A Signaling Theory of Derivatives-Based Hedging*

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Abstract

We model a commodity producing firm that has private information about future volume and requires outside financing to fund a growth opportunity. Due to costly financial distress, a firm's first-best strategy is to sell forward its future production, avoiding any price risk. Low-volume firms, however, have an incentive to mimic, which in equilibrium distorts the hedging strategy of high-volume firms. Under certain conditions, high-volume firms signal their type by hedging more than they would under their first-best strategy. In general, high-volume firms signal by taking on excess risk through derivative positions. When allowing firms to use multiple types of derivatives, we show that high-volume firms use both options and forwards, while low-volume firms only use forwards. The model suggests that heterogeneous and prima facie non-optimal hedging policies may be due to signaling and not speculation or risk shifting.

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1 Introduction

Why do firms hedge? One popular theory argues that risk management allows firms to minimize the costs arising from financial distress, including those that arise from expected bankruptcy costs (Smith and Stulz, 1985). Another theory posits that risk management mitigates the under-investment problem (Froot, Scharfstein, and Stein, 1993); since it is costly to raise external funds, minimizing the fluctuation of internal cash flows allows firms to invest in more NPV-positive projects. Our paper takes a different approach and argues that hedging may serve a role in credibly signaling good business prospects.

To fix ideas let us start with an example. Suppose a gold-mining company privately knows that its mines will deliver a high volume of gold. Furthermore, assume that it is hard—albeit useful—to credibly convey this information to investors, who have limited technical knowledge and are aware of the perverse incentives to report high expectations. In such a setting, a credible signal would be to sell forward a large quantity of gold (or, equivalently, to short a large number of gold futures contracts). The signal is credible because a gold-mining firm with low future volume potentially incurs financial-distress/bankruptcy costs when selling too many forward contracts.[1]

In this way, financial-distress costs are analogous to those in Ross (1977) or costly education in Spence (1973). In the same way that agents in Spence (1973) may acquire too much education relative to the first-best benchmark, so do firms potentially over-hedge in our model. However, in our model hedging is not solely a wasteful signal. Instead, hedging plays a natural role in protecting the firm from costly financial distress that arises in low cash flow states of the world. Thus, in our model some level of hedging adds value to all

[1]Alternatively, the firm could enter into a large volume of long-term supply contracts, as in Almeida, Hankins, and Williams (2017), which would have a similar effect.

firms but it is the degree and type of instruments used that allows for credible signaling.

In our baseline model firms are short hedgers—e.g., gold miners or oil producers—with private information about production volume (results are similar for long hedgers, e.g., airlines). There are two types of firms, one with high volume and the other with low volume. Both types of firms experience financial distress costs if their cash flows are below an exogenous threshold (e.g., due to low spot prices), but these costs can always be avoided by selling forward their production, eliminating any price risk. Thus, in the first-best outcome no firm would experience financial distress.

In addition to having assets in place that generate a commodity output, firms also have a project opportunity, the value of which is type-dependent. Specifically, we assume the project is NPV-positive for high-volume firms only. Firms need to raise funds from outside investors to finance the project. In return, high-volume firms have an incentive to credibly signal their private information about their production volume. To shut down risk-shifting incentives from hedging (as in Smith and Stulz, 1985 or Chidambaran, Fernando, and Spindt, 2001), we assume the firm under analysis does not have limited liability; thus, the firm can be thought of as a division within a bigger organization. This assumption is somewhat realistic, since in some organizations hedging is not a centralized (Belk, 2002).

Importantly, we assume that a firm's derivatives positions are publicly observable to outside investors. This is a realistic assumption because public firms must report their material derivatives exposures in annual filings and this information would likely be made available to potential project financiers. Thus, in our model a firm's publicly observable derivatives trading can act as a signal. This is a significant departure from the models of DeMarzo and Duffie (1991), DeMarzo and Duffie (1995), and Breeden and Viswanathan (1998), where a firm manager uses risk management to reduce the noise in earnings announcements or dividend streams but not to directly signal private information.

Under certain conditions, the first-best hedging strategy is an equilibrium. A firm shorts its optimal number of forward contracts, and only the high-type firm obtains funding for its project. This socially efficient equilibrium obtains if it is too costly for the low-type firm to mimic the high-type firm, i.e., when the benefits of obtaining funding for the project are low compared to the expected financial-distress costs arising from an imperfect hedge. By mimicking with an imperfect hedge, the low-type firm experiences significant cash-flow volatility from its derivatives position increasing its expected financial distress costs, which are not offset by the payoffs from its (small) initial exposure to commodity prices.

Such an efficient equilibrium, however, does not always exist. For instance, if financial-distress costs are low, or if the volume difference across types is small, then the high type's first-best hedging strategy will not prevent mimicking by the low type. In these cases, and under certain parameter conditions, the only equilibria supported by plausible off-equilibrium beliefs are separating equilibria where high-type firms over-hedge relative to their optimal.[2] When over-hedging, the high-type firm has the "wrong" exposure to commodity prices, losing when the price is high. The over-hedging exposure can be significant and even imply a cash-flow volatility that is higher than if no hedging instruments where available. Additionally, for some choices of parameter values or with sufficient transaction costs, efficient separation can also be achieved by having the high type choose less hedging than the low type, or even possibly *buying* forwards instead of selling them (these equilibria also verify the intuitive criterion). In either situation, the expected volatility of the high-type firm will be higher than that of the low-type firm. This result is counter-intuitive because one naturally associates hedging policies with an objective of reducing volatility, and it highlights the perverse effect

[2]We refine off-equilibrium beliefs using the intuitive criterion of (Cho and Kreps, 1987). Using the D1 or "universal divinity" criterion of Banks and Sobel (1987) selects the same equilibria.

that signaling incentives can have on a firm's risk management strategy.

Our model demonstrates that signaling incentives can distort a firm's optimal hedging policies. This suggests that large derivatives positions (or hedge ratios significantly different than industry averages) are not necessarily motivated by speculation (Stulz, 1996) or risk-shifting (Myers, 1977), but potentially an equilibrium outcome of a signaling game. Our results suggests an alternative explanation for derivatives exposures that may appear too big or speculative from an ex post perspective. For instance, Joshi (2003) (page 13) argues:

> In particular, companies have a tendency to over-hedge, that is they buy so many derivative contracts that instead of hedging their risks they cancel out all their risk, and in addition create an exposure in the opposite direction. [...] For example, Ashanti, the gold mining company, "hedged" its exposure to falls in the gold price in such a way that they lost a huge amount of money when the price of gold increased.

The famous Metallgesellschaft case, where the company experienced large losses due to its hedging-related derivatives positions, could also be construed as being due to over-hedging (Edwards and Canter, 1995).

In an extension to the model, we allow firms to manage their risk with both forwards and options. We show that the optimal strategy for a high-type firm is to use a combination of both long put-options and short futures. Moreover, this strategy yields first-best outcomes that may not have been achievable when only forwards were available. Similar to the baseline model, the high-type firm separates by generating additional exposure to price volatility. The key difference is that a high-type firm can now optimally hedge with forwards and use the upfront costs and asymmetric payoffs of the options to more efficiently separate. Thus, a novel empirical implication of our model is that firms with better prospects have incentives to use both options and forwards, while firms with weaker prospects only use forwards.

As noted, our paper is closely related to Ross (1977), where leverage is used as a credible signal due to the existence of bankruptcy costs.[3] Leverage is essentially a way of taking on more risk, and high-type firms have more "risk capacity." Given this argument, one might expect that signaling incentives would, in equilibrium, lead to under-hedging (potentially not hedging at all); the high volatility from such a risk-management policy not being sustainable for the low type. Indeed, although there do exist signaling equilibria in our model with under-hedging, many times such equilibria are not as efficient as over-hedging for the high-type firm.[4] Over-hedging can be a more efficient way to signal because, for the same level of excess volatility imposed on the low type (in order to deter mimicking), there is less distortion of the high type's hedging policy. In other words, the most efficient over-hedging equilibrium entails a deviation from the high type's first-best number of derivatives contracts that is smaller than the most efficient under-hedging equilibrium. This is because, by construction, the high type has a higher volume to be hedged than the low type.[5]

An implication of our model is that it is hard to empirically identify the benefits of risk management. The findings that risk management adds value (Allayanis and Weston, 2001; Carter, Rogers, and Simkins, 2006; Mackay and Moeller, 2007; Gilje and Taillard, 2015) could be driven by unobserved heterogeneity (that is plausibly time-varying). Moreover, the model is consistent with either a positive or negative association between firm quality and hedging amount—sometimes efficient separation entails high-type firms hedging less than low-type firms, and sometimes the opposite. This ambiguity could explain why some papers find no association between risk management and firm value (Jin and Jorion, 2006).

[3]See Guedes and Thompson (1995) and Hennessy, Livdan, and Miranda (2010) for papers that build on the ideas in Ross (1977).

[4]In these cases the under-hedging equilibria do not survive the intuitive criterion of Cho and Kreps (1987).

[5]Our work also shares the spirit of other papers that—like Ross (1977)—adopt a signaling approach to explaining capital structure and dividends: Bhattacharya (1979), Myers and Majluf (1984), or Miller and Rock (1985); amongst others. However, hedging policies and how signaling interacts specifically with risk management is not the focus of these papers.

The arguments above notwithstanding, our model is consistent with Cornaggia (2013) and Gilje and Taillard (2015) who find that increased access to risk-management tools increases investment. In our model, the introduction of these tools allows firms to signal their prospects, resolving an asymmetric information problem that was preventing the financing of positive-NPV projects. Thus, our model suggests that the effect of risk management on investment may be due to signaling reasons and not solely because risk management allows for better liquidity management, as proposed in Froot, Scharfstein, and Stein (1993).

Finally, our paper also relates to other theory work on risk management. DeMarzo and Duffie (1995) and Breeden and Viswanathan (1998) develop models where hedging modulates how outsiders learn about project quality. In contrast to our paper, there is no signaling in these models. Furthermore, in DeMarzo and Duffie (1995) managers reduce their hedging when positions are public. This contrasts with our model, where managers potentially over- or under-hedge when hedging is publicly disclosed. More recently, Rampini and Viswanathan (2010) develop a model where risk management requires costly collateral. We extend the baseline model of our paper to include this type of cost. In this extension, the cost of collateral importantly affects which type hedges less under efficient separation. Adam, Dasgupta, and Titman (2007) develop a model of product-market competition and show that industry equilibrium requires firms to hedge differently from one another, which is a potential explanation for observed within-industry differences in hedging strategies (e.g., Delta and American in recent years). Our model suggests that such inter-industry differences can also be due to signaling motives and not just product market competition.

The paper proceeds as follows. Section 2 presents the main version of the model and characterizes the equilibrium. Section 3 analyzes some natural extensions. Section 4 concludes. All proofs are contained in the appendix.

2 Model

This section sets up the baseline version of the model and characterizes equilibria.

2.1 Setup

The economy comprises of two *types* $i \in \{L, H\}$ (low and high) of firms engaged in the production of the same good (e.g., barrels of oil). There are two *periods* $t \in \{0, 1\}$. High-type firms will end up producing *quantity* Q_H by $t = 1$, while low-type firms will end up producing quantity Q_L. Firm managers are privately informed about firm type and wish to maximize the expected value of the firm.

The *price* p at which the good will be sold at $t = 1$ is a random variable following a uniform distribution with support $[1-\sigma, 1+\sigma]$, with $\sigma \in [0, 1]$ governing *price volatility.* We choose the uniform distribution for ease of exposition, but we do not expect the main results to change under more general distributional assumptions. There is also a frictionless forward market for the good with delivery at $t = 1$. We denote as N_i the *number of contracts* shorted by firm i at time 0. In the baseline model, we rule out other more complex derivatives, such as options (but see section 3.2). This can be justified by higher transaction costs associated with such derivatives and/or managers understanding them less well.[6] Importantly, we allow the derivatives exposure of the firm N_i to be publicly observable. This matches the reality that firms must report their derivatives exposures in their annual reports and reflects that potential financiers of the firm could request this data as part of their due diligence process.[7]

All agents in the economy are risk-neutral and the risk-free rate is normalized to zero.

[6] It is worth noting that options are much less popular for hedging than forwards and swaps. See Bodnar, Giambona, Graham, Harvey, and Marston (2011).

[7] The Securities and Exchange Commission (SEC) requires public firms to disclose their material exposures to market risk sensitive instruments (including derivatives related to interest rates, foreign exchange rates, and commodity prices) in their annual filings. See Item 305 of SEC Regulation S-K.

For ease of exposition, we also abstract from storage costs and convenience yields.[8] Given our assumptions, the equilibrium forward price is then 1 (simply the mean of p).

We define firm i's baseline *cash flow* CF_i as the payoff at time 1 from selling the good, plus the income from the hedge:

$$CF_i := Q_i p - N_i(p-1) = (Q_i - N_i)p + N_i. \tag{1}$$

Baseline cash flow thus follows a uniform distribution as well, with mean Q_i. In particular, note that the support of the distribution for CF_i converges to $[-\infty, +\infty]$ as the number of forward contracts $N_i \to \pm\infty$.

In addition to the baseline cash flow, the firm has two other payoffs at $t = 1$. First, the firm is subject to a *financial distress cost* C if its baseline cash flow is below an exogenous *threshold* D (which we loosely refer to as "leverage"). This threshold could be a function of operational and financial leverage, which are not explicitly modeled. We denote realized financial distress costs by FD_i:

$$FD_i := \mathbb{1}_{\{CF_i < D\}} C. \tag{2}$$

Importantly, we are explicitly allowing firms to have unlimited liability from losses related to their hedging contracts. Thus, firms can be thought of as a division within a larger firm where the parent company is liable for all the losses. We make this assumption because (i) it simplifies thinking about a setting where the focus is valuable risk management, i.e., risk-shifting incentives—not the subject of our paper—are shut down; and (ii) it is not unrealistic, since there is evidence of risk management being done at the division level (see Belk, 2002).

Second, the firm has a *project opportunity* which pays off at $t = 1$ only if the firm is able

[8] Including these elements would complicate the pricing of the forward contract, without materially affecting our main arguments.

to raise and invest one unit of external capital at $t = 0$ (which is immediately spent, for instance by acquiring rights to a set of oil wells). For simplicity, we assume that financing is done via project finance, so the baseline cash flows (or other unspecified assets the firm owns) are disconnected from the claim held by the project financiers. Project value at $t = 1$ is also type- and price-dependent; thus, the project's cash flows at $t = 1$ are $\alpha Q_i p$, where $\alpha > 0$ governs how large the growth option is relative to baseline cash flows.[9] In the main version of the model, we assume this external financing takes the form of equity shares of the growth project. In a later section we explore the implications of having the project financed by debt. The main intuition and equilibrium properties are, however, similar.

Firms sell a *share* or fraction s of future claims on the project's cash flows and retain $1 - s$. The payoff at $t = 1$ associated with this growth option is then

$$G_i := \mathbb{1}_{\{\text{financed at } t=0\}}(1 - s_i)\alpha Q_i p, \tag{3}$$

where s is the equilibrium share that makes financiers break even. Intuitively, if there is separation, then $s_i = 1/(\alpha Q_i)$.

The timeline below summarizes decisions and payoffs.

$t = 0$

- Hedging decision N_i (publicly observable)
- If NPV-positive from investors' perspective, growth option is project-financed

$t = 1$

- Payoff components:

$$\begin{aligned} CF_i &= (Q_i - N_i)p + N_i \\ FD_i &= \mathbb{1}_{\{CF_i < D\}} C \\ G_i &= \mathbb{1}_{\{\text{financed at } t=0\}} \times \\ &\quad \alpha Q_i p(1 - s_i) \end{aligned}$$

[9] The price dependency of project value does not matter for the results in the main version of the model (equity financing), but it will matter if the project is financed with debt. See section 3.3.

An important ingredient for a firm's ex ante payoff is the probability of distress. Expressions in lemma 1 characterize this probability.

Lemma 1 *The probability of incurring financial distress is given by*

$$\Pr\{CF_i < D\} = \begin{cases} \frac{1}{2\sigma}\left[1+\sigma-\min\left\{1+\sigma, \frac{N_i - D}{N_i - Q_i}\right\}\right] & \text{if } N_i > Q_i \quad (4a) \\ \frac{1}{2\sigma}\left[\max\left\{1-\sigma, \frac{D - N_i}{Q_i - N_i}\right\} - (1-\sigma)\right] & \text{if } N_i < Q_i \quad (4b) \\ 0 & \text{if } N_i = Q_i. \quad (4c) \end{cases}$$

In the case when the firm is not hedging perfectly, Equations (4a) and (4b) show that the probability of distress increases in leverage D and volatility σ.

In order to focus on the interesting cases of the model, we make assumption 1 about project value.

Assumption 1 (Negative NPV of Low Types and Positive NPV of High Types) *The low type's project is NPV-negative and the high type's project is NPV-positive:*

$$\alpha Q_L < 1 < \alpha Q_H. \tag{5}$$

In addition to the assumption about project value, and in order to have a clear first-best benchmark, we make two more assumptions. First, we require hedging to be an optimal choice for firms under symmetric information about future production, which is guaranteed with assumption 2.

Assumption 2 (Distress Threshold) *The financial-distress-cost threshold is relatively low, in the sense that*

$$D < Q_L. \tag{6}$$

In this way, some level of hedging increases the chances of avoiding financial-distress costs. Note that if $D > Q_i$, then the firm maximizes the chances of avoiding financial-distress costs by taking a gamble with derivatives (setting $N_i = \pm\infty$ is optimal).

Second, we make an assumption regarding the size of financial-distress costs C. If such costs are not relatively high, then credible signaling is prohibitively expensive and separation may not be possible.[10] An intuitive way of setting such a threshold is to require that even high-type firms would prefer not hedging at all to taking an infinite position in derivatives (i.e., setting $N_i = \pm\infty$), which we think is a reasonable condition. When taking such a position (irrespective of the direction), the likelihood of experiencing financial-distress costs converges to 1/2 because the support of the baseline cash flow distribution converges to $[-\infty, +\infty]$. Therefore, a high-type firm, who retains $1 - 1/(\alpha Q_H)$ of the project with symmetric information, achieves a total payoff of

$$Q_H + \frac{C}{2} + \alpha Q_H \underbrace{\left(1 - \frac{1}{\alpha Q_H}\right)}_{=1-s_H}. \tag{7}$$

Thus, to make infinite positions undesirable, it is enough to impose the condition in assumption 3.

Assumption 3 (Financial Distress Costs) *Financial-distress costs are relatively high, in the sense that*

$$C > 2(\alpha Q_H - 1). \tag{8}$$

[10] It is straightforward to show that if C is smaller than

$$\frac{2Q_L(\alpha Q_H - 1)}{Q_H},$$

then a fully separating equilibrium is not possible.

2.2 Equilibrium characterization

This section studies the equilibrium of the game we set up above. We focus on Perfect Bayesian Equilibria, which are characterized by a level of hedging for each type and the financing contract, if any. Our setting is very similar to Spence (1973).

As is common in signaling games, there may be a large set of equilibria. Pooling equilibria where both types obtain funding, for instance, can be sustained by off-equilibrium-path beliefs that assign enough probability to the low type. However, in our setting, the standard equilibrium refinement proposed in Cho and Kreps (1987) (the intuitive criterion) selects only perfectly-separating equilibria. Using "universal divinity" D1 criterion of Banks and Sobel (1987) selects the same equilibria, since there are only two types. These results are described in proposition 1. Lemmas 2, 3, and 4 contain intermediate results.

Lemma 2 *Define $\overline{N}_i > Q_i$ ($\underline{N}_i < Q_i$) as the maximum (minimum) number of contracts shorted by firm i such that no financial-distress costs are experienced. Then*

$$\overline{N}_i = \frac{(1+\sigma)Q_i - D}{\sigma} \tag{9}$$

$$\underline{N}_i = \frac{D - (1-\sigma)Q_i}{\sigma} \tag{10}$$

Lemma 2 implies that optimal hedging is not uniquely defined, which follows from the fact that the price distribution is uniform. That is, firm i experiences no financial distress as long as its hedging level N_i is sufficiently close to its production level (i.e., $N_i \in [\underline{N}_i, \overline{N}_i]$), as illustrated in Figure 1. Such multiplicity leads to the existence of multiple payoff-equivalent equilibria, but this does not detract from the key economic implications of the model.

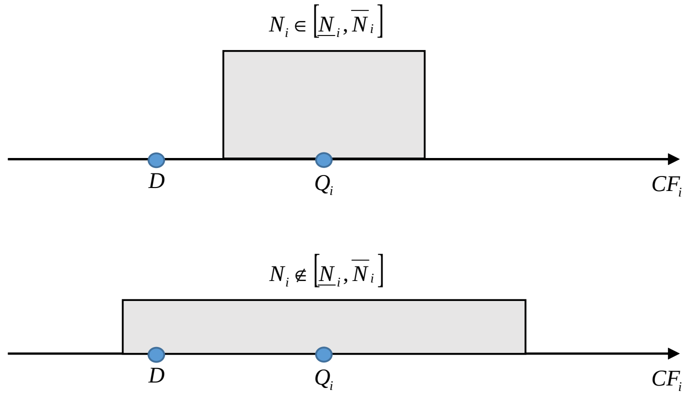

Figure 1: Hedging level, baseline-cash-flow distributions, and distress. The figure plots baseline-cash-flow distributions for two hedging strategies N_i. The top case has a number of forward contracts close enough to Q_i (i.e, $\in [\underline{N}_i, \overline{N}_i]$), such that the likelihood of experiencing financial distress is zero. The bottom case has either a too low or too high number of forward contracts (i.e., $\notin [\underline{N}_i, \overline{N}_i]$), such that the likelihood of financial distress is positive.

Lemma 3 *Define $N_H^{'} > Q_L$ as the number of contracts shorted by the high type that make mimicking a weakly dominated strategy for the low type. Then*

$$N_H^{'} = Q_L + \Delta, \tag{11}$$

where

$$\Delta := \frac{Q_L - D}{\sigma \left[1 + 2\frac{Q_L}{C}\left(\frac{1}{Q_H} - \alpha\right)\right]}. \tag{12}$$

Define $N_H^{''} < Q_L$ as the number of contracts shorted by the high type that make mimicking a weakly dominated strategy for the low type. Then

$$N_H^{''} = Q_L - \Delta. \tag{13}$$

Lemma 3 shows that there are two natural candidates for the least-costly separating equilibrium. The threshold $N_H^{'}$ corresponds to a situation where, in order to mimic, the low type would have to short a higher-than-ideal number of contracts, and where the excess volatility induced by such action would make the firm indifferent between mimicking or not. The threshold N''_H corresponds to the symmetric case, where the excess volatility is induced by requiring the low type's mimicking strategy to trade too few contracts—for example, the high type could choose to almost not use any hedging. Lemma 4 contains some comparative statics on Δ, i.e., the deviation from the first-best hedging strategy that is necessary for not mimicking to be incentive-compatible for the low type.

Lemma 4 *The deviation from the ideal hedging strategy Δ is decreasing in C, σ, and D; and increasing in α, Q_L, and Q_H.*

Most of the results in the above lemma are immediate from inspecting Equation 12. The minimum required deviation from optimal hedging (Δ) decreases if expected financial-distress costs increase, i.e., if costs C, volatility σ, and/or the threshold D are higher (higher signaling costs). On the other hand, if the returns to mimicking are higher, either because the project is relatively more valuable (higher α), or if the share sold to investors is lower (high α and/or high type production Q_H), then the deviation from first-best increases. The deviation also increases in the low type's production Q_L, since higher production by the low type not only makes the project more valuable, but also reduces the baseline probability of it incurring financial-distress costs.

The case where the high type chooses N''_H (low hedging) would seem the natural application of Ross (1977) to risk management, with hedging very little being the analogue of high leverage in Ross (1977). In our setting, the higher "risk capacity" of the high type would let it use *less* hedging. Also to this point, note that a corollary of lemma 2 is $\underline{N}_H < \underline{N}_L$, i.e., the high type can always avoid financial distress with a lower number of contracts than the low type. However, as described in proposition 1 below, these *under-hedging* strategies are not always an efficient way to obtain separation, and thus sometimes are not selected by the intuitive criterion.

Proposition 1 *The following characterizes equilibria that verify the intuitive criterion.*

1. *Such equilibria always exist, they are perfectly-separating, the strategy of the low type is* $N_L^* \in [\underline{N}_L, \overline{N}_L]$ *(optimal hedging), and the high type project is financed.*

2. *(*High-hedging *equilibria) Suppose that* $N_H'' < \underline{N}_H$*. Then the only equilibria that verify the intuitive criterion require the following strategy for the high type:*

   (a) $N_H^* = N_H'$*, if* $N_H' \geq \overline{N}_H$*;*

   (b) $N_H^* \in \left[\max(\underline{N}_H, N_H'), \overline{N}_H\right]$*, otherwise.*

3. *(*Low-hedging *equilibria) Suppose that* $N_H'' \geq \underline{N}_H$*. Then the high-hedging equilibria still exist and verify the intuitive criterion; but there is also a continuum of low-hedging equilibria that are equally optimal and verify the intuitive criterion as well, with* $N_H^* \in [\underline{N}_H, N_H'']$*.*

The key takeaway from proposition 1 is the following: according to the model, it seems reasonable to expect firms to *over-hedge* for signaling reasons, in the sense that there always exist equilibria where the high-type firm hedges more than its production volume ($N_H^* > Q_H$), and, sometimes, there are no other equilibria (point 2.a. in the proposition).[11] Over-hedging is a relatively efficient way to credibly signal private information because, for the same level of volatility imposed on a low-type firm contemplating mimicking, it minimizes

[11] Although in the model over-hedging involves hedging more than production, similar intuition would apply if the hedge-ratio in the first-best strategy is not 1 (i.e., if under full information $N_H^* \neq Q_H$). The high type would still find it weakly more efficient to separate by choosing to hedge more than its optimal relative to less. The hedge-ratio in the first-best strategy is 1 because the futures contracts are fairly priced, contain no basis risk, and managers are risk neutral. These assumptions are maintained for simplicity.

the distance to the high-type firm's first-best strategy. This result suggests that firms can have the "wrong" exposure to commodity prices, losing when the price goes up,[12] for reasons other than speculation or risk shifting.[13]

The argument for equilibrium over-hedging notwithstanding, one cannot ignore efficient low-hedging equilibria, where the high-type firm hedges less than the low-type firm (note that $N_H'' < \underline{N}_L$). Although in such cases high-hedging equilibria also exist, this will change once we introduce additional costs of hedging (see section section 3.1). Also note that in under-hedging equilibria it could be the case that $N_H < 0$, i.e., the high-type firm *buys* forward contracts instead of selling them. This behavior, which is rational in our signaling model, could be potentially misconstrued as speculative.

The model also provides some additional interesting results. First, note that if $N_H^* > 2Q_H$, then the variance of the firm's baseline cash flows is higher than if risk management did not exist. The variance of the baseline company cash flows is

$$Q_H^2 var(p) = \frac{(Q_H \sigma)^2}{3}.$$

[12]Or losing when the price goes down, for long hedgers.

[13]Readers may be concerned that hedging is not an exclusive activity in that low types could always mimic high types when raising funding and then undo risk-management practices immediately following to avoid financial distress. Although a valid concern, we feel that by including additional complications to the model this concern can be avoided without changing the intuition. Specifically, investors could included in their contracts that certain risk-management policies, i.e. hedging levels be maintained. Or, instead of hedging using financial forward firms could sign long-term supply contracts which would have penalties for violating the contract (Almeida et al., 2017). Alternatively, with an additional period between investment and production, managers could be concerned with intermediate changes to their stock price which would result in a negative shock to low-quality firms if the manager attempted to deceive. These additional extensions are currently under consideration.

With risk management, the variance is

$$(Q_H - N_H)^2 var(p) = \frac{(Q_H - N_H)^2 \sigma^2}{3},$$

meaning the variance with risk management is higher if $N_H > 2Q_H$, which obtains in equilibrium with certain parameter configurations. This implies that a company's "hedging strategy" could lead to an increase in the volatility of the company's cash flows.

Another interesting and related result is that even if risk management were not necessary to avoid distress for any firm (i.e., if $D < (1 - \sigma)Q_L$), it still may occur in equilibrium. This suggests that we may observe companies employing what appear to be risk-management strategies even if the risks of facing financial distress appear to be relatively low or nonexistent. Furthermore, it is possible that not only will risk management appear to be used, but it could be valuable for the high-type firm to face some level of expected distress costs, even though without "risk management" distress would never occur.

2.3 Numerical illustration

This section presents a numerical illustration that is useful for providing more intuition for the results. Figure 2 plots key thresholds associated with varying Q_H, the high type's volume. For all Q_H presented in the figure, assumptions (1)-(3) are verified.

The top panel of Figure 2 plots $\overline{N}_H$ (i.e., the maximum number of contracts traded by the high-type firm without incurring financial distress—see lemma 2) and N_H' (i.e., the minimum number of contracts traded by the high type that prevent mimicking—see lemma 3). For low production volume Q_H, $N_H' < \overline{N}_H$, meaning that even without any potential financial distress the high-type firm deters the low-type firm from mimicking. This happens because

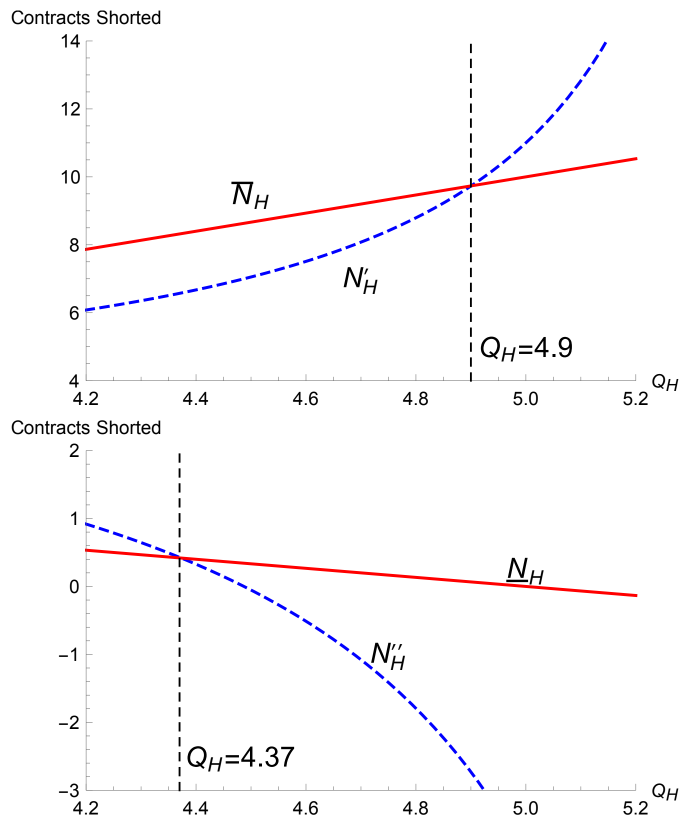

Figure 2: Equilibrium regions — numerical illustration. The top panel plots, for varying Q_H: $\overline{N}_H$, i.e., the maximum number of contracts shorted by the high type such that it does not incur financial distress; N'_H, i.e., the minimum number of contracts (shorted by the high type) above Q_L that is incentive-compatible with the low type not mimicking. The bottom panel plots, for varying Q_H: $\underline{N}_H$, i.e., the minimum number of contracts shorted by the high type such that it does not incur financial distress; N''_H, i.e., the maximum number of contracts (shorted by the high type) below Q_L that is incentive-compatible with the low type not mimicking. Parameter values: $Q_L = 3.5$, $\alpha = 0.24$, $D = 2$, $\sigma = 0.6$, and $C = 0.42$.

the return to mimicking (funding the project) is relatively low when Q_H is low: the share sold to financiers $s_H = 1/(\alpha Q_H)$ is quite high. As Q_H increases, the returns to mimicking increase in a convex fashion, since $s_H = 1/(\alpha Q_H)$ is convex in Q_H.

The threshold $\overline{N}_H$ also increases with Q_H, since the the distribution of baseline cash flows shifts to the right, that is, it becomes less likely to hit the financial-distress threshold D. However, $\overline{N}_H$ increases only linearly with Q_H (see lemma 2).

As the top panel of the figure shows, for high enough production ($Q_H \geq 4.9$), $N'_H \geq \overline{N}_H$.

This means that all (high-hedging) equilibria that verify the intuitive criterion have the high-type firm playing a unique strategy, namely $N_H^* = N_H^{'}$. This is a result of the high-type firm having to incur some expected financial distress to prevent mimicking, and so it will choose the strategy that carries the least amount of expected financial-distress costs. If production is lower ($Q_H < 4.9$), then the high-type firm can choose strategies between the two lines in the top panel of Figure 2: any works well in the sense that (i) it does not imply any financial-distress costs, and (ii) the low type's no-mimicking condition is verified (even if slack).

The bottom panel of Figure 2 shows that for relatively low production ($Q_H < 4.37$), it is possible for the high-type firm to choose a low-hedging strategy that disincentivizes the low-type firm from mimicking (i.e., lower than $N_H^{''}$), while no financial-distress costs are expected (since $\underline{N}_H < N_H^{''}$.) This means that efficient separation can be obtained by choosing N_H between the two lines. While high-hedging efficient equilibria also exist in the region $Q_H < 4.37$, these can disappear once additional costs of hedging are introduced. For instance, suppose that $N_H = 0$ is a low-hedging strategy that achieves efficient separation. Then maybe the high-type firm is better off with this strategy than a high-hedging one, if $N_H = 0$ saves the managerial time and collateral necessary to enter a derivatives' trade in the real world. We explore this issue further in section 3.1.

3 Extensions

3.1 Costly hedging

This section explores the effect of adding a cost to implementing a hedging strategy. Considering such a cost is realistic, since there are opportunity costs associated with (i) allocating worker time to manage derivatives positions, and (ii) allocating valuable collateral to margin accounts or other forms of insuring counter-parties (as argued in Rampini and Viswanathan, 2010). Furthermore, derivatives' trades have other transaction costs.

The main takeaway of the section is that with this new cost, under-hedging can become the unique efficient separating equilibrium.

First, we formalize the costly-hedging setup and derive the new first-best (section 3.1.1). Second, we extend the numerical example of section 2.3 to illustrate the key takeaway with regards to efficient separation (section 3.1.2).

3.1.1 Costly hedging: setup and first-best

We follow the same setup as the original benchmark model, but add a *hedging cost* HC_i, paid at $t = 1$, that scales linearly with the absolute number of contracts

$$HC_i := \eta |N_i|. \tag{14}$$

The company's baseline cash flows then become

$$CF_i := (Q_i - N_i)p + N_i - \eta |N_i|. \tag{15}$$

The remaining setup, specifically with regard to financial distress, project financing, and the value of the growth option, are the same. We also maintain the same assumptions as before and employ an additional one.

Assumption 4 (Hedging Costs) *Hedging is not too costly, in particular*

$$Q_L(1 - \eta) > D. \tag{16}$$

This implies that when fully hedging production, either type of firm can still avoid financial distress. Note that this assumption also implies that $\eta < 1$, since $D > 0$.

The probability of incurring financial distress now depends on the size of the firm Q_i, the level of hedging N_i, the distress threshold D, and the cost of hedging η. The lemma below is presented without proof, since it is analogous to lemma 1 (basically replacing D with $(D - \eta|N_i|)$).

Lemma 5 *The probability of incurring financial distress is given by*

$$\Pr\{CF_i < D\} = \begin{cases} \frac{1}{2\sigma}\left[1 + \sigma - \min\left\{1+\sigma, \frac{N_i - \eta|N_i| - D}{N_i - Q_i}\right\}\right] & \text{if } N_i > Q_i \quad (17a) \\ \frac{1}{2\sigma}\left[\max\left\{1-\sigma, \frac{D - N_i + \eta|N_i|}{Q_i - N_i}\right\} - (1-\sigma)\right] & \text{if } N_i < Q_i. \quad (17b) \\ 0 & \text{if } N_i = Q_i. \quad (17c) \end{cases}$$

Next, we characterize the first-best level of hedging for a given type. To find this, it is useful to note that the expected value of a firm for a given level of hedging N_i is

$$Q_i - \eta N_i - \Pr\{(Q_i - N_i)p + N_i(1-\eta) < D\}C + \mathbb{1}_{\{\text{financed at } t=0\}}(1-s_i)\alpha Q_i. \qquad (18)$$

In the equation above, we write ηN_i instead of $\eta|N_i|$, since naturally the first-best level of hedging cannot be negative, given the existence of hedging costs. From Equation 18, we can see that the level of hedging only affects the expected distress and hedging costs. This is because the firm's expected baseline revenues from production and the value of the growth option are independent of the level of hedging. Therefore, the firm's first-best level is the level that minimizes the combination of expected-distress and hedging costs. Furthermore, we can see that contrary to the case with no hedging costs (i.e., $\eta = 0$), the first-best level of hedging is always less than the production of the firm.

Lemma 6 *The first-best level of hedging is always such that $N_i < Q_i$.*

To find the first-best level of hedging, it is useful to first denote the minimum number of hedging contracts shorted by a given type such that $\Pr\{CF_i < D | N_i\} = 0$ (i.e., no financial distress costs are experienced). Note that this is a rewriting of lemma 2, with the inclusion of hedging costs.

Lemma 7 *Define $\underline{N}_i$ as the maximum of 0 and the minimum number of contracts shorted by firm i such that no financial distress costs are experienced. Then*

$$\underline{N}_i := \begin{cases} \max\left\{0, \dfrac{D-(1-\sigma)Q_i}{\sigma-\eta}\right\} & \text{if } \sigma > \eta \quad (19a) \\ 0 & \text{if } \eta > \sigma. \quad (19b) \end{cases}$$

A corollary of the lemma above is that minimum number of contracts to avoid financial distress for high types ($\underline{N}_H$) is weakly less than that for low types ($\underline{N}_L$). This is because the risk capacity of the high type is higher than that of the low type, meaning that the high type can afford to weakly hedge less and still avoid financial distress. Note however, that it is possible the minimum for both firms is 0. This occurs if either type of firm could avoid financial distress with no risk management.

Finally, we characterize the first-best level of hedging for the firm.

Lemma 8 *The first-best level of hedging for firm i is:*

$$N_i^* := \begin{cases} \dfrac{D-(1-\sigma)Q_i}{\sigma-\eta} & \text{if } \left[\frac{D-(1-\sigma)Q_i}{\sigma-\eta}\right]\eta \leq \frac{C}{2\sigma}\left[\max\left\{1-\sigma, \frac{D}{Q_i}\right\} - (1-\sigma)\right] \quad (20a) \\ 0 & \text{otherwise.} \quad (20b) \end{cases}$$

3.1.2 Costly hedging: numerical example

This section extends the numerical example from section 2.3, to illustrate how hedging costs affect efficient separation. Specifically, we choose the same parameters as those in section 2.3 and focus on the case where production for the high-type firm is $Q_H = 4.3$. Recall that given this Q_H, the low- and high-hedging equilibria are equally efficient when there are no hedging costs (see Figure 2). Once hedging costs are introduced, we show that only the low-hedging strategy separates efficiently.

The solid red lines in Figure 3 plot the (ex ante) payoff for each type for varying levels of hedging N_i, and considering that the project is undertaken. The left panels show the case without hedging costs, to facilitate comparison, and the right panels set hedging costs $\eta = 0.03$. The top (bottom) panels refer to the high (low) type.

Consider first the benchmark case with no hedging costs. The dashed horizontal blue lines in the top and bottom panels represent the "reservation utility" for each type, meaning the payoff associated with the first-best hedging strategy (defined in lemma 8) without the project. We refer to these reservation utilities as $U_{0,i}$. The bottom-left panel shows that the low-type firm has an incentive to deviate and mimic if the hedging level of the high-type firm (N_H) is too close to the low-type firm's production (i.e., $N_H \in [N''_H, N'_H]$). However, the top-left panel shows that the high-type firm can easily choose to hedge an amount outside of this interval and still not experience any financial-distress costs (the flat portion of the solid red line), which corresponds to the efficient separating equilibrium.

When we add hedging costs, the left panels in Figure 3 show that firm payoffs (solid red lines) rotate clockwise; this happens because hedging costs are proportional to the (absolute) number of contracts traded. The interval $[N''_H, N'_H]$ still corresponds to the hypothetical strategies adopted by the high-type firm that would make mimicking a profitable deviation.

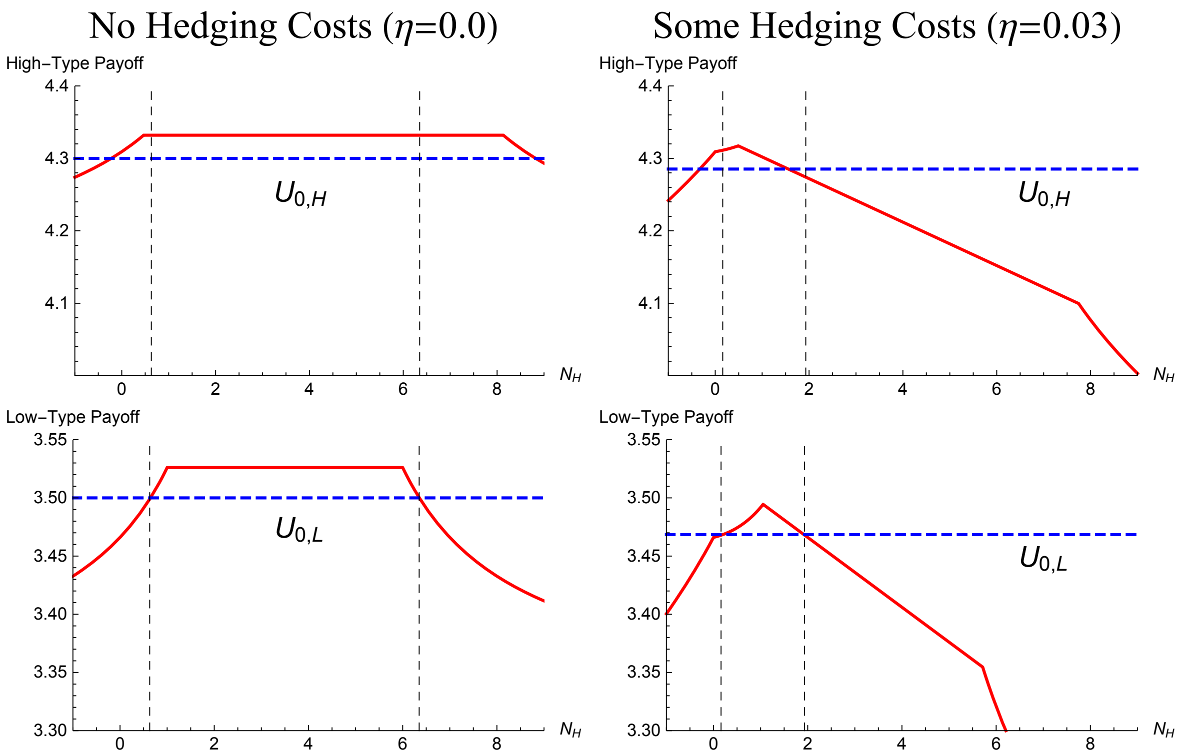

Figure 3: Payoffs and hedging strategy (N_i). This figure plots, for varying levels of hedging N_i, the ex ante payoff for the high type, assuming separation (top panels); and for the low type, assuming successful mimicking (bottom panels). The horizontal dashed lines in blue represent the reservation payoff for each type $U_{0,i}$, where a first-nest level of hedging is chosen, but there is no project. The left panels depict the case from the benchmark model without hedging costs (see Figure 2); the right panels set the hedging cost η at 0.03. Parameter choice: $Q_L = 3.5, Q_H = 4.3, \alpha = 0.24, D = 2, \sigma = 0.6, C = 0.42$. The vertical lines depict the thresholds for the level of hedging by the high type that guarantees separation.

However, unlike the case with no hedging costs, a high-hedging strategy is not desirable by the high-type firm; in fact, for the example given, choosing N_H' is worse than the high-type firm's reservation utility. On the other hand, an efficient separating equilibrium exists at N_H'' (the high-type firm distorts its optimal strategy downwards), which makes the low-type firm indifferent between mimicking or not, and produces a higher payoff for the high-type firm than its reservation utility. Finally, we note that in this equilibrium, the hedging strategy of the high-type firm (≈ 0.2) is well below that of the low-type firm, who picks its first-best hedging strategy (≈ 1.1). Note however, that for different parameter assumptions the over-hedging strategy of the high-type firm is the most efficient and will be the unique LCSE. This result illustrates that either over- or under-hedging can be the expected equilibrium outcome with the inclusion of hedging costs. Further, that with the inclusion of hedging costs, over a wide-range of parameters high-type firms will have a higher expected volatility than low-type firms.

3.2 Hedging with options

We now extend our framework by allowing firms to potentially use both options and forwards/futures to manage their risk. For simplicity, we restrict our analysis to long-put positions only, where each put option gives the firm the right to sell one unit of the commodity at the *strike price* K. These options have a natural application for a producing firm, in that they allow the firm to hedge away risk associated with low price realizations. As we will show, the introduction of put options allows high-type firms to separate from low-type firms in a more efficient way.

First we fix notation. Denote the *number of options* used by type i as O_i and the *price of an option* with a strike price of K as $P(K)$. Given our distributional assumptions on the price of the commodity, and assuming a competitive options market, the price of a put option with strike price K has the following solution:[14]

$$P(K) = \frac{[K - (1-\sigma)^2]}{4\sigma} \tag{21}$$

[14]This is found by taking the expected value of the option conditional on being in the money multiplied by the probability the option is in the money. The algebra is simple and thus is omitted.

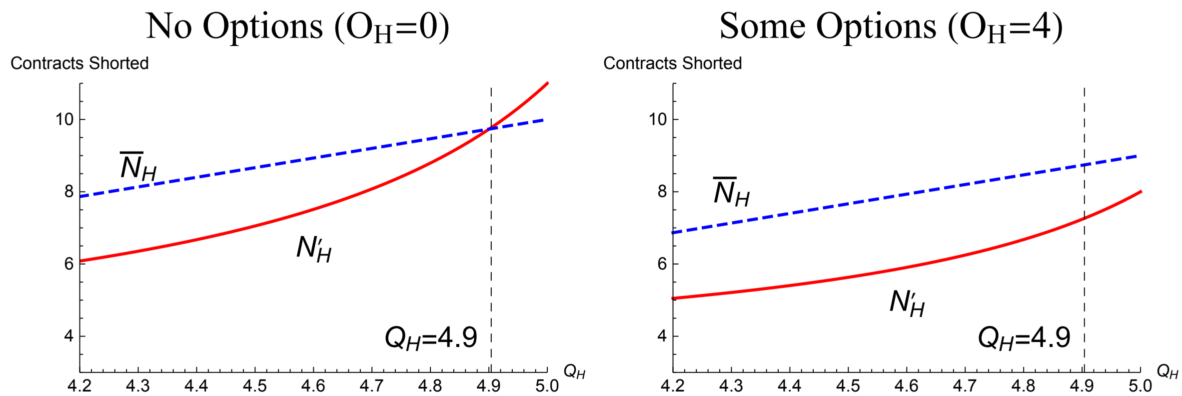

Figure 4: Equilibriums with Options. Both panels plot, for varying Q_H: $\overline{N}_H$, the maximum number of futures contracts shorted by the high type such that financial distress does not occur; and N'_H, the minimum number of contracts shorted by the high type (above Q_L) that is incentive-compatible with the low type not mimicking. In the left panel the high type uses no options, in the right panel 4 put positions are employed. Parameter values: $Q_L = 3.5$, $\alpha = .24$, $D = 2$, $\sigma = .6$, $C = .42$, and $K = 1$.

The baseline cash flows from Equation 1 are thus re-defined as

$$CF_i := (Q_i - N_i)p + N_i - O_i P(K) + \max\{0, K - p\}O_i. \tag{22}$$

Using the same notation as before, we can solve for a new $\overline{N}_i$, i.e., the maximum number of contracts that can be purchased while still avoiding any financial distress (see lemma 2). Similarly, we can re-compute N'_H, i.e., the minimum number of forward contracts that assures no mimicking by the low type (see lemma 3). Note that both $\overline{N}_i$ and N'_H now depend on the number of options O_i and their strike price K. The main result is presented with the aid of a numerical example, depicted in Figure 4. The example follows the one presented earlier in section 2.3.

The left panel of Figure 4 corresponds to the top panel of Figure 2. Recall that in the earlier figure, we showed that for $Q_H > 4.9$, the efficient separating equilibrium has the high-type firm over-hedge ($N_H > \overline{N}_H$), implying some expected financial-distress costs. The right panel shows what happens when options are allowed, and depicts a case where the high-type firm is long 4 put options with strike price $K = 1$.

First, note that there is a downward shift in $\overline{N}_H$, meaning the limit of forward contracts that the firm can short—without experiencing financial distress—goes down. This occurs because options are costly (analogous to the costly-hedging section). If the firm shorts many forward contracts, then it cannot afford to lose much when prices are *high*, and there is less of a cushion for these losses if the firm buys options. However, this strategy makes it easier to disincentivize the low-type firm from mimicking because the number of forwards needed to deter mimicking $N_H^{'}$ also shifts downward, and more so than $\overline{N}_H$. In fact, the high-type firm no longer faces any expected financial distress in the cases when the high-type firm's production is sufficiently high ($Q_H > 4.9$), as depicted in the figure.

The option strategy is efficient at separating because spending funds on options has a much stronger negative effect on the risk capacity of low-type firms than on the risk capacity of high-type firms. This is not an obvious result *a priori*, since one could easily conjecture that with options the intersection of $\overline{N}_H$ and $N_H^{'}$ in Figure 4 would occur at lower, and not higher production, Q_H.

Moreover, we argue that there always exists an efficient hedging strategy for high-type firms such that they can separate from low-type firms and have no exposure to financial distress.

Proposition 2 (First-Best with Options and Futures) *There exists a strike price $\hat{K} \in [1-\sigma, 1+\sigma]$ such that the high-type firm can sell futures $N_H = Q_H$ and purchase put options $\hat{O}_H = \frac{(Q_H - D)4\sigma}{[\hat{K}-(1-\sigma)]^2}$ that provides the first-best outcome.*

The sketch of this proof is as follows. A high-type firm first sells forward all of its production. Selling forward its production guarantees a payoff of at least Q_H. It then can choose an option strategy $\hat{O}_H$ such that given the strike price K, it will never experience financial distress, i.e., $CF_H > D$ for all $p \in [1-\sigma, 1+\sigma]$. Since the options are competitively priced, this means the high type's expected cash flows are Q_H, even though its realized cash flows depend on whether the option expires in or out-of-the-money. Finally, it chooses a strike price $\hat{K}$ such that the low type always has an expected cost of distress greater than its potential benefit from mimicking. This strike price $\hat{K}$ always exists.

The intuition is similar to that of the numerical example. Since the low-type firm is over-hedging if it mimics (i.e., $Q_H = N_H > Q_L$), when prices are high the low-type firm is losing money due to over-hedging its production. This alone may not prevent the low type from mimicking if the likelihood of financial distress for the low type is too low, as discussed in section 2. Options, however, further reduce the low-type firm's cash flows when prices are high because of the initial premium expended on options, which then go on to expire out-of-the-money. In return, option purchases further reduce the low-type firm's cash flows when prices are high. Moreover, because a high-type firm does not face costs from over-hedging, it can always purchase just enough options such that the low type always experiences financial distress if the options expire out-of-the-money, while at the same time still avoiding costly distress. Thus, in expectation the high-type firm's cash flows are still first-best, Q_H, and the low type always finds it too costly to mimic. Intuitively, the asymmetric payoff of the options causes low cash flows to occur precisely when they are most costly for a low-type firm mimicking a high-type firm that is fully-hedging its production via forwards (i.e, when prices are high). Importantly, note that although the high type's expected cash flows are equivalent to the full information benchmark, the volatility of cash flows is higher than the full-information benchmark.

Finally, we want to point out that this extension suggests a novel empirical implication.

Specifically, an efficient separating equilibrium has the low-type firm eliminate price risk using forwards, but the high-type firm potentially using both forwards and options in order to separate. Thus, we should see firms that use options show subsequent better performance (revealing their type); and, that firms that use options invest relatively more.

3.3 Debt project finance

In this section we replace the mode of project financing—equity in the main version of the model—with debt. If the project is financed, investors provide one unit of capital against the promise of a (maximal) payment of F in the future, where F represents the *face value* of debt. Therefore, the value of the project for the firm is now

$$G_i := \mathbb{1}_{\{\text{financed at } t\,=\,0\}} \max\left\{0, \alpha Q_i p - F_i\right\}, \tag{23}$$

where F_i is the equilibrium face value of debt that makes competitive financiers break even.

To facilitate exposition, we focus on risk-free debt,[15] meaning that even at the worst possible realization of p, the project pays off at least 1 (the initial investment). This assumption implies $F = 1$ and places a restriction on the maximum level of volatility:

Assumption 5 *Project debt is risk-free. Formally,*

$$\alpha Q_H(1-\sigma) \geq 1 \Leftrightarrow \sigma < 1 - \frac{1}{\alpha Q_H}. \tag{24}$$

Also, in the interesting case to analyze, the most successful project outcome produces some value for the low type. This imposes a restriction on the minimum level of volatility:

[15] Results with risky debt (not shown) are similar.

Assumption 6 *The project is valuable for the low type ex ante. Formally,*

$$\alpha Q_L(1+\sigma) > 1 \Leftrightarrow \sigma > \frac{1}{\alpha Q_L} - 1. \tag{25}$$

In a separating equilibrium, the value of the project for the high-type firm is the same as in the main version of the model (the NPV). The value of the project for the low-type firm is, however, different. This affects mimicking incentives.

Lemma 9 *The expected value of the project for the low-type firm is*

$$\overline{G}_L := \Pr\{G_L > 0\}\mathrm{E}[G_L | G_L > 0] = \frac{[\alpha(1+\sigma)Q_L - 1]^2}{4\alpha\sigma Q_L}. \tag{26}$$

Furthermore, it is true that: (i) the value of the project $\overline{G}_L$ *increases with volatility; and (ii) the value of the project* $\overline{G}_L$ *is lower than with equity financing.*

The fact that project value increases with volatility is intuitive, since the project is an option. The fact that project value is lower with debt financing (for the low type) follows from there being a low probability that the option is exercised, since the project has negative NPV. This result makes separation potentially less costly than with equity financing (echoing the pecking-order theory of Myers and Majluf, 1984), as described in proposition 3.

Proposition 3 *Under debt financing, the number of contracts shorted by the high-type firm that (just) prevent the low-type firm from mimicking are* $Q_L \pm \Delta_d$*, with*

$$\Delta_d := \frac{Q_L - D}{\sigma - \frac{[\alpha Q_L(1+\sigma)-1]^2}{2\alpha C Q_L}}. \tag{27}$$

Furthermore, this is a lower distortion than with project equity financing, i.e., $\Delta \geq \Delta_d$*.*

3.4 Long hedgers

As written above, our model describes a situation where firms are net producers of a commodity, implying their hedge is to sell forward contacts. In order to show that this is not crucial to the implications of the model, this section discusses the case where firms are net buyers of the commodity (e.g., an airline). The setup is quite similar to the case where the agents are producers and the implications are the same.

The timing of the model and distribution of commodity prices are the same as before. The key difference is now the firm's baseline cash flows consist of some additional revenue that is not directly tied to the price of the commodity. For simplicity, we assume that the revenues of the firm are independent of the price of the commodity but dependent on the production volume of the firm, which is still private information. We denote the baseline revenues in $t = 1$ as γQ_i. Further, we assume that a firm with production Q_i needs to buy Q_i units of the commodity in order to make revenues of γQ_i by $t = 1$. If we think of firms as airlines, then Q_i represents the future fuel requirements, with a simplifying assumption that an airline needs one unit of fuel to produce revenues of γ per unit of fuel bought.

Similar to before, the firm can enter into forward contracts to hedge the price at which they buy the commodity. Thus, the firm purchases N_i forward contracts, and the equilibrium forward price is still 1. We define firm i's baseline cash flow as the payoff at time 1 from selling their good, buying the commodity, plus the income from the hedge

$$CF_i := Q_i\gamma - Q_i p + N_i(p - 1). \tag{28}$$

In addition to the baseline cash flow, we still assume the firm has two other payoffs at $t = 1$. The first are the financial distress costs C if baseline cash flows are below the threshold

D. The second is the firm's growth option if the firm is able to raise and invest one unit of external capital at $t = 0$. Project value at $t = 1$ is still type- and price-dependent with the following modification. The value of the firm's growth option, however, is now: $\alpha Q_i(\gamma - p)$, where $\alpha > 0$ governs how large the growth option is relative to baseline cash flows. Again, we assume project-level financing and focus on the case with equity financing. Firms issue a share s of future claims on the project's cash flows and retain $1 - s$. The payoff at $t = 1$ associated with this growth option is then

$$G_i := \mathbb{1}_{\{\text{financed at } t=0\}}(1 - s_i)[\alpha Q_i(\gamma - p)], \tag{29}$$

where s is the equilibrium share that makes competitive financiers break even. Intuitively, with separation, $s_H = 1/[\alpha Q_H(\gamma - 1))]$.

To avoid repeating analyses that are very similar to the baseline version of the model, we argue somewhat informally that long-hedging and short-hedging produce the same results. First, note that if we define a variable

$$Q_i^{'} := Q_i(\gamma - 1), \tag{30}$$

then the value of the project for both types has the same functional form as before (take expectations in Equations 3 and 29). Using $Q_i^{'}$, it is also straightforward to adapt the NPV conditions from assumption 1 and the leverage condition from assumption 2.

Finally, compare the baseline-cash-flow processes for the two cases (from Equations 1 and 28, respectively):

$$\text{(short hedging) } CF_i = Q_i + Q_i(p - 1) - N_i(p - 1) \tag{31}$$

$$\text{(long hedging) } CF_i = Q_i(\gamma - 1) - Q_i(p - 1) + N_i(p - 1). \tag{32}$$

Although the signs on $(p - 1)$ flip, in both cases we have baseline cash flows following a uniform distribution, and where the support converges to $[-\infty, +\infty]$ as $N_i \to \pm\infty$. This means that in both cases (short- and long-hedging), when $N_i \to \pm\infty$, the probability of distress converges to 1/2. With our assumption 3 about a minimum level for costs of financial distress C, this implies that a separating equilibrium is sure to exist for both short and long hedging (see proposition 1 and its proof for details).

In short, the equilibrium properties of long-hedging are the same as those of short-hedging.

4 Conclusion

We develop a simple model to understand how signaling incentives may distort hedging behavior. In equilibrium high-volume firms may take on large derivatives positions, even though there is no speculative or risk-shifting motive to do so. Our model emphasizes the difficulty associated with empirically measuring the benefits of risk management; and is also consistent with mixed evidence, in earlier empirical studies, on whether increased risk management adds value. In addition, we provide a new channel through which the introduction of risk management instruments can increase investment by firms, a result supported by previous empirical studies. The new instruments allow firms with postive-NPV investments to signal their prospects and overcome information asymmetry problems that would have prevented them from receiving needed external financing.

We show that our model produces an additional and novel testable empirical prediction with regard to the types of derivatives a firm uses in its risk management strategy. Specifically, firms with better prospects have an incentive to use both forwards and options, while firms with reduced prospects will only use forwards. Thus, our model provides an explanation for both heterogeneity in the degree and types of instruments used in risk management, as noted in other empirical studies of risk management practices.

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Appendix (Proofs)

Proof of lemma 1.

If $N_i = Q_i$, then baseline cash flows become non-random. Using Equation 1, and if $N_i < Q_i$, then the probability that financial distress is incurred is given by

$$\Pr\{(Q_i - N_i)p + N_i < D\} = \Pr\left\{p < \frac{D - N_i}{Q_i - N_i}\right\} = \int_{1-\sigma}^{\max[1-\sigma,(D-N_i)/(Q_i-N_i)]} \frac{1}{2\sigma}\, dp,$$

which simplifies into Equation 4b. Calculation is similar for Equation 4a, and thus its explicit proof is omitted.■

Proof of lemma 2.

The expressions are obtained by equalizing the worst realization of baseline cash flow to D.■

Proof of lemma 3.

To obtain Equation 11, we simply set the low type's payoff associated with efficient hedging equal to the payoff of mimicking:

$$Q_L = Q_L - \frac{1}{2\sigma}\left(1 + \sigma - \frac{N_H' - D}{N_H' - Q_L}\right) C + \alpha Q_L \left(1 - \frac{1}{\alpha Q_H}\right), \tag{A.1}$$

where the LHS of Equation A.1 is the payoff from perfect hedging ($N_i = Q_i$) and the RHS is the payoff from mimicking the high type, which entails some financial-distress costs plus the value associated with the project. Simplifying yields Equations 11 and 12. Equation 13 is obtained similarly.■

Proof of lemma 4.

Note that

$$\frac{1}{Q_H} - \alpha < 0,$$

following assumption 1 (high types have positive-NPV projects). Signing the derivatives is then trivial.■

Proof of proposition 1.

First we rule out all equilibria where both types play the same strategy N^* with positive probability (this takes care of pooling and semi-pooling equilibria). Conjecture the existence of a pooling equilibrium where the project is financed. Denote by s^* the equilibrium share sold to investors, when they observe a firm hedge N^*. Then the utility of each type U_i is given by

$$\begin{aligned} U_L^* &:= Q_L - C\,\Pr\{CF_L < D\} + \alpha Q_L(1 - s^*) \qquad \text{(A.2)} \\ U_H^* &:= Q_H - C\,\Pr\{CF_H < D\} + \alpha Q_H(1 - s^*). \qquad \text{(A.3)} \end{aligned}$$

We claim that we can find an $N_0 \geq Q_H$ such that

$$U_L^* = Q_L - \frac{1}{2\sigma}\left[1 + \sigma - \left(\frac{N_0 - D}{N_0 - Q_L}\right)\right] C + \alpha Q_L(1 - s_{min}), \qquad \text{(A.4)}$$

where s_{min} is the best possible share asked by investors (i.e., $1/(\alpha Q_H)$, corresponding to the belief that the firm is a high type with probability 1). To show that such N_0 exists, it is enough to show that, as $N_0 \to \infty$ (maximal distress costs), the LHS of Equation A.4 is greater than the RHS. Furthermore, since it must be that $U_L^* \geq Q_L$ in equilibrium (the low type can always choose an efficient hedging strategy), then it is enough to show

$$0 > -\frac{C}{2} + \alpha Q_L(1 - s_{min}),$$

which is true given assumption 3 on the minimum level of C:

$$C > 2\alpha Q_H(1 - s_{min}) > 2\alpha Q_L(1 - s_{min}).$$

Given the existence of N_0, then by continuity there will also exist a slightly higher N_1; such that N_1 is equilibrium-dominated for the low type, in the sense defined in Cho and Kreps (1987), but where N_1 is not equilibrium-dominated for the high type. Specifically, playing N_1 produces strictly less utility than U_L^* in Equation A.2, even under the most optimistic beliefs by investors. This does not happen with the high type because, naturally, the probability of financial distress is lower for the high type at N_1: the probability of financial distress for the low type is strictly positive at N_1, and it is either zero for the high type, or positive but smaller:

$$\frac{1}{2\sigma}\left[1 + \sigma - \left(\frac{N_1 - D}{N_1 - Q_H}\right)\right] < \frac{1}{2\sigma}\left[1 + \sigma - \left(\frac{N_1 - D}{N_1 - Q_L}\right)\right].$$

Therefore, under the intuitive criterion, investors should assign a probability of 1 to the high type when observing a deviation to N_1, which, in turn, would make such deviation profitable for the high type. Therefore, there cannot be any equilibrium that verifies the intuitive criterion and has all players choose such N^* with positive probability. This concludes showing that pooling equilibria with project financing do not verify the intuitive criterion. Consider now a pooling equilibrium where the project is not financed. Then it is straightforward that the same arguments for high type deviation apply (now there is an added incentive to deviate, namely undertaking the project), and such equilibrium would not verify the intuitive criterion.

Next we turn to perfectly-separating equilibria. First note that in such equilibria it cannot be the case that $N_L \notin [\underline{N}_L, \overline{N}_L]$; the low type is not obtaining funding, so the best response is to pick an efficient hedging strategy and turn off financial-distress costs. To establish existence of separating equilibria, it is enough to show that (i) there always exists $N_H^{'}$, i.e., a level of of hedging by the high type that makes non mimicking by the low type incentive-compatible, and (ii) that $N_H^{'}$ verifies a "participation constraint" for the high type, in the sense that it prefers $N_H^{'}$ to efficient hedging without the project (which yields the "reservation" payoff Q_H). The condition for the existence of an $N_H^{'}$ that verifies condition (i) is that $\Delta > 0$, which, using Equation 12 and simplifying yields

$$C > \frac{2Q_L(\alpha Q_H - 1)}{Q_H}.$$

The above threshold clearly is below the threshold for C from assumption 3, so (i) holds. To prove that (ii) also holds, note that for any $N_L > Q_H$ (as is the case of $N_H^{'}$), the financial distress of the low type is higher than that of the high type. Since at $N_H^{'}$ the joint value of project plus financial distress is zero for the low type, then it mus be strictly positive for the high type. Therefore, the payoff at $N_H^{'}$ for the high type (with the project) needs to be strictly greater than Q_H.

Finally, the proposition rules out any separating equilibria that are unnecessarily costly from the perspective of the high type, for instance playing $N_H > N_H^{'}$, which follows from applying the intuitive criterion. For example, consider an equilibrium where indeed $N_H^* > N_H^{'}$. Then a deviation to $N_H^{'}$ is equilibrium-dominated for the low type, but, obviously, not for the high type; which means that investors need to assign a probability of 1 to it being the high type deviating from equilibrium play. Since $N_H^{'}$ carries fewer financial-distress costs than the conjectured equilibrium play, then the high type would then prefer to deviate, meaning that the equilibrium does not verify the intuitive criterion. The reasoning and steps are similar for ruling out other unnecessarily costly separating equilibria, and thus are omitted.■

Proof of lemma 6.

Suppose not, such that $\exists$ an $N_i^* \geq Q_i$ that maximizes Equation 18. First note that by assumption when $N_i = Q_i$ the cash flows are such that $CF_i > D$, so $\Pr\{CF_i < D | N_i = Q_i\} = 0$. This implies there exists some region of N_i above and below Q_i, such that $CF_i = D$ for the maximum price if $N > Q_i$ and $CF_i = D$ for the minimum price if $N < Q_i$. This means that for either of these two levels of N_i, $\Pr\{CF_i < D | N_i\} = 0$. Denoting the maximum N_i such that $\Pr\{CF_i < D | N_i\} = 0$ as $\overline{N}_i$. At any $N_i > \overline{N}_i$ the firm will experience positive distress costs and increased hedging costs, meaning that this cannot be an optimum. Therefore, it must be such that $N_i^* < \overline{N}_i$. Additionally, since at $N_i = Q_i$ it is such that $\Pr\{CF_i < D\} = 0$, it will never be optimal to hedge more than Q_i since expected distress costs are still 0 at $N_i = Q_i$ and any higher N_i will just induce higher hedging costs. Finally, at $N_i = Q_i$, by assumption the expected cash flows are such that $Q_i(1 - \eta) > D$, meaning that the firm can slightly decrease the hedging level by $\epsilon \to 0$ and still have $\Pr\{CF_i < D | N_i = Q_i - \epsilon\} = 0$. Any ϵ decrease in N_i will result in the firm having the same distress costs (equal to 0) but will reduce its hedging costs. Therefore, the expected cash flows are higher for $N_i = Q_i - \epsilon$ than for $N_i = Q_i$ meaning $N_i^* \geq Q_i$ cannot be optimal, a contradiction.■

Proof of lemma 7.

Since we want the absolute minimum we can focus on under-hedging.

First, note that if $D < (1-\sigma)Q_i$ then the absolute minimum number of contracts the firm can hedge and avoid distress costs is $N_i = 0$, since with no risk management they can still avoid distress costs even under the worst price scenario.

Examining the case when $\sigma > \eta$, we can see from Equation 17b that $\Pr\{CF_i < D|N_i\} = 0$ if $\frac{D-N_i(1-\eta)}{Q_i-N_i} \leq 1-\sigma$. This occurs whenever $N_i \geq \frac{D-(1-\sigma)Q_i}{\sigma-\eta}$. Thus the minimum N_i is $\frac{D-(1-\sigma)Q_i}{\sigma-\eta}$. Note that this could be negative if $D-(1-\sigma)Q_i < 0$, but, as noted above, when $D < (1-\sigma)Q_i$ the absolute minimum for the firm to hedge and avoid distress costs is $N_i = 0$. Therefore, the absolute minimum is $\max\left\{0, \frac{D-(1-\sigma)Q_i}{\sigma-\eta}\right\}$.

If $\eta > \sigma$, then this implies that $Q_i(1-\sigma) > Q_i(1-\eta)$, and by assumption 4, $Q_i(1-\eta) > D$. Thus, this also implies $(1-\sigma)Q_i > D$, meaning the absolute minimum that avoids distress cost is $N_i = 0$, as noted above. Therefore, when $\eta > \sigma$, $N_i = 0$ is always the absolute minimum.■

Proof of lemma 8.

Denote that the first-best level of hedging as N_i^*. First, note that by lemma 6, the first-best level of hedging must be $N_i^* < Q_i$. Second, it must also be such that the first-best level of hedging $N_i^* \leq \underline{N}_i$. To see this, note that for any N_i such that $Q_i > N_i > \underline{N}_i$, the firm has zero expected distress costs. However, for an $N_i > \underline{N}_i$, the firm will have higher hedging costs but the same expected distress costs as when $N_i = \underline{N}_i$, meaning it is always better off choosing $\underline{N}_i$. Therefore, the first-best level of hedging must be $N_i^* \in [0, \underline{N}_i]$.

Note that if $\eta > \sigma$, then this set is a singleton since $\underline{N}_i = 0$. Therefore, if $\eta > \sigma$ then $N_i^* = 0$. Assuming that $\sigma > \eta$, then this is a closed and bounded set. Further, note that CF_i is a continuous function over this set, so that by the extreme value theorem a maximum must exist.

We will now show that the first-best level of hedging is not in the interior of $(0, \underline{N}_i)$. First, note that CF_i is a smooth function over this set. Thus, the second order condition is a necessary condition for a local maximum. Further, note that in this set there is always some expected distress costs. Taking the second derivative of CF_i with respect to N_i and simplifying yields

$$\frac{d^2 CF_i}{dN_i^2} = \frac{C[D - Q_i(1-\eta)]}{\sigma(N_i - Q_i)^3}. \tag{A.5}$$

Noting that $N_i < Q_i$ on this set, then the second order condition for an interior maximum only holds if $D > Q_i(1-\eta)$ which violates assumption 4, that full-hedging can remove distress costs. Thus, a local maximum cannot exist on the interior of this set.

Therefore, the maximum, must be a corner solution, $N_i^* \in \{0, \underline{N}_i\}$. The expected cash flow from hedging $N_i = \underline{N}_i$, is $\underline{N}_i\eta$; and, the expected cash flow from hedging $N_i = 0$, is $\frac{C}{2\sigma}\left[\max\left\{1-\sigma, \frac{D}{Q_i}\right\} - (1-\sigma)\right]$. The firm's first-best is $N_i \in \{0, \underline{N}_i\}$ that maximizes these expected cash flows. ■

Proof of Proposition 2.

The high type's cash flows with full-hedging $N_H = Q_H$ and option strategy O_H with strike price K are

$$Q_H - O_H \frac{[K-(1-\sigma)]^2}{4\sigma} + \max\{0, K-p\}O_H - \mathbb{1}_{\{CF_H < D\}}C. \tag{A.6}$$

Choose O_H such that $CF_H > D$ for all $p \in [1-\sigma, 1+\sigma]$. This can be found by setting O_H such that the total cost of the option is $Q_H - D$ and rearranging to solve for O_H. Denote this $\hat{O}_H$

$$\hat{O}_H = \frac{(Q_H - D)4\sigma}{[K-(1-\sigma)]^2}. \tag{A.7}$$

Note that the expected benefit from the low type mimicking the high type is $\alpha Q_L \frac{\alpha Q_H - 1}{\alpha Q_H}$, where s was calculated if investors believe the firm to be a high type with probability 1. We can see that if $Pr(CF_L < D)C > \alpha Q_L \frac{\alpha Q_H - 1}{\alpha Q_H}$, then the low type will not have an incentive to mimic. Using Assumption 3, C is at least $2(\alpha Q_H - 1)$. In return, if $Pr(CF_L < D) \geq 1/2$, then low types will not mimic.

Therefore, to show that the low type will not mimic, we just need to show that there exists a $K \in [1-\sigma, 1+\sigma]$ such that the expected probability of distress for the low type from mimicking is greater than 1/2. Substituting N_H and $\hat{O}_H$ into the low type's cash flows and simplifying, yields the probability of the low type experiencing distress from mimicking $N_L = Q_H$ and $O_L = \hat{O}_H$ for some K

$$Pr\left((Q_H - Q_L)p > \max\{0, K-p\}\frac{(Q_H - D)4\sigma}{[K-(1-\sigma)]^2}\right). \tag{A.8}$$

Note that the above always holds for $p > K$ because $Q_H > Q_L$. Therefore, if $K \leq 1$, then the low type's probability of distress will be at least 1/2. Therefore we can always find a $K < 1$ such that the above probability is greater than 1/2, meaning the low type does not have an incentive to mimic (its IC is satisfied), and the high type does not have any positive probability of distress. Thus, this yields the first-best outcome. ■

Proof of lemma 9.

Given assumptions 5 and 6, it is true that

$$1+\sigma > \frac{1}{\alpha Q_L} > 1-\sigma.$$

Therefore, the probability that the growth option G_L is in-the-money is given by

$$\Pr\{\alpha Q_L p > 1\} = \Pr\left\{p > \frac{1}{\alpha Q_L}\right\} = \frac{1}{2\sigma}\left(1 + \sigma - \frac{1}{\alpha Q_L}\right). \tag{A.9}$$

Conditional on being in-the-money, the growth option has an expected value of

$$\alpha Q_L \mathrm{E}\left[p \middle| p > \frac{1}{\alpha Q_L}\right] - 1 = \alpha Q_L \frac{1}{2}\left(\frac{1}{\alpha Q_L} + 1 + \sigma\right) - 1. \tag{A.10}$$

Combining Equations A.9 and A.10 we obtain Equation 26.

Formally, to show that project value $\overline{G}_L$ increases with volatility, we need to prove that

$$\frac{\partial}{\partial \sigma}\left(\frac{[\alpha(1+\sigma)Q_L - 1]^2}{4\alpha\sigma Q_L}\right) \geq 0.$$

After a few steps of algebra, the above simplifies to

$$\sigma^2 \geq 1 + \frac{1 - 2\alpha Q_L}{(\alpha Q_L)^2}.$$

It is then enough to show that the above condition is verified for the lower bound of σ from assumption 6:

$$\frac{(1-\alpha Q_L)^2}{(\alpha Q_L)^2} \geq 1 + \frac{1 - 2\alpha Q_L}{(\alpha Q_L)^2} \Leftrightarrow (1-\alpha Q_L)^2 \geq (\alpha Q_L)^2 + 1 - 2\alpha Q_L,$$

which holds with equality.

Next, to show that $\overline{G}_L$ is smaller than with equity financing, we need to prove that

$$\frac{[\alpha(1+\sigma)Q_L - 1]^2}{4\alpha\sigma Q_L} \leq \left(1 - \frac{1}{\alpha Q_H}\right)\alpha Q_L.$$

Since $\overline{G}_L$ increases in σ, it is enough to prove the above for the upper bound of σ from assumption 5:

$$\frac{[\alpha\left(1+1-\frac{1}{\alpha Q_H}\right)Q_L-1]^2}{4\alpha\left(1-\frac{1}{\alpha Q_H}\right)Q_L} \leq \left(1-\frac{1}{\alpha Q_H}\right)\alpha Q_L.$$

After a few steps of algebra, the above simplifies to $Q_H \geq Q_L$.■

Proof of proposition 3.

To obtain Δ_d we follow the same steps as in lemma 3, where condition in Equation A.1 becomes

$$Q_L = Q_L - \frac{1}{2\sigma}\left(1+\sigma-\frac{N_H^{'}-D}{N_H^{'}-Q_L}\right)C + \frac{[\alpha(1+\sigma)Q_L-1]^2}{4\alpha\sigma Q_L}.$$

To see that $\Delta_d \leq \Delta$ simply note that the returns to mimicking are lower, since the value of the project is lower (lemma 9).■