 500 dividend-price ratio—as well as NBER recession dates—over the 1900–2015 period. There is a clearly visible connection between π_t and the US dividend-price ratio, our proxy for the conditional equity premium; the sample correlation

[8] To simplify the presentation, we use π_t to denote both the "true" disaster probability and our filtered estimate of that probability.

Table I: Estimation Results

	Full sample		95% Confidence interval
	Estimate	S.E.	
Normal Times:			
Average growth in consumption μ	0.020	0.002	(0.014, 0.025)
Volatility of consumption growth σ	0.027	0.002	(0.023, 0.031)
Disaster probability:			
Long-term disaster probability $\bar{\pi}$	0.038	0.016	(0.015, 0.075)
Persistence ρ	0.818	0.064	(0.675, 0.942)
Volatility parameter ν	0.099	0.301	(0.072, 0.561)
Disaster size:			
Shape parameter α	1.898	0.255	(1.321, 2.701)
Scale parameter β	0.067	0.009	(0.048, 0.093)
Shifting parameter θ	0.046	0.003	(0.038, 0.057)

Notes. This table presents maximum likelihood estimates of the time-varying disaster probability model. Log consumption growth evolves according to

$$\Delta c_{i,t} = \mu_i + \sigma_i \varepsilon_{i,t} + v_{i,t},$$

where μ_i and σ_i are constants, $\varepsilon_{i,t}$ is a standard normal random variable, and $v_{i,t} = J_{i,t}\mathbf{1}_{\Delta n_{i,t}>0}$. Here $\Delta n_{i,t}$ follows a Poisson distribution with time-varying probability π_t:

$$\pi_t - \bar{\pi} = \rho(\pi_{t-1} - \bar{\pi}) + \nu\sqrt{\pi_{t-1}}u_t.$$

Finally, $J_{i,t}$ follows a shifted gamma distribution that takes a minimum value of θ and gamma parameters α and β. The log consumption growth mean μ_i and variance σ_i are estimated from US data while excluding four consumption growth disasters (in 1920, 1921, 1930, and 1932). Disasters are defined according to Eq. (9). The remaining parameters are estimated via maximum likelihood from an international data set of 42 countries for the period 1900 to 2015. The likelihood for the non-Gaussian state-space model is computed by simulation (see Section II.C).

is 0.58.[9] In terms of magnitudes, regressing the dividend-price ratio on π_t reveals that a 1% increase in π_t raises that ratio by about 0.4%. We have $0.58^2 \approx 0.34$, which suggests that about a third of the variance observed in the dividend-price ratio can be attributed to disaster risk variance. This elasticity remains highly significant when the regressions incorporate a time trend and changes little in post-war data. Finally, Figure 2 shows the dividend-price ratio in levels whereas Eq. (8) expresses that ratio in logs. Either approach yields similar results, although the latter finds a slightly lower correlation between the two series.[10]

Figure 2 shows that both the dividend-price ratio and π_t are larger in the first half than in the second half of the twentieth century. Both series spike during the Great Depression and the two world wars yet also around less prominent events such the 1907 Knickerbocker Crisis and the 1936–1939 Spanish Civil War. As others have noted (e.g., Barro and Ursúa 2008), the world wars were not macroeconomic disasters for the United States, which actually prospered economically during both periods. That the dividend-price ratio rose (i.e., prices fell) during these episodes of higher disaster risk is a good example of the mechanism we propose—namely, that markets react to disaster risk even when disasters do not materialize. This correlation remains remarkably high in the post-war period. Both the dividend-price ratio and disaster risk increase following WWII, which corresponds to the Korean War (that country experienced three disasters between 1950 and 1952). The series are relatively high and volatile between 1974 (year of the Chilean coup d'état) and 1989 (year of the fall of the Berlin Wall). The Great Recession is associated with only two disasters (both in Iceland).[11] So for the 2008–2009 period, the associated increase in π_t is modest but has the same order of magnitude as the increase in the dividend-price ratio.

We observe at least two sequences during which changes in disaster risk are not as-

[9] Because π_t is itself strongly correlated by contemporaneous disasters Δn_t, the correlation between disasters and π_t is about as high (0.54).

[10] Results are reported in Table A.II. Formally, we add linear and quadratic time trends and then interact π_t with a dummy set equal to 1 only for post-war data. Results are presented with the dividend-price ratio in levels and in logs.

[11] For example, consumption contracted by 6.2% in Spain in 2009, which is not a non-normal event given the volatility (8.0%) of Spain's consumption growth over the century.

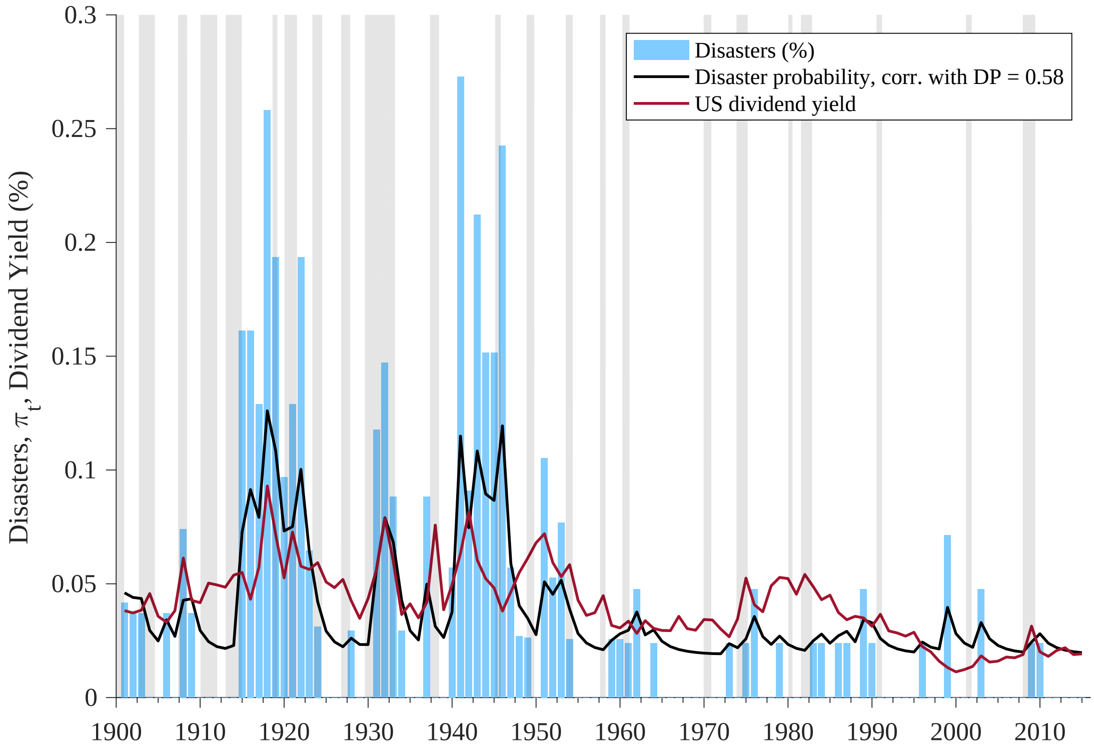


Figure 2: Disasters, Disaster Risk, and the Dividend Yield
This graph plots the share of countries entering a disaster, the estimated disaster probability π_t, and the S&P 500 dividend-price ratio. Shaded areas denote NBER recessions.

sociated with meaningful changes in the dividend-price ratio. First, π_t increases in 1960 following three macroeconomic disasters in China (during 1959–1961). These disasters correspond to the Great Leap Forward, a catastrophic episode in China's history whose consequences were, however, mostly confined to China. This instance reveals the inescapable limit of our empirical strategy: we treat all disasters the same way regardless of their likely informativeness about average disaster risk. Thus we privilege an Occam's razor approach that follows Barro (2006) over a more sophisticated one that could well run the risk of "data snooping". The second instance of a disconnect between the two series is the 1997 Asian financial crisis, which spawned disasters in Indonesia, Korea, and Malaysia in 1998. Unlike the earlier deviant sequence, the Asian financial crisis did have systemic implications—suggesting that the risk of a disaster had increased in the United States. The non-response of US stock prices in 1997 and thereafter is a genuine anomaly, which is usually explained in terms of the dot-com "bubble".

F. Predicting Disasters, Consumption, and Stock Returns

Our paper's main hypothesis is that the risk of a disaster occurring varies over time. Changes in the forecasts of future disasters induce fluctuations not only in expected future cash flows but also in the premium required to hold stocks. The second of these effects dominates and so stock prices appear to be cheaper under a high risk of disaster, which leads to predictable returns. This dynamic holds especially in the absence of disasters, when all variation in the dividend-price ratio and in expected returns seems unrelated to future cash flows. Therefore, in this section we ask whether π_t predicts future disasters and whether π_t can be used to forecast consumption growth and excess returns. And since the only source of dividend-price ratio variability in our model is predictable disaster risks, we repeat this forecasting exercise while using that ratio. Later, in Section III, we shall compare these empirical results with the simulated moments generated by our model.

Figures 3a and 3b plot the slope coefficients (with 90% confidence intervals) and R^2 statistics for OLS regressions of realized disasters on π_t and on the log dividend-price ratio.[12] Table II and Table III regress (respectively) US consumption growth and S&P 500 excess stock returns—measured over horizons of 1, 3, 5, and 10 years—again on π_t and on the log dividend-price ratio. We report slope estimates, t-statistics, and R^2 statistics for the full sample (1900–2015) as well as individually for the pre-war (1900–1945) and post-war (1946–2015) periods. Inference is conducted using Hansen and Hodrick (1980) standard errors, which explicitly account for the moving average structure that overlapping time periods introduces into error terms.

We first verify that π_t and the log dividend-price ratio forecast future disasters. We present results for international disasters (Δn_{t+1}) and also for the 7 countries with at least one disaster. We find that both π_t and the dividend-price ratio are strongly predictive of international disasters.

It may not be surprising that one can forecast the number of international disasters Δn_{t+1}. After all, our estimation strategy amounts to maximizing the ability of π_t to predict Δn_{t+1}. It seems more challenging to predict $\Delta n_{i,t+1}$, the indicator variable set equal to 1 only when the focal country undergoes a disaster. We thus seek to establish whether country disasters are forecastable. (We naturally expect less precise estimates given the small number of disasters per country; e.g., for the United States we observe only 4 disasters over the sample period.) Recall that our specification for consumption dynamics allows disaster risk to vary across countries (i.e., we let ϱ differ from unity in Eq. (2)); that specification is motivated by our interest in whether predictability varies across countries. Figures 3a and 3b reveal that both π_t and the dividend-price ratio forecast US disasters with positive slopes. Although individual estimates are not precise, they are positive for all countries and significant in the majority (6) of them. Also interesting is that the slopes cannot be distinguished from unity in 6 out of 7 countries. Figure 3b shows that results are robust to forecasting with the dividend-price ratio instead of with π_t.

Although we find disaster risk to be predictable, this does not imply that consumption

[12]The results are similar when we use probit or Poisson regressions. Observe that our specification does not include an intercept when forecasting with π_t because that term already captures the unconditional disaster probability (Eq. (2)).

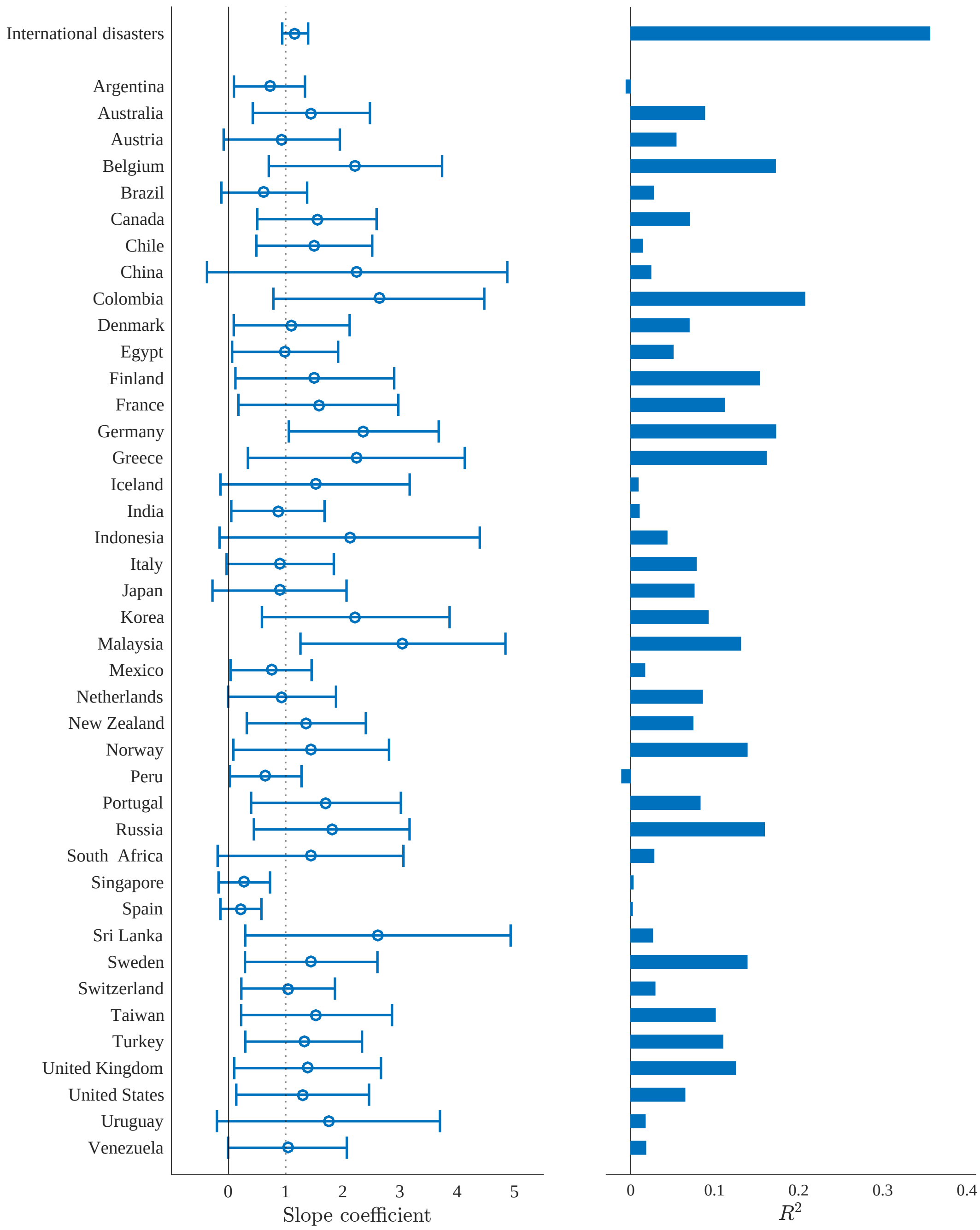


Figure 3a: Forecasting Disasters with π_t
This figure presents slope coefficients (with 90% confidence intervals) and R^2 statistics for OLS regressions of next-year macroeconomic disasters on the disaster probability π_t: $\Delta n_{i,t+1} = b_i \pi_t + u_{i,t+1}$. The first line presents results for the number of international disasters (Δn_{t+1}), and the rest of the figure presents results for individual country disaster dummies ($\Delta n_{i,t+1}$). Results are reported for countries with at least one disaster over the sample period. Inference is conducted using heteroskedasticity- and autocorrelation-consistent robust standard errors.

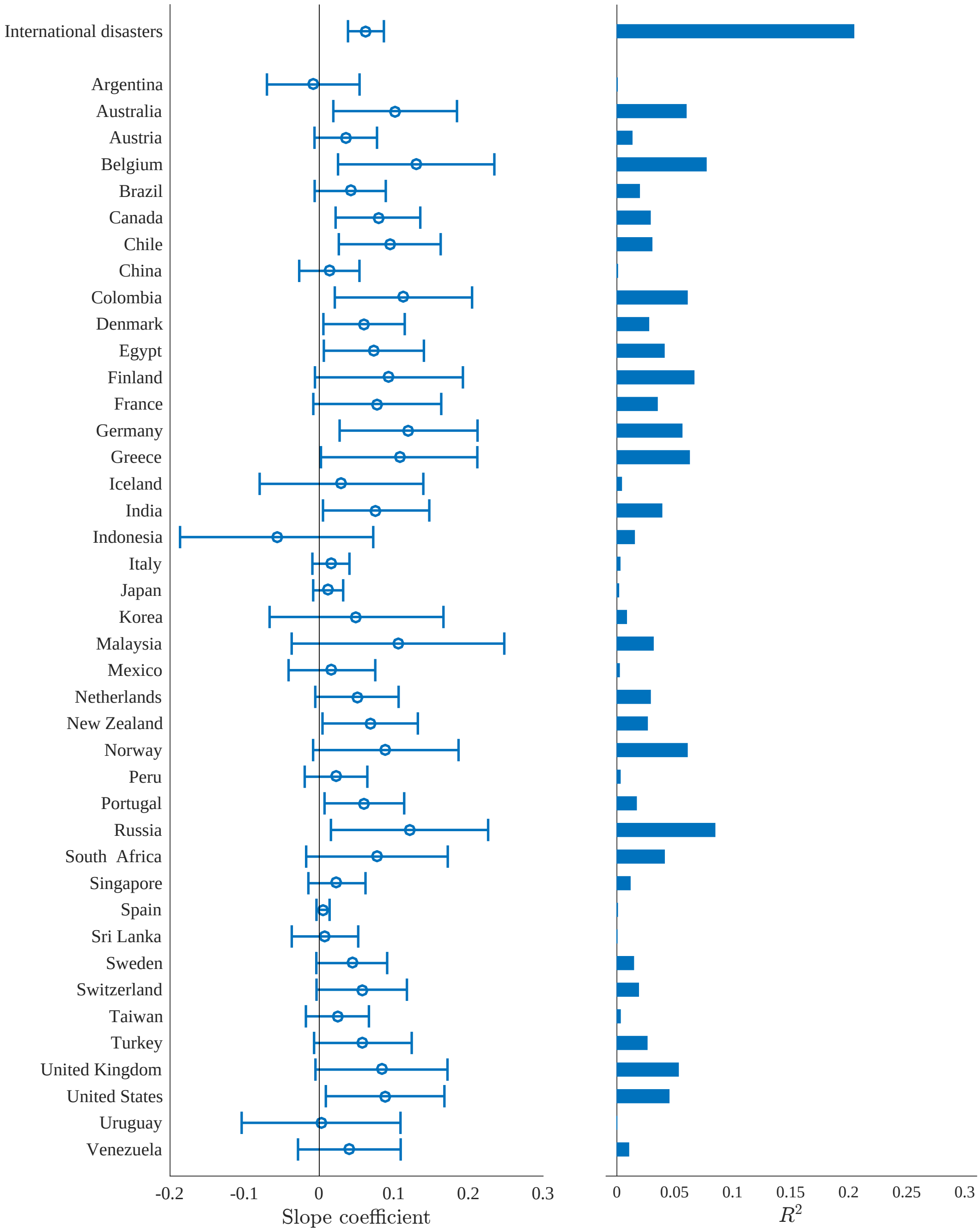


Figure 3b: Forecasting Disasters with the log dividend-price ratio
This figure presents slope coefficients (with 90% confidence intervals) and R^2 statistics for OLS regressions of next-year macroeconomic disasters on the log dividend-price ratio $d_t - p_t$: $\Delta n_{i,t+1} = a_i + b_i(d_t - p_t) + u_{i,t+1}$. The first line presents results for the number of international disasters (Δn_{t+1}), and the rest of the figure presents results for individual country disaster dummies ($\Delta n_{i,t+1}$). Results are reported for countries with at least one disaster over the sample period. Inference is conducted using heteroskedasticity- and autocorrelation-consistent robust standard errors.

growth is predictable. The literature has emphasized the dividend-price ratio's limited power to forecast future consumption (e.g. Beeler and Campbell 2012) and dividend growth (e.g. Chen 2009) in post-war data. The regression results given in Table II establish that, in line with prior evidence, neither π_t nor the dividend-price ratio have strong predictive power for US consumption growth.[13] Both the dividend-price ratio and disaster risk predict a decline in consumption growth, but the respective correlations are not statistically significant. In the full sample, slope estimates decrease in absolute value as the forecasting horizon increases. Results for the subsamples indicate that that most of the predictability in consumption growth can be attributed to the pre-war sample; consumption growth predictability is practically non-existent in the post-war sample. This result echoes findings, reported in Chen (2009), that dividend growth was predictable during pre-war years but essentially unpredictable after World War II.

Table II: Predictability of Consumption Growth

	Full sample (1900-2015)				Prewar (1900-1945)				Postwar (1946-2015)			
K	1	3	5	10	1	3	5	10	1	3	5	10
$\sum_{k=1}^{K} \Delta c_{t+k} = a + b\pi_t + u_{t+k}$												
b	-0.09	-0.29	-0.01	-0.04	-0.17	-0.42	0.16	1.16	-0.13	-0.05	-0.29	-1.47
s.e.	0.12	0.38	0.58	0.89	0.21	0.65	0.96	1.11	0.26	0.75	1.15	1.72
R^2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.02
$\sum_{k=1}^{K} \Delta c_{t+k} = a + b(d_t - p_t) + u_{t+k}$												
b	-0.02	-0.02	-0.02	-0.03	-0.09	-0.08	-0.02	-0.02	-0.00	0.00	0.01	0.04
s.e.	0.01	0.02	0.04	0.06	0.02	0.06	0.09	0.13	0.00	0.02	0.03	0.05
R^2	0.07	0.02	0.00	0.01	0.28	0.04	0.00	0.00	0.00	0.00	0.00	0.05

Notes. This table presents slope coefficients, t-statistics, and R^2 statistics for predictive regressions of US consumption growth on disaster risk and the log dividend-price ratio. Results are reported for the full sample as well as for subsamples covering (respectively) the pre- and post-war periods. Inference is conducted using Hansen and Hodrick's (1980) standard error correction for overlapping observations.

Table III presents our third key result—namely, that disaster risk predicts future

[13] Consumption growth rates and dividend growth rates are highly correlated in the data. Results for dividend growth rates (not shown) are qualitatively similar.

excess returns. We have just shown that disaster risk and the dividend-price ratio are strongly correlated; this relation suggests that the conditional equity premium is proportional to disaster risk, as hypothesized by Gabaix (2012) and as predicted by Eq. (8). Another test is to regress future excess returns on π_t, as suggested by Eq. (7). Table III shows that both π_t and the dividend-price ratio are positively related to excess returns, with magnitudes that increase with the forecast horizon. Predictability is statistically stronger when the dividend-price ratio is used, especially for one- to three-year horizons. At the ten-year horizon, both variables exhibit the same predictive power, with R^2 values of 20% for disaster risk and 22% for the dividend-price ratio. From this we conclude that nearly a fifth of long-term return variance can be explained by changes in the disaster risk premium.

Table III: Predictability of Excess Returns

	Full sample (1900-2015)				Prewar (1900-1945)				Postwar (1946-2015)			
K	1	3	5	10	1	3	5	10	1	3	5	10
$\sum_{k=1}^{K} r_{m,t+k} - r_{f,t+k} = a + b\pi_t + u_{t+k}$												
b	0.30	1.80	4.31	9.36	0.64	2.90	5.19	7.60	3.16	9.79	18.65	32.21
s.e.	0.75	1.87	2.74	3.87	1.16	2.77	3.69	3.38	2.49	5.65	8.32	13.28
R^2	0.00	0.01	0.06	0.20	0.00	0.03	0.10	0.29	0.01	0.07	0.16	0.24
$\sum_{k=1}^{K} r_{m,t+k} - r_{f,t+k} = a + b(d_t - p_t) + u_{t+k}$												
b	0.08	0.19	0.34	0.59	0.17	0.51	0.80	0.75	0.10	0.24	0.41	0.70
s.e.	0.04	0.11	0.17	0.28	0.14	0.27	0.34	0.39	0.04	0.12	0.19	0.35
R^2	0.02	0.06	0.13	0.22	0.01	0.10	0.19	0.21	0.06	0.13	0.22	0.30
$\sum_{k=1}^{K} r_{m,t+k} - r_{f,t+k} = a + b\tilde{\pi}_t + u_{t+k}$												
b	0.45	1.08	2.60	5.27	0.49	1.15	2.28	3.61	3.45	8.73	16.01	27.39
s.e.	0.43	1.01	1.45	2.08	0.60	1.30	1.66	1.45	1.50	3.84	5.80	9.51
R^2	0.00	0.01	0.07	0.19	0.00	0.01	0.07	0.28	0.06	0.16	0.32	0.45

Notes. This table presents slope coefficients, t-statistics, and R^2 statistics for predictive regressions of the excess returns on π_t, the log dividend-price ratio, and $\tilde{\pi}_t$. Results are reported for the full sample as well as for subsamples covering (respectively) the pre- and post-war periods. Inference is conducted using Hansen and Hodrick's (1980) standard error correction for overlapping observations.

Noteworthy is the greater predictability characteristic of the post-war sample (which includes no US disasters). At a three-year horizon, for instance, disaster risk (resp., the dividend-price ratio) accounts for 7% (resp., 13%) of return variance. Predictability is weaker within the pre-war sample when we use either the dividend-price ratio or π_t, especially at short horizons. Disaster risk offers a natural interpretation of the pre-war sample's weaker return predictability. The US sample contains four disasters (in 1920, 1921, 1930, and 1932), all of which unfolded during the pre-war period. Disasters are positively related to π_t and are usually associated with negative returns. Hence it is not surprising to find weaker predictability in the presence of disasters. In Section III.C, we return to this point in the context of our model.[14]

Combining information from past disasters with the dividend-price ratio There is no question that π_t has less predictive power for excess returns than does the dividend-price ratio (see Table III). This weaker statistical evidence is unsurprising when one considers that we construct π_t using only past and current disasters. In reality, market participants have access to other sources of information that improve the accuracy of their predictions concerning disaster risk. It is therefore likely that asset prices capture additional information about future disasters and also about future excess returns. We illustrate this point by estimating an alternative measure of disaster risk that makes use of information from past disasters and from the dividend-price ratio. Let us use $\tilde{\pi}_t$ to denote this alternative probability. Remaining consistent with this paper's general methodology requires that we not use a return forecasting equation to estimate $\tilde{\pi}_t$. The most natural way to seek an improved predictor of future disasters is to regress those disasters Δn_{t+1} on π_t and on $d_t - p_t$, the log dividend-price ratio. The fitted values derived using this model give us $\tilde{\pi}_t$. To account for the Poisson distribution of disasters and to ensure that $\tilde{\pi}_t$ is always positive, we estimate it using a generalized Poisson regression model (although results are qualitatively similar when using OLS-fitted values). In panel (b) of Figure 5

[14]Manela and Moreira (2017) find that likewise "news implied volatility" predicts returns in the post-war sample yet not in the pre-war sample. These authors also interpret their findings in terms of disaster concerns.

(in Section II.H) we plot both $\tilde{\pi}_t$ and the OLS-fitted values.

We find that $\tilde{\pi}_t$ is an even better predictor of disaster than is π_t. At a one-year horizon, McFadden's pseudo-R^2—in a univariate specification with π_t on the right-hand side—is 32% (this value is slightly lower than the R^2 for the OLS regression in Figure 3a). The pseudo-R^2 increases to 39% for regressions that include the log dividend-price ratio, and we find that both coefficients are highly significant. By construction, $\tilde{\pi}_t$ is also more strongly correlated with the dividend yield: the sample correlation is now 0.75. The bottom panel of Table III reports predictability results for excess returns. The improvement is striking for post-war data, where $\tilde{\pi}_t$ is a better predictor than the dividend-price ratio over all horizons.

G. *Disaster Risk and Option Prices*

We have estimated disaster risk using macroeconomic data, but another natural approach would be to use option prices. The first paper to do so is Backus et al. (2011), which has been followed by a growing literature (e.g., Gao and Song 2015, Seo and Wachter 2015). In parallel, a number of papers have emphasized the connection between option prices and the equity premium. Martin (2017) derives a lower bound on the equity premium based on an options price index that is almost perfectly correlated with the the S&P 500 volatility index (VIX). There is also a strong empirical connection between disaster risk and option prices. Siriwardane's (2015) measure of disaster risk is highly correlated with the VIX. Manela and Moreira (2017) use front-page articles of the *Wall Street Journal* to construct forecasts of the VIX and show that much of the variance in forecasted VIX is related to disasters such as wars. Although both the dividend-price ratio and the VIX are reasonable predictors of excess returns, in the data these factors are themselves not highly correlated. Moreover, the VIX is generally much less persistent than the dividend-price ratio, with a half-life of 10 months as compared with a half-life of about 6 years for that ratio (in logs).

Because we postulate a unique state variable, our model produces a unit correlation between the VIX, the log dividend-price ratio, and π_t; see Section III. We have emphasized the empirical correlation with the log dividend-price ratio, but there is also a strong connection between the VIX and π_t. Figure 4 plots π_t over time along with VIX and NVIX, where the latter is Manela and Moreira's (2017) long-term proxy for volatility as implied by newspaper headlines. There is a remarkable correlation (0.43) between π_t and VIX during the sample period in which the VIX has been calculated (viz., ever since January 1990); yet over that same period, the correlation between the VIX and the dividend-price ratio is statistically indistinguishable from zero. The NVIX is less volatile than the VIX, which is expected because NVIX captures only the variation in volatility that can be gleaned from the *Journal*'s headlines. The correlation between π_t and NVIX is positive but is lower than the correlation between π_t and VIX.

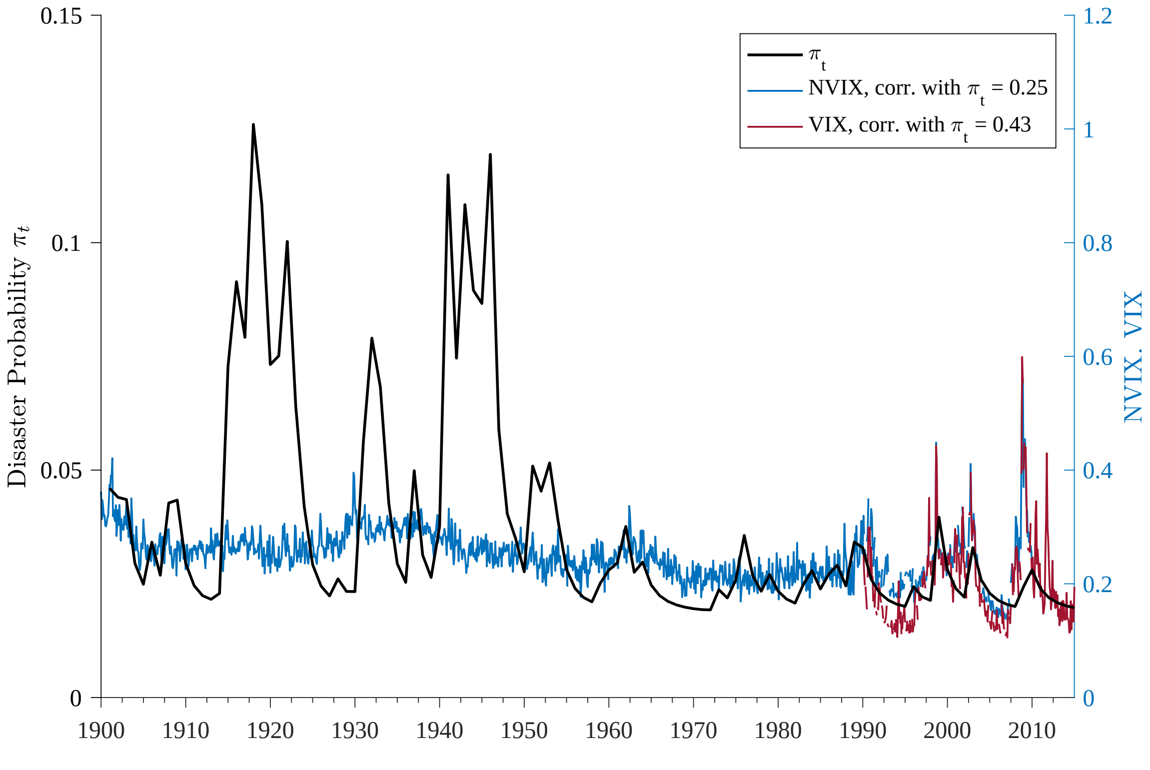


Figure 4: Disaster Risk and Option Prices
This graph plots π_t, Manela and Moreira's (2017) news-implied volatility index (NVIX), and the VIX. The π_t term is sampled annually, whereas volatility series are sampled monthly. Correlations are based on end-of-year values.

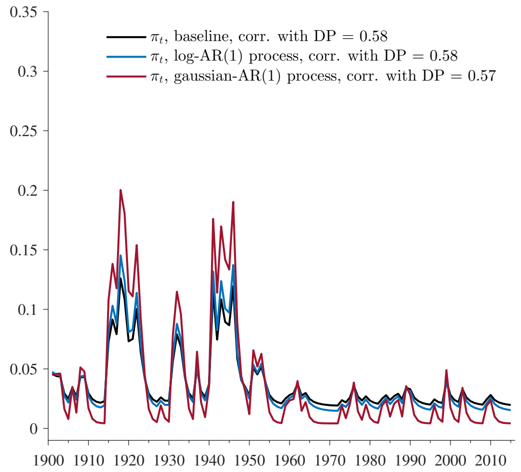
In short, π_t is correlated with two distinct and widely used financial indicators. The correlation is stronger with the component related to the price of stocks (the dividend-price ratio) than with the component related to the price of options (the VIX). Our results therefore strengthen the case that disaster concerns matter for high-frequency fluctuations in uncertainty, and they also document a new link between disaster risk and low-frequency fluctuations in the equity premium.

H. Robustness

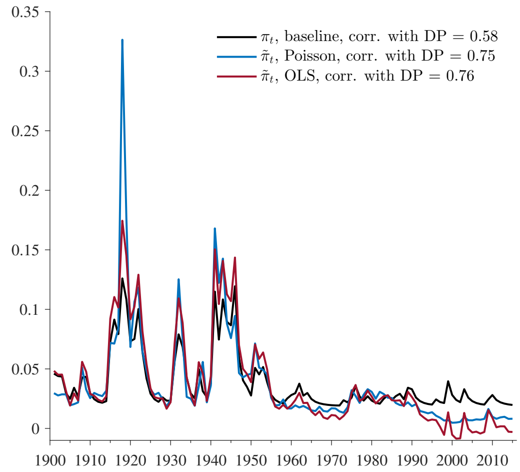
Figure 5 illustrates that the strong connection between disaster risk and π_t does not depend on how π_t is constructed. Panel (a) of the figure examines alternative econometric models: one with Poisson disasters but log-autoregressive latent dynamics, the other a standard linear Gaussian model. Panel (b) considers specifications that exploit information from the dividend yield and also from past disasters. Panel (c) assumes less frequent disasters: 2.5 and 3 standard deviations from average consumption growth rate, instead of the 2-SD cutoff in Eq. (9). Panel (d) presents results based on OECD data only (25 countries) and GDP data. In all cases, correlation with the dividend-price ratio is near (or greater than) 0.50. Tables A.III and A.IV present estimation results corresponding to Panels (c) and (d) of Figure 5. Parameter estimates are close to the baseline values reported in Table I.

III. Asset Pricing Implications

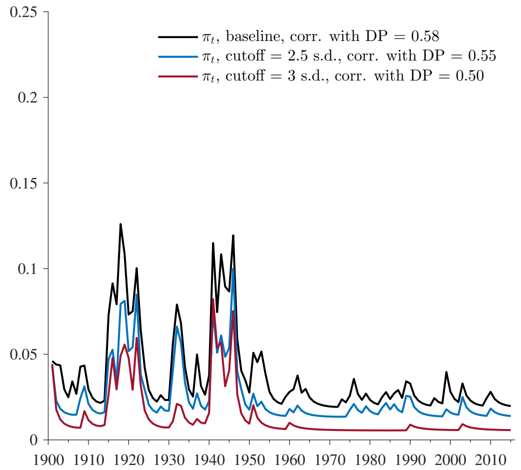
This section presents a simple equilibrium asset pricing model with time-varying disaster risk. The model is a discrete-time version of the one offered by Wachter (2013) and belongs to the discrete-time affine class proposed in Drechsler and Yaron (2011). After describing the economy and the dynamics of aggregate consumption, we derive equilibrium asset prices. Solution details are provided in Appendix B.



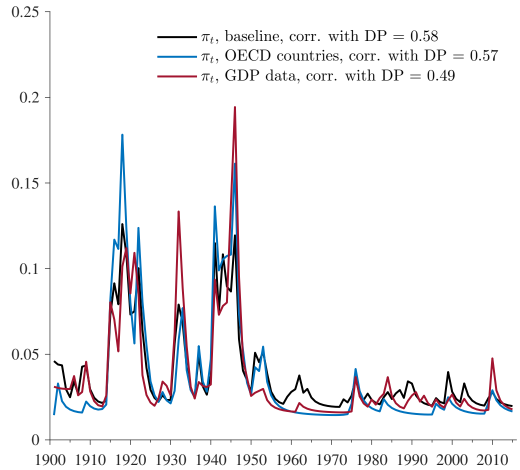
(a) Alternative econometric models



(b) Bivariate specifications



(c) Alternative cutoffs



(d) Alternative data sets

Figure 5: Alternative disaster probabilities

This figure shows estimates of disaster risk under several alternative specifications. Panel (a) plots estimates based on a model with Poisson-distributed disasters but with log-autoregressive latent dynamics (blue) and a standard linear Gaussian model (red). Panel (b) shows estimates of $\tilde{\pi}$, the alternative disaster probability that uses information from the log dividend-price ratio; estimates based on a Poisson model are show in blue, and OLS estimates are plotted in red. Panel (c) assumes a more severe cutoff at which consumption declines are treated as disasters (defined as 2.5 and 3 standard deviations from average consumption growth rate, instead of 2 as in Eq. (9)). Panel (d) plots estimates based on OECD consumption data only (25 countries) in blue and GDP data in red. Estimation results are given in Tables A.III and A.IV.

A. The Economy

We consider a pure exchange economy, à la Lucas (1978), populated by a representative investor with recursive preferences (Epstein and Zin, 1989):

$$V_t = \left[(1-\delta)C_t^{1-1/\psi} + \delta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1-1/\psi}{1-\gamma}}\right]^{1/(1-1/\psi)}.$$

In this expression, δ is the time discount factor, $\gamma \neq 1$ is the relative risk aversion, and ψ is the elasticity of intertemporal substitution. To ensure tractability, we focus on the case of ψ equal to one. Similarly to Collin-Dufresne et al. (2016), we normalize utility V by consumption level C such that the log value function $vc_t \equiv \log V_t/C_t$ is given by

$$vc_t = \frac{\delta}{1-\gamma} \log \mathbb{E}_t[e^{(1-\gamma)(\Delta c_{t+1}+vc_{t+1})}]. \tag{10}$$

The aggregate dividend paid by the equity claim is modeled in a parsimonious way (after Abel 1999, Campbell 2003, and Wachter 2013): $\Delta d_{t+1} = \phi \Delta c_{t+1}$. Although this model is simplistic, when $\phi > 1$ it predicts that dividends fall by more than consumption in the event of a disaster—which is consistent with US data (Longstaff and Piazzesi, 2004).

B. Equilibrium

We solve for asset prices by expressing the stochastic discount factor in terms of the investor's value function, which is affine in the disaster probability π_t. We can then solve for the return on the equity claim via the investor's Euler equation up to the usual Campbell and Shiller (1988a) log linearization. As in Section I.B, we reason at the level of a typical country and so the country disaster probability is $\pi_{i,t} = \pi_t$. The stochastic discount factor is given by

$$M_{t+1} = \underbrace{\delta e^{-\gamma \Delta c_{t+1}}}_{\substack{\text{discounting of} \\ \text{expected utility}}} \times \underbrace{\frac{e^{-(1-\gamma)vc_{t+1}}}{\mathbb{E}_t[e^{(1-\gamma)(\Delta c_{t+1}+vc_{t+1})}]}}_{\substack{\text{discounting of} \\ \text{continuation utility}}} \tag{11}$$

(Collin-Dufresne et al., 2016) and the value function satisfies

$$vc_t = v_0 + v_\pi \pi_t, \tag{12}$$

where

$$\begin{aligned} v_0 &= \frac{\delta}{1-\delta}\left(\mu + 2\bar{\pi}(1-\rho)v_\pi - \frac{\gamma\sigma^2}{2}\right), \\ v_\pi &= \frac{(1-\delta)(1-\gamma) + \sqrt{(1-\gamma)^2(1+\delta(\delta+2\delta\sigma^2-2) - 2\delta\sigma^2\varphi(1-\gamma))}}{\delta(1-\gamma)^2\sigma^2}. \end{aligned}$$

Note that the stochastic discount factor variance (i.e., the price of risk in the economy) increases with the disaster probability:

$$\mathrm{var}_t(\log M_{t+1}) = \gamma^2\sigma^2 + (\gamma-1)^2 v_\pi^2 \nu^2 \pi_t + \gamma^2 \mathbb{E}_t[J_{t+1}^2]\pi_t,$$

where $\mathbb{E}_t[J_{t+1}^2]$ is given by the second-order derivative of the moment-generating function evaluated at 0.

Let W_t be the present value of the future aggregate consumption stream. If the elasticity of intertemporal substitution is equal to 1 then investor wealth is proportional to consumption, $W_t = C_t\frac{\delta}{1-\delta}$, in which case the log return on wealth satisfies $r_{c,t+1} = -\log\delta + \Delta c_{t+1}$.

The log risk-free rate, $r_{f,t} = -\log \mathbb{E}_t[M_{t+1}]$, is affine in the disaster probability:

$$r_{f,t} = -\log\delta + \mu - \gamma\sigma^2 + \pi_t(\varphi(1-\gamma) - \varphi(-\gamma)). \tag{13}$$

This risk-free rate is stationary and decreases linearly with disaster probability π. An increased likelihood of disaster increases the consumption risk, which the investor can hedge with risk-free investments. The resulting increase in holdings of the risk-free asset entails an increased price for that asset and hence a reduction in the risk-free rate. This effect increases in magnitude with relative risk aversion.

To solve for the equity claim price, we log-linearize returns around the unconditional mean of the log dividend yield $dp \equiv \mathbb{E}[d_t - p_t]$ with $d_t - p_t \equiv \log D_t/P_t$:

$$\begin{aligned} r_{d,t+1} &= \log(e^{-d_{t+1}+p_{t+1}} + 1) + d_t - p_t + \Delta d_{t+1} \\ &\approx k_0 - k_1(d_{t+1} - p_{t+1}) + d_t - p_t + \Delta d_{t+1}; \end{aligned}$$

where the endogenous constants k_0 and k_1 satisfy

$$k_0 = -k_1 \log(k_1) - (1 - k_1)\log(1 - k_1) \qquad \text{and} \qquad k_1 = e^{-dp}/(1 + e^{dp}).$$

Campbell et al. (1997) and Bansal et al. (2012) document the high accuracy of such a log linearization, which we hereafter assume to be exact. We can use the Euler equation $1 = \mathbb{E}_t[M_{t+1}e^{r_{d,t+1}}]$ to recover that the log dividend yield is affine in the disaster probability:

$$d_t - p_t = A_0 + A_\pi \pi_t, \tag{14}$$

where

$$\begin{aligned} A_0 &= \log(1 - k_1) - \log(k_1) - A_\pi \bar{\pi} \quad \text{and} \\ A_\pi &= \frac{1}{k_1^2 \nu^2}\left(\sqrt{\Omega^2 + 2k_1^2\nu^2(\varphi(1-\gamma) - \varphi(\phi - \gamma))} - \Omega\right) \end{aligned}$$

with $\Omega = 1 - (1-\gamma)k_1^2\nu^2 v_\pi \rho$.

The dividend yield is stationary and increases (resp., decreases) with the disaster probability π_t when γ is (resp., is not) greater than 1. These dynamics reflect a preference for the early (resp., late) resolution of uncertainty about time variation in π_t. An increase in disaster probability makes it more likely that disasters will affect future consumption. An investor who prefers early resolution of uncertainty ($\gamma > 1$) is worried about current disaster risk and also about uncertainty in future disaster risk. Hence prices are low, relative to dividends, when π_t is high (and vice versa). Note that the substitution effect and the income effect offset each other when the elasticity of intertemporal substitution is equal to one, as we assume here.

The log equity premium is given by

$$\log \mathbb{E}_t[e^{r_{d,t+1}}] - r_{f,t} = \underbrace{\gamma\phi\sigma^2}_{\text{non-disaster risk}} + \underbrace{(1-\gamma)k_1 A_\pi v_\pi \nu^2 \pi_t}_{\text{disaster probability risk}} + \underbrace{(\varphi(\phi) + \varphi(-\gamma) - \varphi(\phi-\gamma) - 1)\pi_t}_{\text{disaster size risk}}, \tag{15}$$

and the return variance is

$$\mathrm{var}_t(r_{d,t+1}) = \underbrace{\phi^2\sigma^2}_{\text{non-disaster risk}} + \underbrace{k_1^2 A_\pi^2 \nu^2 \pi_t}_{\text{disaster probability risk}} + \underbrace{\phi^2 \pi_t \frac{\partial^2}{\partial u^2}\varphi(u)\Big|_{u=0}}_{\text{disaster size risk}}. \tag{16}$$

Both the equity premium and the return variance are given by three terms. The first of these terms concerns non-disaster risk and gives rise to the usual consumption-CAPM compensation (Lucas, 1978). Variation in disaster probability gives rise to the second component of the equity premium and return variance. This term increases with current disaster probability and also with its persistence and volatility, whereas it disappears for $\gamma = 1$; this term also captures the excess volatility of returns over dividends. The third term is associated with disaster realizations and increases with both disaster size variance and current disaster probability.

Finally, the risk-neutral return variance (i.e., the model-implied VIX) has a similar form:

$$\mathrm{var}_t^{\mathbb{Q}}(r_{d,t+1}) = \underbrace{\phi^2\sigma^2}_{\text{non-disaster risk}} + \underbrace{k_1^2 A_\pi^2 \nu^2 \pi_t}_{\text{disaster probability risk}} + \underbrace{\phi^2 \pi_t \frac{\partial^2}{\partial u^2}\varphi(u)\Big|_{u=-\gamma}}_{\text{disaster size risk}}, \tag{17}$$

where only the third term differs from the physical return variance. Here the variance risk premium is proportional to π_t.

C. *Asset Pricing Results*

In this section we discuss the calibration of unobserved parameters and spell out the model's main predictions about consumption growth and asset prices. We show that our estimated parameters governing the dynamics of π_t offer quantitative support for the asset pricing predictions of our variable disaster model.

In view of the given consumption parameters, we set the dividends' leverage parameter to $\phi = 2.6$. This value implies that dividend growth and consumption growth are perfectly correlated (the empirical correlation does, in fact, exceed 60%) and that dividends are more volatile than consumption. Evidence from the Depression era suggests that disasters have a much larger effect on dividends than on consumption (Longstaff and Piazzesi, 2004). Wachter (2013) argues that a value of 2.6 is conservative and captures well the extra risk of dividends relative to consumption in both normal times and disaster periods.

We complete the calibration by setting the preference parameters δ and γ to fit the main asset pricing moments. We find that low values (between 2 and 3) of relative risk aversion suffice to generate a high equity premium that conforms with the data. For greater values of relative risk aversion, a good match to the equity premium implies an implausibly high risk-free rate. Hereafter we consider the pair ($\delta = 97.8\%$, $\gamma = 2.5$) as our baseline calibration. Table IV shows that this calibration leads to a good fit of the most important unconditional moments of asset prices. That outcome is hardly unexpected because our estimated disaster risk parameters are close to those used by Wachter (2013), which are calibrated to match actual asset prices. Panel A of Table IV reports historical asset pricing moments and their model counterparts for several preference settings. The risk-free rate is about 2.9% with 0.9% unconditional volatility. The equity premium is approximately 6.2%, and the return volatility is about 16.9%; hence the Sharpe ratio is 36.5%. The dividend yield is about 4.5% with 1.0% unconditional volatility—values that are quite close to the actual data. In Panel B of the table, we study the effect of a change in either the persistence ρ or the disaster scale ν (while holding all else constant). Raising either persistence or scale increases both the equity premium and return volatility, notwithstanding the low level of risk aversion ($\gamma = 2.5$). Conversely, a decrease in persistence or scale significantly lowers the first two moments of stock returns.

Table IV: Asset Pricing Moments

Panel A

%	Data 1900-2009	Data 1946-2009	$\gamma = 2$ $\delta = 98.3\%$	$\gamma = 2.5$ $\delta = 97.8\%$	$\gamma = 3$ $\delta = 97.4\%$
Risk-free rate level	1.4	1.2	2.6	2.9	3.2
Risk-free rate volatility	4.9	3.4	0.8	0.9	1.1
Equity premium	6.8	7.0	4.7	6.2	9.1
Return volatility	19.4	16.9	15.8	16.9	19.2
Sharpe ratio	35.2	41.5	29.5	36.5	47.2
Dividend yield level	4.1	3.4	2.8	4.5	7.4
Dividend yield volatility	1.6	1.4	0.5	1.0	2.1

Panel B

%	$\rho = 0.90$	$\rho = 0.75$	$\nu = 0.15$	$\nu = 0.05$
Risk-free rate level	2.9	2.9	2.9	2.9
Risk-free rate volatility	1.2	0.8	1.4	0.5
Equity premium	10.7	4.2	23.5	1.9
Return volatility	26.1	13.9	38.3	12.4
Sharpe ratio	41.1	30.3	61.4	15.5
Dividend yield level	7.2	2.9	17.2	0.8
Dividend yield volatility	4.1	0.3	12.7	0.1

Notes. Panel A reports unconditional moment statistics from S&P 500 real returns and three-month US Treasury real rates—in addition to model-implied steady-state moments based on the Table I parameters—for several preference settings. Panel B reports model-implied steady-state moments using all but one of the parameters from Table I together with the preference parameters $\gamma = 2.5$ and $\delta = 97.8\%$.

We note that variation in disaster risk is essential for generating a large equity premium. Agents who prefer the early resolution of uncertainty dislike bad news yet also (and importantly in this context) dislike the possibility of more bad news in the future. Thus agents price not only disasters but also the uncertainty associated with variation in the probability of disaster. In the baseline calibration, 76% of the equity premium reflects compensation for disaster probability risk at the steady state (see Eq. (15)). Likewise, disaster probability risk accounts for about 48% of return variance in the steady state (Eq. (16)). These results indicate that a preference for the early resolution of uncertainty is critical for quantitatively reconciling disaster risk and asset prices, and they are in line with our previous finding that disaster risk is correlated with the US dividend yield in post-war data (i.e., in the absence of disasters).

Now we investigate the extent of predictability that is implied by our model. We start by verifying that disaster probability predicts one-year-ahead disasters with a highly significant coefficient close to unity (0.95, t = 2.62), in line with evidence for the United States. Then we show in Table V that π_t is a good predictor of returns but not of consumption growth. For the sake of consistency with Section II.F's empirical analysis, we simulate 1,000 paths of the economy with 116 yearly observations for each simulation. To avoid generating a negative probability of disaster, we simulate monthly series and then convert them to annual frequency.[15] We then run regressions using a horizon of 1, 3, 5, and finally 10 years. The leftmost four data columns of Table V report average regression estimates for each horizon. We can see that disaster probability does not forecast consumption growth. This lack of a relation is not surprising given the assumed consumption dynamics of Eq. (1); it also accords with actual data, as documented in the middle (pre-war) panel of Table II. Disaster probability is a reasonably good predictor of excess returns, with a positive slope and explanatory power increasing with the horizon. Although the explanatory power is due to our model's simplifying assumptions, that model captures the positive intertemporal relationship—between disaster probability and

[15]Simulations of π_t at monthly frequency (i.e., $\pi_{t+\triangle} = (1-\rho^{\triangle})\bar{\pi} + \rho^{\triangle}\pi_t + \nu\sqrt{\pi_t \triangle} u_t$ for $\triangle = 1/12$) have but a negligible likelihood of realizations lower than 0.1%, which we replace by a small positive threshold.

excess returns—found in the data. Model results are indeed close to US post-war data, which are reported in the bottom panel of Table III.

Results reported in the center and the right side of Table V allow for a more accurate comparison between model-implied predictability and the empirical evidence. The center four columns report predictability results from simulations of the economy using a sample of 46 years in which four disasters occur; in the rightmost four columns we consider the case of a 70-year economy in which no disasters take place. In this way we characterize scenarios comparable to the US experience during the pre- and post-war periods, respectively. In neither period does disaster probability forecast consumption growth, yet it forecasts excess returns in both periods. We find that excess returns are predicted more reliably in the absence of realized disasters, which is also consistent with empirical evidence from the post-war United States.

Overall, our measure of disaster probability—although estimated from macroeconomic data—enables a quantitative assessment of variable disaster models and strongly supports the notion that asset prices depend heavily on the time-varying nature of disaster risk.

IV. Conclusion

The rare disaster model is one of the few workhorses of empirical asset pricing. The model is based on the idea that equity markets compensate investors' fear of rare but large macroeconomic events. In this paper we use consumption data to assess that model by directly estimating its unique and all-important state variable: the time-varying probability of a macroeconomic disaster. We use a latent variable approach to derive a formal estimate of the time-varying expected probability of a macroeconomic disaster; for this purpose, we use a data set comprising 42 countries and covering the period from 1900 to 2015. The results reported here constitute strong evidence in support of the rare dis-

Table V: Model-Implied Long-Horizon Regressions

	Unrestricted (116 years)				Four disasters (46 years)				No disasters (70 years)			
K	1	3	5	10	1	3	5	10	1	3	5	10
					Consumption growth							
$\sum_{k=1}^{K} \Delta c_{t+k} = \alpha + \beta\pi_t + \epsilon_{t+k}$												
β	0.13	0.30	0.40	0.49	0.23	0.52	0.68	0.68	-0.00	0.00	0.00	0.00
s.e.	0.12	0.21	0.26	0.38	0.27	0.45	0.58	0.82	0.08	0.14	0.17	0.24
R^2	0.02	0.04	0.05	0.06	0.04	0.09	0.12	0.14	0.01	0.04	0.05	0.08
					Excess returns							
$\sum_{k=1}^{K} r_{m,t+k} - r_{f,t+k} = \alpha + \beta\pi_t + \epsilon_{t+k}$												
β	1.49	3.57	4.86	6.36	1.29	3.12	4.24	5.93	2.11	4.93	6.52	7.99
s.e.	0.54	0.85	1.03	1.36	1.07	1.72	2.13	2.82	0.58	0.84	0.98	1.27
R^2	0.07	0.16	0.20	0.22	0.05	0.13	0.17	0.22	0.16	0.35	0.42	0.42

Notes. This table presents slope coefficients, standard errors, and R^2 statistics for predictive regressions of consumption growth and excess returns on disaster probability. The economy has been simulated 1,000 times at monthly frequency with 116 yearly observations for each simulation. Estimates reported in the left group of columns are obtained by pooling regression results from all simulations. Regression results reported in the center columns are based on 1,000 simulations, each of 46 years' length, and four realized disasters. Results reported in the rightmost four columns are based on 1,000 simulations, each of 70 years' length, and no realized disasters.

aster hypothesis. Disaster risk is volatile, persistent, and strongly correlated with the US dividend yield. Our model generates a large and volatile equity premium whose coefficient of relative risk aversion is 2.5 and whose elasticity of intertemporal substitution is 1.

Appendix A. Estimation Procedure

This appendix details the procedure used to estimate the latent disaster probability π_t. We proceed in two steps: the first estimates an approximation π_t^*, which is assumed to follow a log-autoregressive process; the second step recovers π_t.

Let $\lambda_t \equiv \log(N\pi_t^*)$, and assume that λ_t follows a first-order autoregressive or AR(1) process

$$\lambda_{t+1} - \bar{\lambda} = \rho^*(\lambda_t - \bar{\lambda}) + \nu^* e_t. \tag{A1}$$

According to Eq. (5), the number of countries experiencing a disaster at time t (Δn_t) follows a Poisson distribution parameterized by $N\pi_t$. With λ_t so defined, if we assume that $\pi_t^* \approx \pi_t$ then

$$p(\Delta n_{t+1}|\lambda_t) = \exp[\Delta n_{t+1}\lambda_t - \exp(\lambda_t) - \log(\Delta n_{t+1}!)], \tag{A2}$$

from which it follows that the expected number of disasters in $t+1$ is $\exp(\lambda_t)$.

Our state-space model is given by Eqs. (A1) and (A2). The latter equation expresses an observed value and is our measurement equation; the former equation expresses an unobserved value and will be estimated using the Kalman filter. It would be ideal theoretically for π_t to replace λ_t as the state variable, in which case the law of motion for π_t would be given by (3). From an econometric perspective, however, the log-autoregressive specification is preferable because it ensures that π_t^* remains positive. We remark that, at annual frequency, the discretized square-root process has a non-negligible probability of generating negative values.

Our model consists of three parameters: $\psi = (\bar{\lambda}, \rho^*, \nu^*)$. Analytical methods cannot be used because the observations are not normally distributed, so we rely on simulation techniques. Thus we adopt a method, developed in Durbin and Koopman (1997), that consists of estimating the model with maximum likelihood techniques and then using a traditional linear Gaussian model to approximate the non-Gaussian one. The true likelihood can then be computed by adjusting the likelihood—which is available in closed form—of the approximating Gaussian model. The adjustment term, which we obtain via simulation, corrects for the departure of the true likelihood from the Gaussian likelihood.

Formally, the likelihood is defined as $\mathbb{L}(\psi) = p(\Delta n_{t+1}|\psi)$ and corresponds to the joint density of the model defined by Eqs. (A1) and (A2). Dropping time subscripts, we can write

$$\begin{aligned} \mathbb{L}(\psi) = p(\Delta n|\psi) &= \int_\lambda p(\Delta n, \lambda|\psi)\,\mathrm{d}\lambda \\ &= \int_\lambda p(\Delta n|\lambda, \psi)p(\lambda|\psi)\,\mathrm{d}\lambda. \end{aligned} \tag{A3}$$

The first line of Eq. (A3) expresses the likelihood as the joint distribution of the observed disasters Δn and the unobserved state variables λ, given ψ. In the second line, the joint distribution of Δn and λ is expressed as the conditional density of Δn (given λ) multiplied by the marginal density of λ.

A closed-form solution for the integral in (A3) does not exist if $p(\Delta n, \lambda|\psi)$ follows a Poisson distribution. In principle, we could simulate trajectories of λ from the density $p(\lambda|\psi)$ and then evaluate Eq. (A3) by the sample mean of the corresponding values of $p(\Delta n|\lambda, \psi)$. Yet we choose to rely instead on the importance sampling technique (e.g., Ripley 1987), which has been shown to be much more efficient in this context. Importance sampling involves choosing a density $g(\lambda|\Delta n, \psi)$ that is as close to $p(\lambda|\Delta n, \psi)$ as possible, after which one can evaluate the likelihood (A3) while making an appropriate adjustment to correct for the difference between the true density and its approximation. Observe that, provided $g(\lambda|\Delta n, \psi)$ is positive everywhere, Eq. (A3) can be written as

$$\begin{aligned}\mathbb{L}(\psi) &= \int_\lambda p(\Delta n|\lambda, \psi)\frac{p(\lambda|\psi)}{g(\lambda|\Delta n, \psi)}g(\lambda|\Delta n, \psi)\,\mathrm{d}\lambda \\ &= \mathbb{E}_g\left[p(\Delta n|\lambda, \psi)\frac{p(\lambda|\psi)}{g(\lambda|\Delta n, \psi)}\right], \end{aligned} \tag{A4}$$

where $\mathbb{E}_g$ denotes expectation with respect to the *importance density* $g(\lambda|\Delta n, \psi)$. Although $g(\lambda|\Delta n, \psi)$ can be any conditional density a priori, we choose a Gaussian distribution—one parametrized to be as close as possible to $p(\lambda|\Delta n, \psi)$—in order to maximize the estimation's efficiency. The importance density is selected using the mode estimation method described in Chapter 10 of Durbin and Koopman (2012).

Next we point out that, although the *conditional* densities differ between the exact and approximate models, the *marginal* densities must be the same in both models. As a result, $g(\lambda|\Delta n, \psi) \neq p(\lambda|\Delta n, \psi)$ but $g(\lambda|\psi) = p(\lambda|\psi)$. We can therefore rewrite the importance density as

$$g(\lambda|\Delta n, \psi) = \frac{g(\Delta n, \lambda|\psi)}{g(\Delta n|\psi)} = \frac{g(\Delta n|\lambda, \psi)g(\lambda|\psi)}{g(\Delta n|\psi)} = \frac{g(\Delta n|\lambda, \psi)p(\lambda|\psi)}{g(\Delta n|\psi)}.$$

This expression implies, in turn, that

$$\frac{p(\lambda|\psi)}{g(\lambda|\Delta n, \psi)} = \frac{g(\Delta n|\psi)}{g(\Delta n|\lambda, \psi)}. \tag{A5}$$

Observe that $g(\Delta n|\psi)$ is the model's likelihood function under the approximating Gaussian distribution g; hence we write $\mathbb{L}_g(\psi) = g(\Delta n|\psi)$. Substituting Eq. (A5) into Eq. (A4) now yields

$$\mathbb{L}(\psi) = \mathbb{E}_g\left[p(\Delta n|\lambda, \psi)\frac{\mathbb{L}_g(\psi)}{g(\lambda|\Delta n, \psi)}\right] = \mathbb{L}_g(\psi)\mathbb{E}_g\left[\frac{p(\Delta n|\lambda, \psi)}{g(\Delta n|\lambda, \psi)}\right]. \tag{A6}$$

Equation (A6) expresses the non-Gaussian likelihood $\mathbb{L}(\psi)$ as an adjustment to the linear Gaussian likelihood $\mathbb{L}_g(\psi)$, which is easily computed with the Kalman filter. An important advantage of this expression is that it requires simulation only of the second term and not of the likelihood itself.

To summarize: our first step filters out λ_t by maximizing the log of (A6), which is given as the sum of a standard Gaussian log likelihood and a correction term.

Our second step recovers π_t, ρ, and ν by running the OLS regression

$$\frac{\pi_t^* - \bar{\pi}}{\sqrt{\pi_{t-1}^*}} = \rho \frac{\pi_{t-1}^* - \bar{\pi}}{\sqrt{\pi_{t-1}^*}} + \nu u_t^*, \tag{A7}$$

where $\pi_t^* = \exp(\lambda_t)/N$ and $\bar{\pi}$ is taken to equal the sample average of disasters in the data. We find that this approximation produces only small errors: the root-mean-square error is 0.0055, and the correlation between the filtered series π_t^* and the series for π_t obtained in the second stage is greater than 0.999 (see Panel (a) of Figure 5).

Appendix B. Solving the Model

Recall that the dynamics of consumption belong to the affine class and are given by

$$\begin{aligned} \Delta c_t &= \mu + \sigma \varepsilon_t + v_t, \\ \pi_t &= (1-\rho)\bar{\pi} + \rho \pi_{t-1} + \nu \sqrt{\pi_{t-1}} u_t. \end{aligned}$$

Therefore, the following expectation has exponential affine solution (Drechsler and Yaron, 2011):

$$\mathbb{E}_t\left[e^{u_1 \Delta c_{t+1} + u_2 \pi_{t+1}}\right] = e^{g_0(u) + g_1(u)'[\Delta c_t, \pi_t]'}, \qquad u = (u_1, u_2)' \in \mathbb{R}^2,$$

where

$$\begin{aligned} g_0(u) &= \left(\mu - \tfrac{\sigma^2}{2}\right) u_1 + \bar{\pi}(1-\rho) u_2 + \tfrac{1}{2}\sigma^2 u_1^2, \\ g_1(u) &= \left[0, \tfrac{1}{2}\nu^2 u_2^2 + \rho u_2 + \varphi(u_1) - 1\right]'. \end{aligned}$$

The representative agent has recursive utility of the form

$$V_t = \left[(1-\delta)C_t^{1-1/\psi} + \delta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1-1/\psi}{1-\gamma}}\right]^{1/(1-1/\psi)}.$$

Normalized utility obtains if we take the limit as $\psi \to 1$, divide by C_t, rearrange, and then take the logarithm:

$$vc_t = \frac{\delta}{1-\gamma}\log(\mathbb{E}_t[e^{(1-\gamma)(\Delta c_{t+1}+vc_{t+1})}]).$$

Hypothesizing an affine form for vc_t,

$$vc_t = v_0 + v_c\Delta c_t + v_\pi \pi_t,$$

we can substitute to obtain

$$v_0 + v_c\Delta c_t + v_\pi\pi_t = \frac{\delta}{1-\gamma}\log(\mathbb{E}_t[e^{(1-\gamma)(\Delta c_{t+1}+v_0+v_c\Delta c_{t+1}+v_\pi\pi_{t+1})}])$$

and then compute the expectation on the right-hand side:

$$\begin{aligned} v_0 + v_c\Delta c_t + v_\pi\pi_t = \frac{\delta}{1-\gamma}\big((1-\gamma)v_0 &+ g_0([(1-\gamma)(1+v_c),(1-\gamma)(v_\pi)]') \\ &+ g_1([(1-\gamma)(1+v_c),(1-\gamma)v_\pi]')'[\Delta c_t,\pi_t]'\big). \end{aligned}$$

Finally, we solve for the coefficients:

$$\begin{aligned} v_c &: \; v_c = 0, \\ v_\pi &: \; v_\pi = v_\pi\delta + \delta(1-\gamma)v_\pi^2\nu^2/2 + \tfrac{\delta}{1-\gamma}(\varphi(1-\gamma)-1), \\ v_0 &: \; v_0 = v_0\delta + \mu\delta + v_\pi\delta(1-\rho)\bar{\pi} + \delta(1-\gamma)v_\pi^2\nu^2/2, \end{aligned}$$

where we take the negative root to solve for v_π.

In order to derive asset prices, we first solve for the stochastic discount factor:

$$\begin{aligned} M_{t+1} &= \delta \frac{e^{-\gamma \Delta c_{t+1} + (1-\gamma) v c_{t+1}}}{\mathbb{E}_t[e^{(1-\gamma)(\Delta c_{t+1} + v c_{t+1})}]} \\ &= \delta e^{-\gamma \Delta c_{t+1} + (1-\gamma) v c_{t+1} - (1-\gamma) v_0 - g_0([1-\gamma, (1-\gamma) v_\lambda]') - g_1([1-\gamma, (1-\gamma) v_\pi]')[\Delta c_t, \pi_t]'}. \end{aligned}$$

Since the risk-free rate satisfies $e^{-r_{f,t}} = \mathbb{E}_t[M_{t+1}]$, we obtain

$$\begin{aligned} e^{-r_{f,t}} &= \delta e^{(1-\gamma) v_0 + g_0([-\gamma, (1-\gamma) v_\pi]') + g_1([-\gamma, (1-\gamma) v_\pi]')[\Delta c_t, \pi_t]'} \\ &\quad \times e^{-(1-\gamma) v_0 - g_0([1-\gamma, (1-\gamma) v_\pi]') - g_1([1-\gamma, (1-\gamma) v_\pi]')[\Delta c_t, \pi_t]'} \end{aligned}$$

and therefore

$$r_{f,t} = -\log \delta + \mu - \gamma \sigma^2 + \pi_t(\varphi(1-\gamma) - \varphi(-\gamma)).$$

Recall that dividends can be expressed as $\Delta d_t = \phi \Delta c_t$. Stock returns are therefore given by

$$\begin{aligned} r_{d,t+1} &= \log \tfrac{P_{t+1} + D_{t+1}}{P_t} = \log \tfrac{P_{t+1}/D_{t+1} + 1}{P_t/D_t} + \log \tfrac{D_{t+1}}{D_t} \\ &= \log(e^{-d_{t+1} + p_{t+1}} + 1) + d_t - p_t + \Delta d_{t+1} \\ &\approx k_0 - k_1(d_{t+1} - p_{t+1}) + d_t - p_t + \Delta d_{t+1} \end{aligned}$$

for some endogenous constants k_0 and k_1 to be derived later. We posit an affine form for the logarithm of the dividend yield:

$$d_t - p_t = A_0 + A_c \Delta c_t + A_\pi \pi_t.$$

The Euler equation for the stock (i.e., the claim asset on D_t) is $1 = \mathbb{E}_t[M_{t+1} e^{r_{d,t+1}}]$. We now plug in M_{t+1}, the log-linearized $r_{d,t+1}$, and our affine guesses for $d_t - p_t$ and

$d_{t+1} - p_{t+1}$:

$$1 = \mathbb{E}_t[\delta e^{-\gamma\Delta c_{t+1}+(1-\gamma)vc_{t+1}-(1-\gamma)v_0-g_0([1-\gamma,(1-\gamma)v_\pi]')-g_1([1-\gamma,(1-\gamma)v_\pi]')[\Delta c_t,\pi_t]'} \\ \times e^{k_0-k_1(A_0+A_c\Delta c_{t+1}+A_\pi\pi_{t+1})+(A_0+A_c\Delta c_t+A_\pi\pi_t)+\phi\Delta c_{t+1}}].$$

Then we rearrange terms and solve the expectation as follows:

$$\begin{aligned} 1 &= \delta e^{-g_0([1-\gamma,(1-\gamma)v_\pi]')-g_1([1-\gamma,(1-\gamma)v_\pi]')[\Delta c_t,\pi_t]'+k_0-k_1A_0+(A_0+A_c\Delta c_t+A_\pi\pi_t)} \\ &\quad\times \mathbb{E}_t[e^{-\gamma\Delta c_{t+1}+(1-\gamma)v_\pi\pi_{t+1}-k_1(A_c\Delta c_{t+1}+A_\pi\pi_{t+1})+\phi\Delta c_{t+1}}] \\ &= \delta e^{-g_0([1-\gamma,(1-\gamma)v_\pi]')-g_1([1-\gamma,(1-\gamma)v_\pi]')[\Delta c_t,\pi_t]'+k_0-k_1A_0+(A_0+A_c\Delta c_t+A_\pi\pi_t)} \\ &\quad\times e^{g_0([\phi-\gamma-k_1A_c,(1-\gamma)v_\pi-k_1A_\pi]')+g_1([\phi-\gamma-k_1A_c,(1-\gamma)v_\pi-k_1A_\pi]')[\Delta c_t,\pi_t]'}. \end{aligned}$$

Finally, we solve for the coefficients of the dividend yield and the log linearization constants:

$$\begin{aligned} k_0 &= -k_1\log(k_1) - (1-k_1)\log(1-k_1), \\ \log k_1 &= \log(1-k_1) - A_0 - A_c\mathbb{E}[\Delta c_t] - A_\pi\mathbb{E}[\pi_t]. \end{aligned}$$

The coefficients satisfy

$$\begin{aligned} A_c &: \quad A_c = 0, \\ A_\pi &: \quad A_\pi = g_1([1-\gamma,(1-\gamma)v_\pi]')[0,1]' - g_1([\phi-\gamma-k_1A_c,(1-\gamma)v_\pi-k_1A_\pi]')[0,1]', \\ A_0 &: \quad A_0 = -k_0 + k_1A_0 - g_0([\phi-\gamma-k_1A_c,(1-\gamma)v_\pi-k_1A_\pi]') + g_0([1-\gamma,(1-\gamma)v_\pi]'). \end{aligned}$$

Note that one must solve simultaneously for k_1 and A_π (we take the positive root) and then for k_0 and A_0. Hence the equity premium is given by

$$\begin{aligned}
&\log \mathbb{E}_t[e^{r_{d,t+1}}] - r_{f,t} \\
&\quad = \log(\mathbb{E}_t[e^{r_{d,t+1}}]\mathbb{E}_t[M_{t+1}]) \\
&\quad = \log \mathbb{E}_t[e^{r^J_{d,t+1}}] + \log \mathbb{E}_t[M^J_{t+1}] - \log \mathbb{E}_t[e^{r^J_{d,t+1}} M^J_{t+1}] - \mathrm{cov}_t[r^C_{d,t+1}, m^C_{t+1}] \\
&\quad = [\phi, -k_1 A_\pi] \begin{bmatrix} \sigma^2 & 0 \\ 0 & \nu^2 \pi_t \end{bmatrix} [\gamma, (\gamma - 1)v_\pi]' + \pi_t[\varphi(\phi) + \varphi(-\gamma) - \varphi(\phi - \gamma) - 1],
\end{aligned}$$

where the superscripts C and J denote (respectively) the normal and non-normal components. The return variance can be expressed as

$$\mathrm{var}_t[r_{d,t+1}] = [\phi, -k_1 A_\pi] \begin{bmatrix} \sigma^2 & 0 \\ 0 & \nu^2 \pi_t \end{bmatrix} [\phi, -k_1 A_\pi]' + \phi^2 \big(\tfrac{\partial^2}{\partial u^2} \varphi(u)\big|_{u=0}\big) \pi_t.$$

Finally, the risk-neutral return variance satisfies

$$\mathrm{var}^{\mathbb{Q}}_t[r_{d,t+1}] = [\phi, -k_1 A_\pi] \begin{bmatrix} \sigma^2 & 0 \\ 0 & \nu^2 \pi_t \end{bmatrix} [\phi, -k_1 A_\pi]' + \phi^2 \big(\tfrac{\partial^2}{\partial u^2} \varphi(u)\big|_{u=-\gamma}\big) \pi_t$$

(Drechsler and Yaron, 2011).

Appendix C. Additional Tables

Table A.I: List of Disasters

Year	Disasters		
1900	Argentina (-0.22)	1945	Austria (-0.28), Germany (-0.20), Italy (-0.09), Japan (-0.43), Korea (-0.20), New Zealand (-0.12), Sweden (-0.09), Taiwan (-0.51)
1901	Switzerland (-0.08)	1946	Austria (-0.18), India (-0.11)
1902	Argentina (-0.14)	1947	Denmark (-0.15)
1905	Taiwan (-0.18)	1948	India (-0.09)
1907	Argentina (-0.13), Brazil (-0.17)	1950	Iceland (-0.13), India (-0.08), Korea (-0.20), South Africa (-0.08)
1908	Canada (-0.12)	1951	Korea (-0.13), Singapore (-0.11)
1914	Austria (-0.32), Belgium (-0.22), Canada (-0.11), Germany (-0.19), Switzerland (-0.08)	1952	Korea (-0.14), Malaysia (-0.13), New Zealand (-0.11)
1915	Belgium (-0.27), Chile (-0.27), France (-0.14), Portugal (-0.07), Turkey (-0.35)	1953	Egypt (-0.13)
1916	Germany (-0.11), Mexico (-0.11), Portugal (-0.08), United Kingdom (-0.09)	1958	Venezuela (-0.17)
1917	Australia (-0.15), Belgium (-0.23), Finland (-0.14), Germany (-0.13), Norway (-0.10), Russia (-0.23), Turkey (-0.19), United Kingdom (-0.08)	1959	China (-0.09)
1918	Austria (-0.27), Finland (-0.27), Malaysia (-0.14), Netherlands (-0.47), Norway (-0.08), Russia (-0.38)	1960	China (-0.05)
1919	Austria (-0.21), Canada (-0.13), Malaysia (-0.20)	1961	China (-0.06), Sri Lanka (-0.09)
1920	Egypt (-0.16), Malaysia (-0.21), Russia (-0.27), United States (-0.05)	1963	Sri Lanka (-0.08)
1921	Brazil (-0.16), Canada (-0.18), Denmark (-0.20), Norway (-0.17), Sweden (-0.14), United States (-0.07)	1972	Sri Lanka (-0.09)
1922	Chile (-0.20), New Zealand (-0.16)	1974	Chile (-0.23)
1923	Germany (-0.14)	1975	Chile (-0.18), Portugal (-0.09)
1927	Chile (-0.22)	1978	Peru (-0.11)
1930	Colombia (-0.18), Malaysia (-0.12), Peru (-0.11), United States (-0.07)	1982	Chile (-0.23)
1931	Australia (-0.22), Chile (-0.32), Malaysia (-0.17), New Zealand (-0.13), Venezuela (-0.23)	1983	Uruguay (-0.17)
1932	Canada (-0.10), Mexico (-0.16), United States (-0.10)	1985	South Africa (-0.06)
1933	Venezuela (-0.24)	1986	Malaysia (-0.13)
1936	Portugal (-0.09), Spain (-0.56), Switzerland (-0.07)	1988	Indonesia (-0.10), Peru (-0.11)
1939	Australia (-0.11), Greece (-0.32)	1989	Peru (-0.19)
1940	Belgium (-0.29), Colombia (-0.13), Denmark (-0.26), Egypt (-0.19), Finland (-0.18), Greece (-0.32), Portugal (-0.08), Sweden (-0.08), United Kingdom (-0.10)	1995	Mexico (-0.12)
1941	Belgium (-0.40), France (-0.37), Switzerland (-0.07)	1998	Indonesia (-0.08), Korea (-0.15), Malaysia (-0.13)
1942	Colombia (-0.12), France (-0.23), Germany (-0.11), Greece (-0.24), Netherlands (-0.38), Russia (-0.55), Taiwan (-0.16)	2002	Argentina (-0.17), Uruguay (-0.18)
1943	Australia (-0.10), Colombia (-0.11), France (-0.15), Italy (-0.14), Korea (-0.15)	2008	Iceland (-0.12)
1944	Austria (-0.19), Japan (-0.21), Korea (-0.12), Taiwan (-0.33), Turkey (-0.18)	2009	Iceland (-0.17)

This table lists disasters based on consumption data in our panel of 42 countries from 1900 to 2015. Disaster sizes are indicated in parentheses.

Table A.II: Disaster Risk and the Dividend-Price Ratio

Dependent variable:	Dividend-price ratio				Log dividend-price ratio			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
π_t	0.390	0.228	0.355	0.233	9.137	3.709	7.878	3.826
t-stat	(4.75)	(3.09)	(5.40)	(3.49)	(4.05)	(2.60)	(4.28)	(2.88)
$\pi_t \times t > 1945$			-0.155	0.072			-5.574	1.485
t-stat			(-0.74)	(0.54)			(-0.89)	(0.55)
t, t^2	No	Yes	No	Yes	No	Yes	No	Yes
R^2	0.342	0.603	0.359	0.605	0.257	0.658	0.286	0.659
N	115	115	115	115	115	115	115	115

This table presents regression results of the (log) dividend-price ratio on π_t. Variants introduce linear and quadratic time trends, and interact π_t with a dummy variable that equals one for postwar data.

Table A.III: Estimation Results: Sensitivity to the Disaster Cutoff

	Cutoff = 2.5 s.d.		Cutoff = 3 s.d.	
	Estimate	S.E.	Estimate	S.E.
Normal Times:				
Average growth in consumption μ	0.019	0.003	0.018	0.003
Volatility of consumption growth σ	0.027	0.002	0.029	0.002
Disaster probability:				
Long-term disaster probability $\bar{\pi}$	0.025	0.012	0.014	0.012
Persistence ρ	0.812	0.068	0.893	0.177
Volatility parameter ν	0.080	0.130	0.091	0.176
Disaster size:				
Shape parameter α	1.658	0.364	1.715	0.725
Scale parameter β	0.086	0.018	0.101	0.036
Shifting parameter θ	0.059	0.006	0.072	0.016

This table presents maximum likelihood estimates of the time-varying disaster probability model, under stricter conditions to treat consumption declines (defined as 2.5 and 3 standard deviations from average consumption growth rate, instead of 2 as in Eq. (9)). Corresponding π_t-estimates are plotted in panel (c) of Figure 5.

Table A.IV: Estimation Results: Alternative Datasets

	OECD		GDP	
	Estimate	S.E.	Estimate	S.E.
Normal Times:				
Average growth in consumption μ	0.020	0.002	0.024	0.004
Volatility of consumption growth σ	0.027	0.002	0.042	0.003
Disaster probability:				
Long-term disaster probability $\bar{\pi}$	0.035	0.023	0.036	0.019
Persistence ρ	0.893	0.135	0.856	0.113
Volatility parameter ν	0.137	0.251	0.114	0.229
Disaster size:				
Shape parameter α	1.952	0.348	1.959	0.245
Scale parameter β	0.068	0.012	0.071	0.009
Shifting parameter θ	0.045	0.004	0.020	0.003

This table gives maximum likelihood estimates of the time-varying disaster probability model based on OECD consumption data only (25 countries) and GDP data. Corresponding π_t-estimates are plotted in panel (d) of Figure 5.

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